FREQUENCY RESPONSE OF AMPLIFIERS

We have so far concentrated on the performance of the amplifier in the mid frequency region. Although the coupling and by pass capacitors used behave like short circuits in the mid frequency range, at low frequencies their impedances are large enough to reduce the gain appreciably. The device capacitances are in the pico Farads range and approximate open circuits in the mid frequency region. However, at high frequencies their impedances become small, causing the gain to drop. Accordingly, there are three distinct frequency regions.

In the low-frequency region the micro Farad coupling and bypass capacitors must be included in the incremental models. In the mid frequency region the model is resistive, with the micro Farad capacitance treated as shorts and the pico Farad capacitances as opens. At high frequencies we must consider the effects of the device capacitance while the micro Farad capacitances are shorts.

Poles and Zeros

For a linear network with no independent sources other than that at the input, the gain A is the ratio of the output voltage or current to an input voltage or current. The complex frequency of the signal is s, and each passive element can be replaced with the impedance R, sL, $\frac{1}{sC}$. Analysis can be made by means of a set of node or loop equations expressed in terms of s. the gain can be obtained from these as a ratio of determinants and each determinant can be expanded into a polynomial. The result has the form

$$A(s) = \frac{a_0 + a_1 s + a_2 s^2 + \cdots}{b_0 + b_1 s + b_2 s^2 + \cdots} \dots\dots\dots\dots\dots\dots\dots\dots\dots.. (1)$$

The real coefficients a and b are functions of the RLC circuit constants and the constants associated with any dependent sources. By factoring the polynomials the gain A becomes

$$A(S) = \frac{K(s - z_1)(s - z_2)(s - z_3)\dots}{(s - p_1)(s - p_2)(s - p_3)\dots.} \dots\dots\dots\dots\dots\dots.. (2)$$

Each singularity z_1, z_2, p_1, p_2, etc can be real or complex. However, because the a and b coefficients are real, a complex root must be one of a pair of complex conjugates. For example, if the polynomial is a cubic, one root certainly is real and the other two are either real or a pair of complex conjugates. For a source with a complex frequency s equal to z_1, the gain A of equation (2) equals zero. Hence z_1, z_2, z_3, …. etc are called the zeros of the gain function. On the other hand, if s equals p_1 or one of the other p- singularities of the denominator, the gain is infinite, and these are constants are referred to as the poles of A.

Frequency Response Curves

Gain versus frequency curves are often used to study and compare the performance of amplifiers. Such plots are drawn with a **logarithmic frequency scale** in order to expand the frequency range. When the gain is plotted in **decibels (dB)**, we can employ straight-line approximations to obtain the performance characteristics; the resultant curves are known as **Bode plots**. To illustrate the Bode method, consider a transfer function written as

$$A = \frac{K(jf/f_1)(1+jf/f_2)}{(jf/f_3)(1+jf/f_4)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

Response curves are often plotted in decibels. A decibel of voltage gain is defined as

$$dB = 20\log_{10}\left|\frac{V_0}{V_1}\right| \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(4)$$

When equation (3) is transformed to decibel of gain magnitude, we have

$$|A| = 20logK + 20log\frac{f}{f_1} + 10log\left[1+(f/f_2)^2\right] - 20log\frac{f}{f_3} - 10log\left[1+(f/f_4)^2\right]$$

Each of the terms of this expression may be plotted against frequency and the total gain obtained as the sum of the dB ordinates of the several curves. Thus for:

Constants: the constant gain K is frequency-invariant, so

$dB = 20logK$ plots as a horizontal straight line. When the second factor $dB = 20\log\frac{f}{f_1}$ is plotted against a log-frequency scale, it appears as a straight line with a slope of $+20dB$ per decade or $6dB$ per octave. The curves passes through 0dB at $f = f_1$. For the factor $dB = -20logf/f_3$ the plot is similar to the one just described but with a slope of $-20dB$ per decade. The curve also passes through $0dB$ at $f = f_3$. For the factor $dB = 10log[1+(f/f_2)^2]$

