Project Proposal Report

Tentative project title: Quantifying Symmetry using Autocorrelation
Type: Research
Team member(s):

Name	Level
Ben Shaw	Graduate

Project summary:

In this project, we will seek to quantify various symmetries of a dataset using autocorrelation. For example, if a dataset is generated with all points lying, approximately, on the 2-dimensional surface of a sphere of radius r centered at the origin, the data should exhibit an approximate rotational symmetry about any line through the origin. To quantify rotational symmetry, a cluster of points in three dimensions (centered at the origin) can be rotated about a particular line with rotation parameter t, so that each point is "rotated onto" a 2-dimensional plane through the origin containing the original line. The result is a dataset which is sequential (in t) and otherwise 2-dimensional. For each of the two dimensions, autocorrelation can be used to quantify the extent to which the data is stationary: if the data can be said to be stationary in each dimension, the original 3-dimensional dataset can be said to exhibit a rotational symmetry about the given line. It is proposed that programs are written to test for rotational symmetries, translational symmetries, and perhaps any symmetries induced by a user-defined function.

These programs would be tested on benchmark synthetic datasets, such as the s-curve dataset, the swiss roll dataset, a spherical dataset, and other synthetically-generated datasets. Time-permitting, the symmetries found in these synthetic datasets would be used to explicitly reduce the dimension of the original dataset and assign a Riemannian metric to the dataset, thereby giving an explicit manifold representation of the data (in lower dimensions).

References:

The following papers are similar in nature to the desired scope of the proposed project, though differ in key ways. The full references will be made a part of the final project write-up.

"Machine Learning on a Manifold," available at https://doi.org/10.48550/arXiv.2112.07673
"Detecting Symmetries with Neural Networks," available at https://doi.org/10.48550/arXiv.2003.13679
"Symmetry Discovery with Deep Learning," Desai et al. (NeurIPS 2021).
"Partial and Approximate Symmetry Detection for 3D Geometry," Mitra et al. (2006)
"Reflection and Rotation Symmetry Detection via Equivariant Learning," Seo et al (CVPR 2022).

Availability of dataset/code:

The code that will be written will be tested on synthetic datasets, many of which are available in libraries in Python: for instance, the s-curve dataset and the swiss roll dataset. Other synthetic datasets will be easily generated manually. The project will also make use of the autocorrelation function, which is believed to be available in a Python library.