Applications of Shape Data Analysis to Time Series Data and Manifold Learning

Ben Shaw

The Data

- A set of distinct 2D points corresponding to points on an object (the boundary, usually).
- Alternatively: a continuous curve in the plane.
- The shape of an object is the geometric information, invariant under translations, rotations, scaling.
- Example: the circle. The geometric shape doesn’t change under said transformations.

Similarity of Shape data

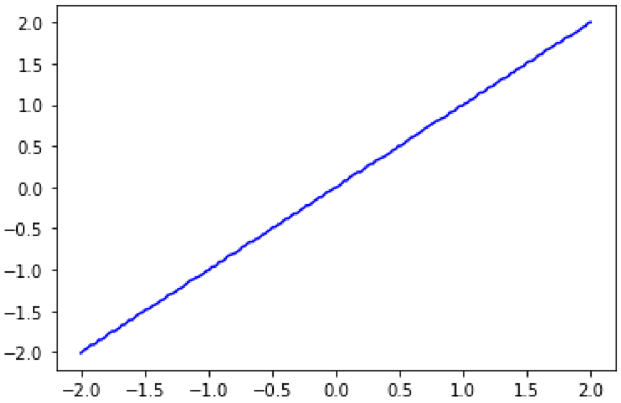
- In one paper (Bharath and Kurtek, 2019), two methods are outlined:
  - Landmark-based approach, or picking landmark points and comparing distances;
  - Curve-based approach, or treating the shape as a continuous curve (discretized in implementation)
- The curve-based approach may be messier, but picking landmarks requires domain expertise.
- Curve similarity: distances in “shape space.”

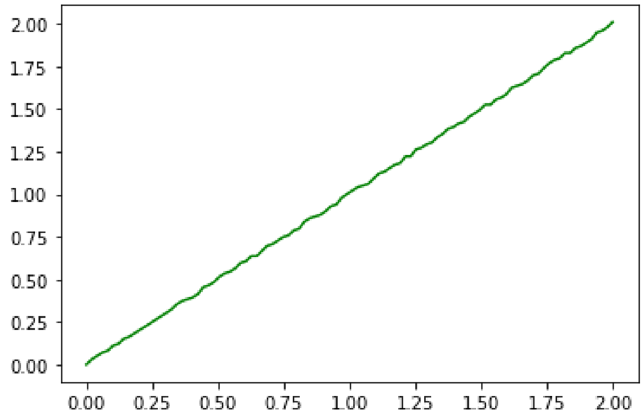
Application to Time series

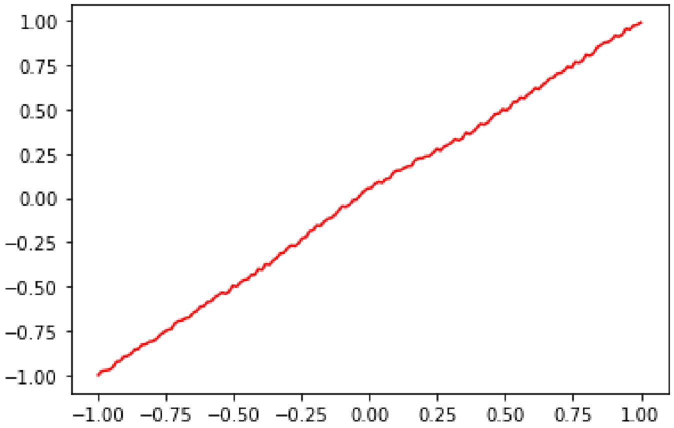
- Shapes can be converted to time series, as we have seen in class.
- Time series can be converted to shapes.
- In this project, we convert time series to shapes and compare differences using tools from shape analysis.

The Time series

Which time series is the green time series more similar to: Blue or Red? Different lengths.