When $f \ll f_2$

$10log[1+(f/f_2)^2] \approx 10\log 1 = 0$

Thus the plot at small frequencies is the 0-dB line. For $f \gg f_2$. The factor becomes

$dB = 10log(f/f_2)^2 = 20\log(f/f_2)$

The asymptotes for the plot of $10log[1+(f/f_2)^2]$ are, therefore, two straight lines, one horizontal at $0dB$ below $f/f_2 = 1$, and one rising at $+20dB$ per decade above $f/f_2 = 1$. The asymptotes intersect at $f/f_2 = 1$ and 0dB.

The asymptotes for the factor $-10log\left[1+(f/f_4)^2\right]$ plot similarly to those described above but with a slope of $-20dB$ per decade above $f/f_4 = 1$. The individual and composite plots are shown in figure 1.

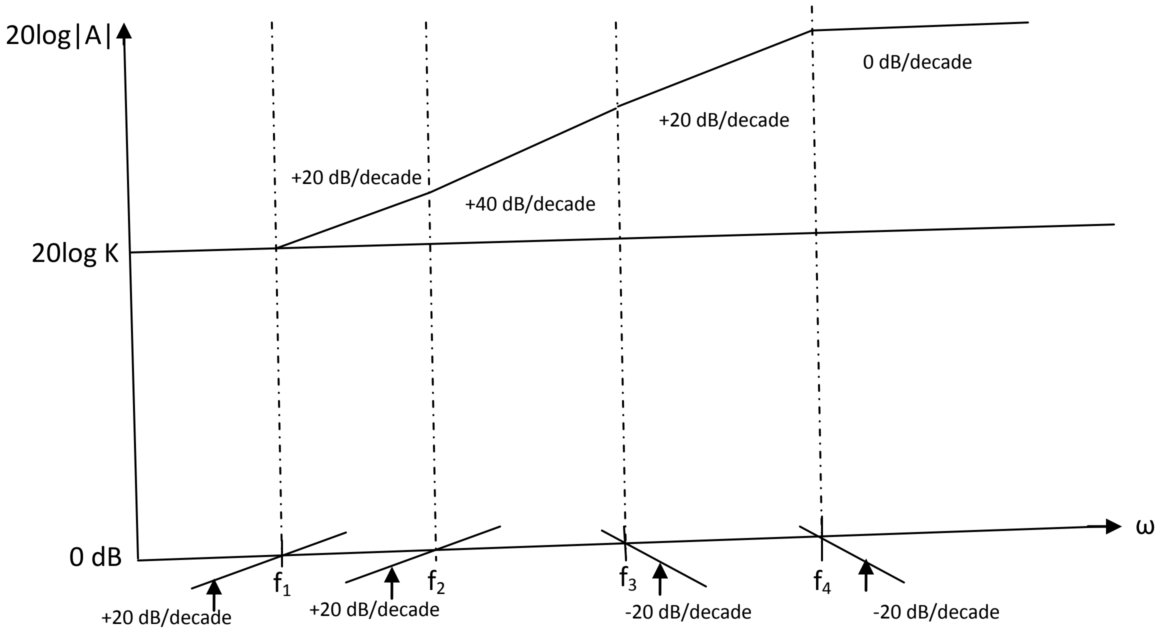

Fig. 1

Consider the function expressed as

$$A(w) = \frac{K}{1 + jf/f_1}$$

Which can be expressed in decibel form as

$$20log|A| = 20logK - 10log\left[1 + (f/f_1)^2\right]$$

At $f = f_1$

$$20log|A| = 20logK - 10log2 = 20logK - 20log2^{\frac{1}{2}}$$

$$\therefore A = \frac{K}{\sqrt{2}} = 0.707K$$

For a resistive load the output power is proportional to the square of the voltage. Therefore, with a constant input, the power to the load at this frequency is one-half that in the mid band range. Thus the break frequency, defined as that at which the output voltage drops to $\frac{1}{\sqrt{2}}$ times its mid band value, is also called the 0.707 frequency, the $3dB$ frequency, the half-power point, the corner frequency, or the band edge.

PHASE PLOTS

A graph for the phase-angle variation of the amplifier may be readily constructed from the gain plot. We note that for a function:

1. $F_1 = K,$ $\theta = 0° \; or \; 180°$
2. $F_2 = jf/f_1$ $\theta = +90°$
3. $F_3 = -jf/f_3$ $\theta = -90°$
4. $F_4 = 1 + jf/f_2$ $\theta = tan^{-1} f/f_2$
5. $F_5 = \frac{1}{1+jf/f_4}$ $\theta = -tan^{-1} f/f_4$

In (4) for $f/f_2 << 1, \theta \approx 0°$ and for $f/f_2 = 1, \theta = 45°$

While for $f/f_2 \gg 1, \theta = +90°$

In (5) for $/f_4 << 1, \theta \approx 0°$, for $f/f_4 = 1, \theta = -45°$

While for $f/f_4 \gg 1, \theta = -90°$

For any frequency f_a, between $f/f_a = 0.1$ and $f/f_a = 10$, the phase angle can be approximated by a straight line to an accuracy of about 5°.

Home Assignment

For the function given in equation (5) do the magnitude and phase plots.

$$T(s) = \frac{100(s+10)}{(s+100)(s+500)(s+1000)} \; . \; . \; . \quad (5)$$

Amplifier Models at Various Frequency Ranges

Consider the single stage amplifier circuit depicted in figure 2.

The complete increment model of the amplifier that is valid at all frequency ranges is shown in figure 3. At the low frequency range, the device capacitances are open circuits while the coupling and bypass capacitors have enough reactances to influence circuit behaviour. The model valid at low frequency is therefore as shown in figure 4.

At the mid frequency range the coupling and bypass capacitors have approximately zero reactances i.e. they are short circuits while the device capacitances are still open circuits. Thus the mid frequency model is as shown in figure 5.

At the high frequency range the coupling and bypass capacitors are still short circuits while the device capacitances are now considered. The high frequency model is shown in figure 6.