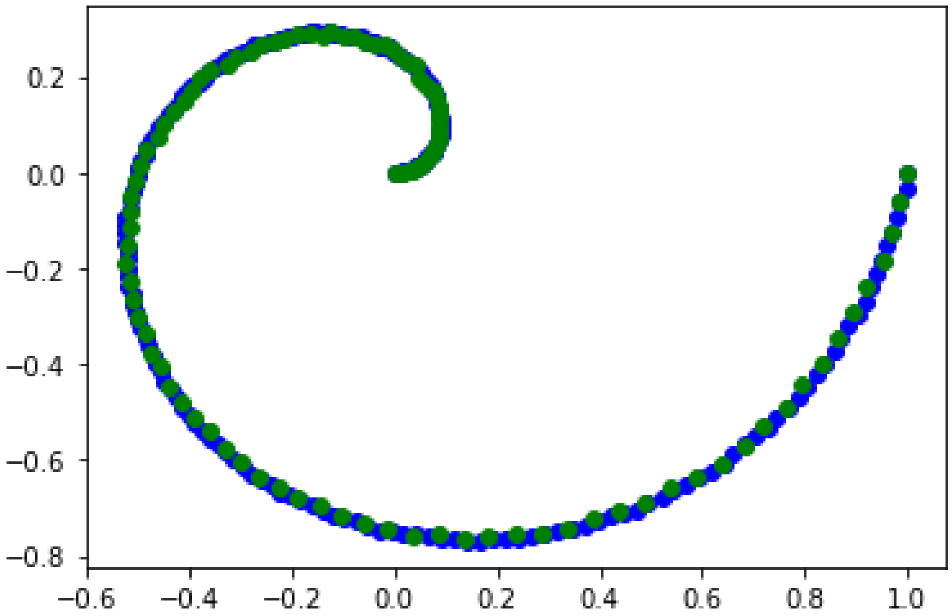


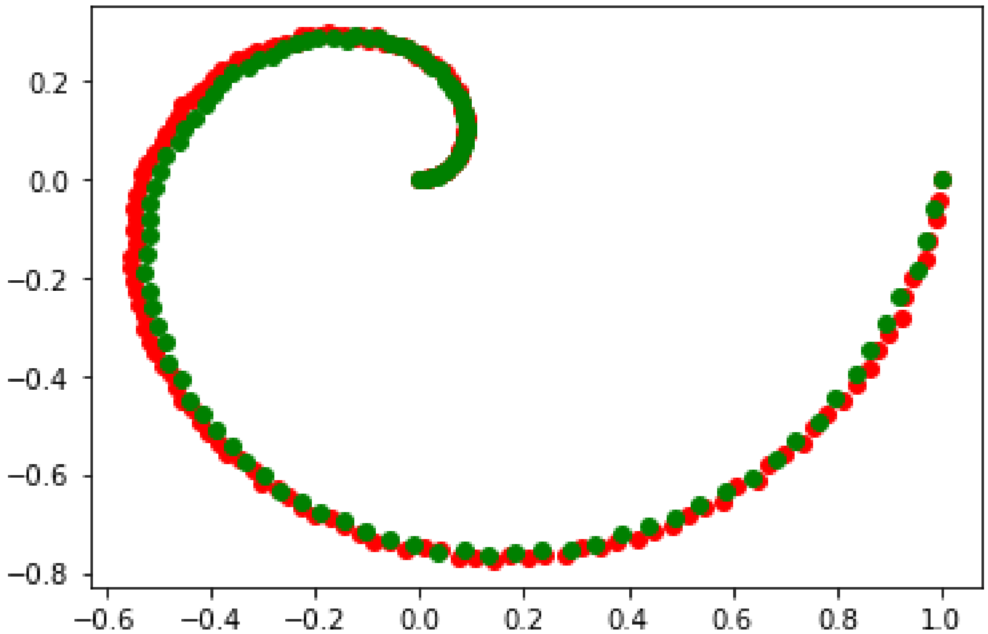


Getting the shape

- The temporal component encodes information about the angle theta: transform t to theta.
- The remaining component of the ts data encodes information about the distance from the origin: transform y(t) to r.
- Final transformation: u=r*cos(theta), v=r*sin(theta), a “Cartesian graph.”