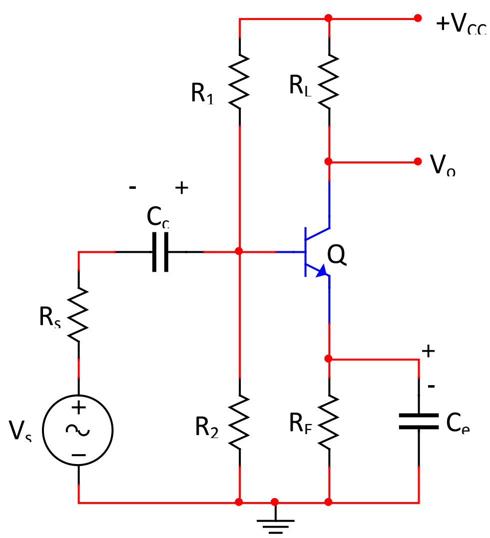

Fig. 2

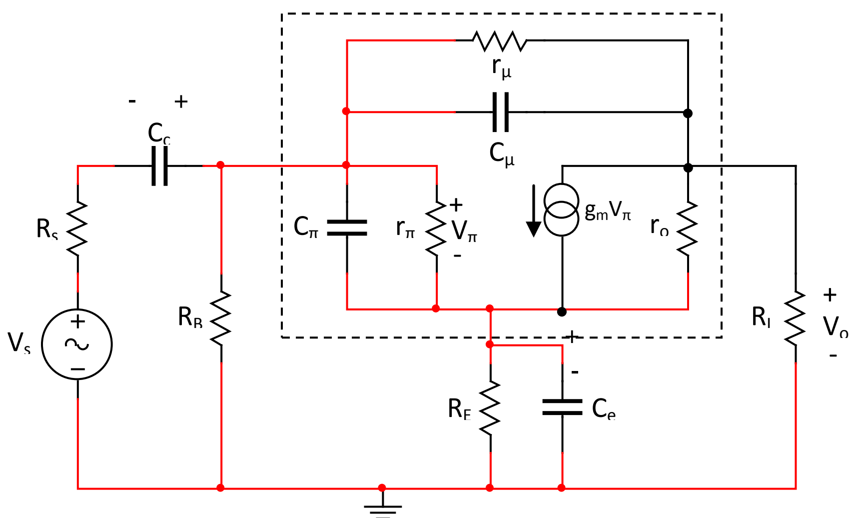

Fig. 3

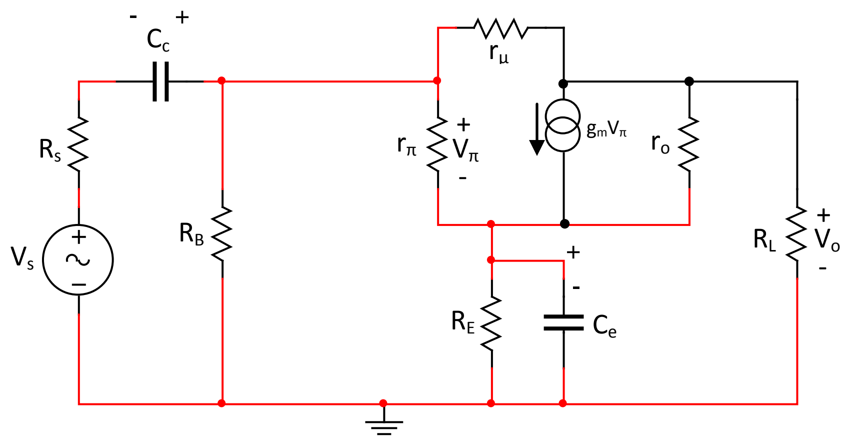

Fig. 4

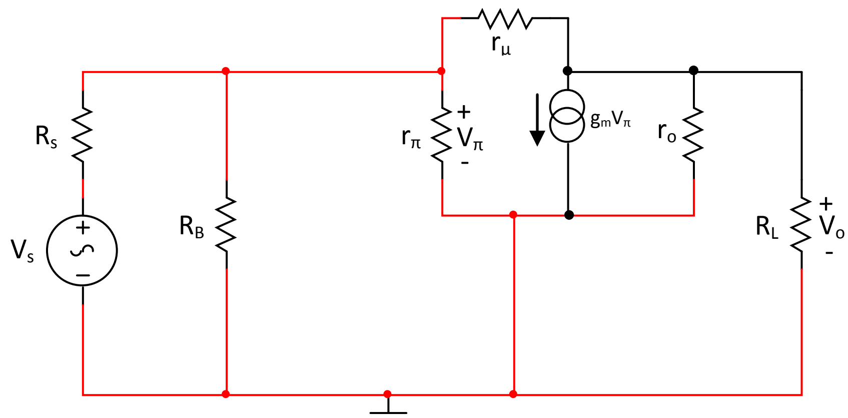

Fig. 5

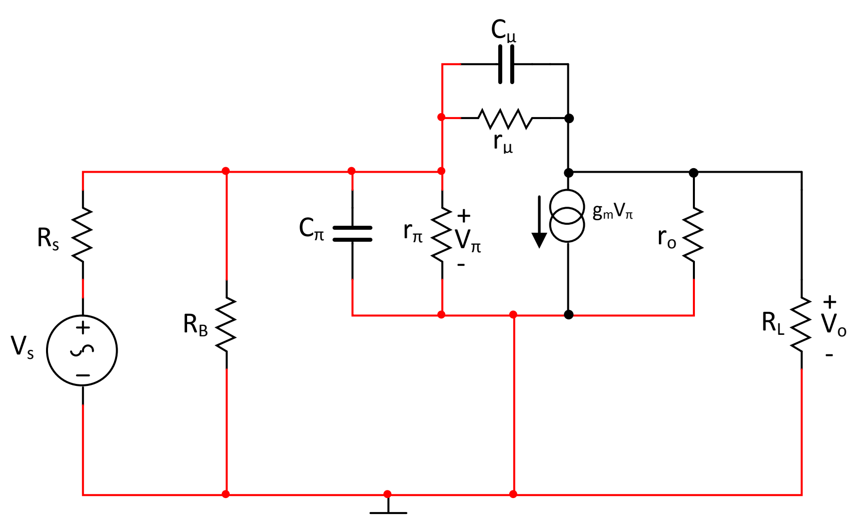

Fig. 6

POLES AND TIME CONSTANTS

There are important relations between the poles of a gain function and the time constants of the network. Suppose the amplifier model has three capacitors C_1, C_2, and C_3. With the source at the input replaced with its internal impedance, let R_{1s} denote the effective resistance in parallel with C_1 when C_2 and C_3 are replaced with short circuits. Symbols R_{2s} and R_{3s} denote the corresponding resistances associated with C_2 and C_3. The short-circuit time constants are defined as

$$\tau_{1s} = R_{1s}C_1, \qquad \tau_{2s} = R_{2s}C_2, \qquad \tau_{3s} = R_{3s}C_3 \; . \; . \; . \; (6)$$

The sum of the reciprocals of these is

$$\sum_{j=i}^{3} \frac{1}{\tau_{js}} = \frac{1}{\tau_{1s}} + \frac{1}{\tau_{2s}} + \frac{1}{\tau_{3s}} \quad . \; . \; . \; (7)$$

A relation quite useful in the design and analysis of low-frequency amplifier circuits is

$$-(p_1 + p_2 + \cdots) = \sum_j \frac{1}{\tau_{js}} \dots \dots \dots \dots \dots (8)$$

This mathematical statement says that the negative of the sum of the poles equals the sum of the reciprocals of the short circuit time constants. It applies to a network with any number of capacitors.

For the network with three capacitors the open-circuit time constants are

$$\tau_{1o} = R_{1o}C_1, \qquad \tau_{2o} = R_{2o}C_2, \qquad \tau_{3o} = R_{3o}C_3 \quad \dots\dots\dots (9)$$

Here, R_{1o} denotes the effective resistance in parallel with C_1 when the other capacitors are removed, or open-circuited, and R_{2o} and R_{3o} are similarly defined.

The following relation is useful in the design and analysis of high-frequency amplifier circuits:

$$-\left(\frac{1}{p_1} + \frac{1}{p_2} + \cdots\right) = \sum_j \tau_{jo} \dots \dots \dots \dots \dots \dots \dots \dots (10)$$

Equation (10) states that the negative of the sum of the reciprocals of the poles equals the sum of the open-circuit time constants.

APPLICATION OF TIME CONSTANTS IN DESIGN

Consider the low-frequency design of the single stage CE amplifier shown in figure 7 in which both the coupling and bypass capacitors are present. The incremental model of the circuit at low frequency is shown in figure 8. Note that we have assumed that both $r_\mu and\ r_o \to \infty$ i.e. open circuit.

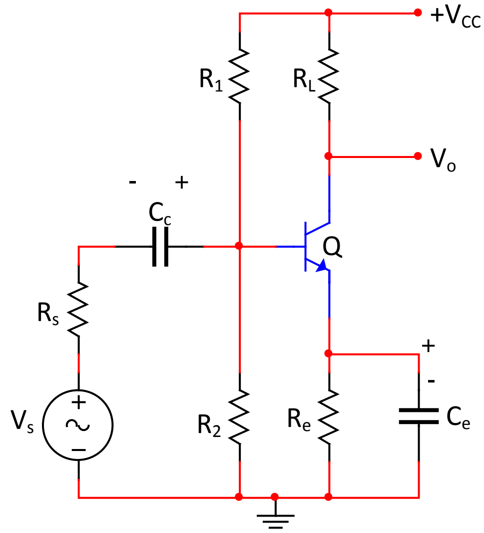

Fig. 7

R_s = 5k, $R_B = R_1//R_2$ = 10k,
r_π = 0.4k, g_m = 100 mMhos,
R_e = 0.4k, β_o = 40