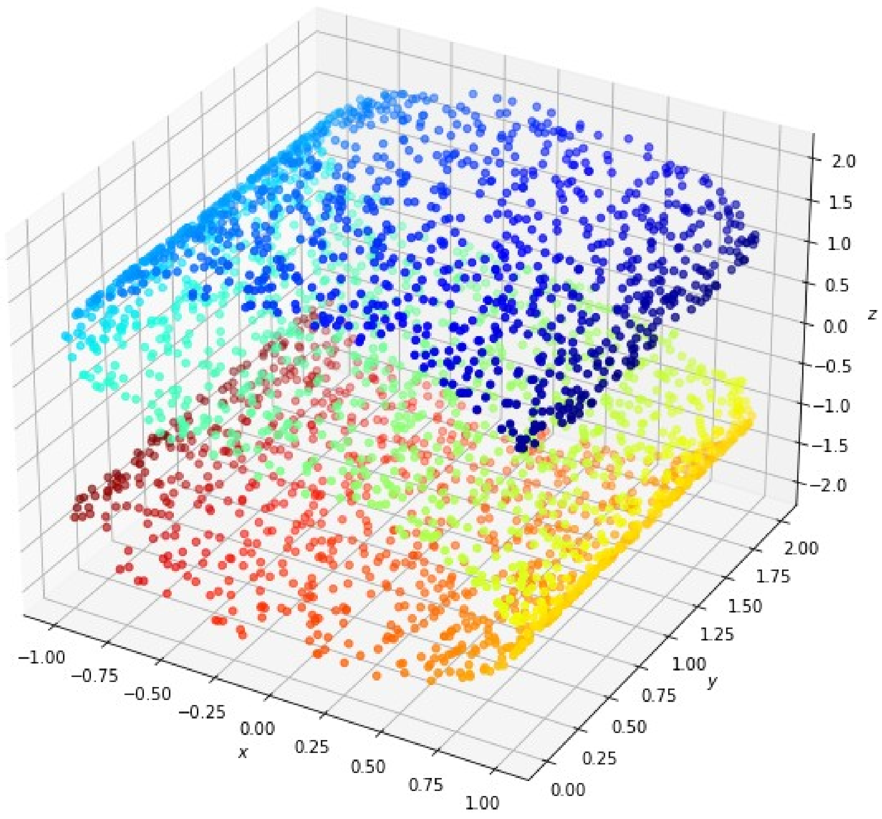
The Shape Data Representative

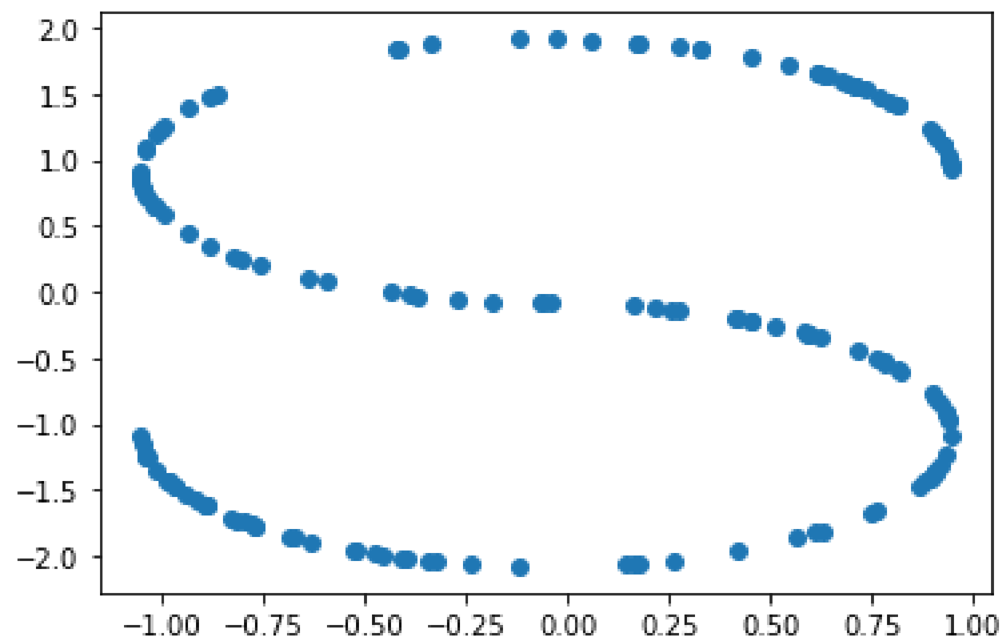




Visually, we see that the green curve is more similar to the blue curve than the red curve. A suitable distance metric in shape space will quantify this. One idea is theta-based euclidean comparison.

Application to Symmetry detection





Symmetry -> Manifold Learning

- The s-curve dataset is (approximately) generated by translation of a 2D s-curve. The data may thus be represented as a point in the translated direction and a point on the 2D s-curve (dimensionality reduction)
- Useful geometric information can be approximated, such as the “Riemannian Metric”
- Other symmetries are useful (i.e. rotational).

Thank you

- Questions?