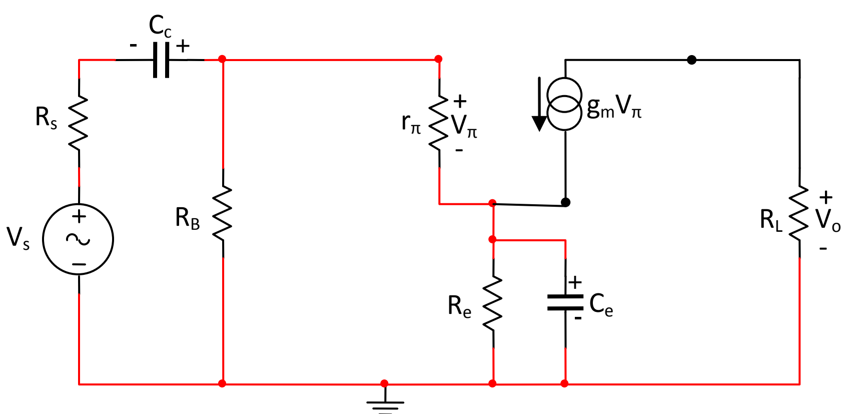

Fig. 8

It is desired to design the network so that the lower $3dB$ frequency is $30Hz$.

The circuit has two poles and two zeros. The zeros are at

$$Z_1 = 0 \text{ and } Z_2 = -\frac{1}{R_e C_e}$$

The short-circuit time constants are $R_{Cs}C_c$ and $R_{es}C_e$. To find R_{cs} we short-circuit the source and C_e. R_{Cs} is then given as

$$R_{cs} = [R_s + R_B//r_\pi] = 5.38\ \text{k}\Omega$$

To find R_{es}, we short-circuit the source and C_c giving the circuit of figure 9 from which we can write the following relations:

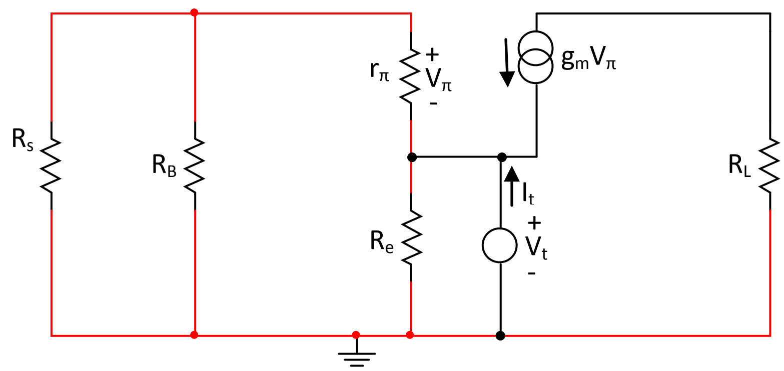

Fig. 9

$$I_t + g_m V_\pi - \frac{V_t}{R_e} + \frac{V_\pi}{r_\pi} = 0\ .\ \ .\ \ .\ (11)$$

$$Also\ \ -V_t - V_\pi - \frac{V_\pi}{r_\pi}(R_s\ //\ R_B) = 0$$

From which

$$\frac{V_\pi}{r_\pi} = \frac{-V_t}{r_\pi + R_s//R_B}\ \ .\ \ .\ \ .(12)$$

Substitute equation (12) into equation (11) and collect terms to obtain

$$\therefore R_{es} = \frac{V_t}{I_t} = \frac{1}{\dfrac{g_m r_\pi + 1}{r_\pi + R_s//R_B} + \dfrac{1}{R_e}} = 0.0758\ k\Omega$$

By using equation (8) we have

$$-(p_1 + p_2) = \frac{1}{5.38C_c} + \frac{1}{0.0758C_e} \dots\dots\dots\dots\dots\dots\dots\dots (13)$$

We wish to select suitable values for C_c and C_e to obtain a $3dB$ frequency of $30Hz$ or $w_l = 0.1885\ krad/s$. In a situation where there is a dominant pole, this one pole approximately equals the sum of the poles, hence

$$w_l = -\sum_j p_j = \sum_j \frac{1}{\tau_{js}} \dots\dots\dots\dots\dots\dots\dots..(15)$$

In our particular situation in which we do not have a dominant pole, a good design procedure is to assume that the sum of the poles is w_l or

$$w_l \approx -\sum_j p_j = \sum_j \frac{1}{\tau_{js}} \dots\dots\dots\dots\dots\dots\dots (16)$$

Using equations (13) and (16) for $w_l = 0.1885\ krads^{-1}$ we obtain

$$\frac{1}{5.38\mathrm{C_c}} + \frac{1}{0.0758\mathrm{C_e}} = 0.1885$$

A good initial trick is to pick values that make the time constants equal or nearly so.

$$\therefore let \frac{1}{5.38C_c} = \frac{1}{0.0758C_e} = \frac{0.1885}{2} = 0.09425$$

$$\therefore C_c = \frac{1}{5.48 \times 0.09425} = 1.94\mu F$$
$$and\ \mathrm{C_e} = \frac{1}{0.0758 \times 0.09425} = 140\mathrm{\mu F}$$

We observe that this initial choice produces values that are too far apart thus producing a costly circuit. It, however, gives us a good idea of the contributions of each time constants and can be used as a trial to enable us obtain closer and, hence, better values of the capacitors. To see if the design meets our objective, an analysis would have to be carried out.

HIGH FREQUENCY AMPLIFIER DESIGN

At high frequencies the device capacitances are included in the incremental model of an amplifier and they give poles and zeros that cause the gain to drop off in this frequency region. In this case we choose the open-circuit time constants such that

$$\frac{1}{w_h} \approx -\sum_j \frac{1}{p_j} = \sum_j \tau_{j0} \dots\dots\dots\dots\dots\dots\dots (17)$$

Let us now apply this design technique to multi-stage amplifiers of various types.

Two-stage RC-Coupled CE Amplifier.

A two-stage RC-coupled common-emitter amplifier is shown in figure 10. Neglect the base bias resistances so that small signal model at high frequency is as given in figure 11. To carry out the analysis we first obtain the open circuit time constants associated with each capacitor.

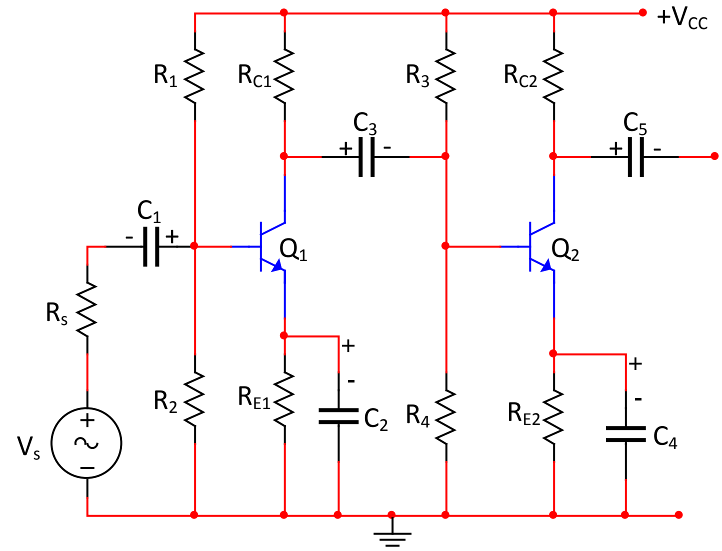

Fig. 10

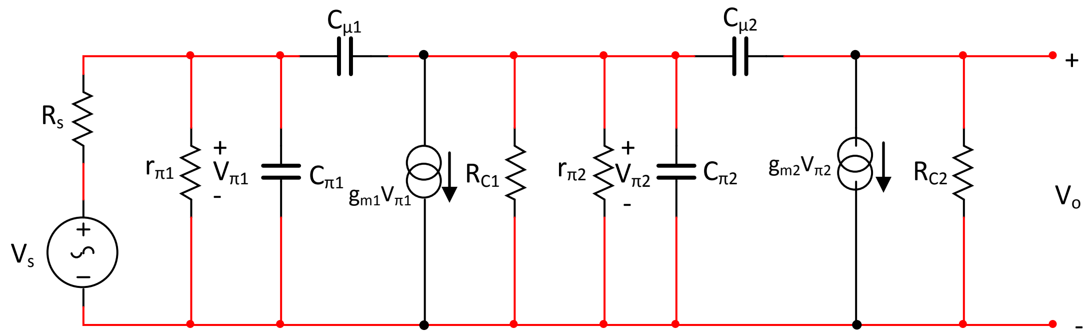

Fig. 11

Open circuit time constant associated with $C_{\pi 1}$ is obtained from figure 12.

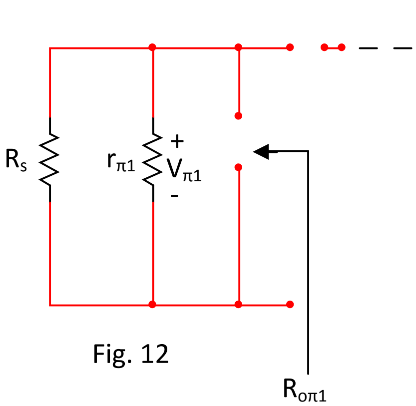

Fig. 12

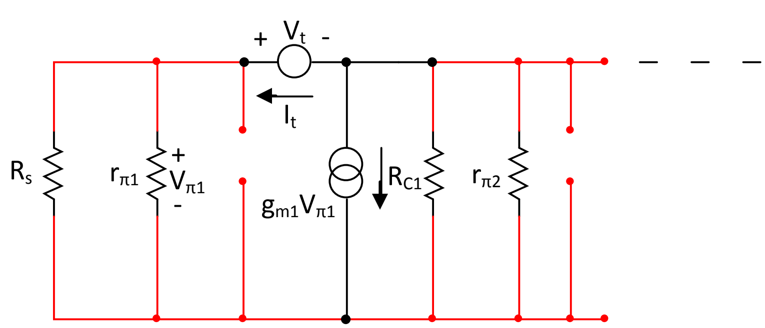

Fig. 13

From this figure:

$$R_{0\pi1} = R_s//r_{\pi1}$$

$$\tau_{10} = R_{0\pi1}C_{\pi1} = (R_s//r_{\pi1})C_{\pi1} \ldots (18)$$

Open circuit time constant associated with $C_{\mu1}$ is obtained from figure 13. Note that the expression for this time constant cannot be obtained explicitly by observation so we use a test source and find the ratio of V_t to I_t.

$$R_{0\mu1} = \frac{V_t}{I_t}$$

From the circuit,

$$-V_{\pi1} + V_t - g_{m1}V_{\pi1}(R_{C1}//r_{\pi2}) - I_t(R_{C1}//r_{\pi2}) = 0$$

$$\text{Also}\,\frac{V_{\pi1}}{r_{\pi1}} = I_t\frac{R_s}{R_s + r_{\pi1}} \text{ by current divider principle}$$

$$\therefore V_t = V_{\pi1}[1 + g_{m1}(R_{C1}//r_{\pi2})] + I_t(R_{C1}//r_{\pi2})$$

$$= I_t\frac{R_s r_{\pi1}}{R_s + r_{\pi1}}[1 + g_{m1}(R_{C1}//r_{\pi2})] + I_t(R_{C1}//r_{\pi2})$$

$$\therefore R_{0\mu1} = \frac{V_t}{I_t} = (R_s//r_{\pi1})[1 + g_{m1}(R_{C1}//r_{\pi2})] + (R_{C1}//r_{\pi2}) \ldots \quad (19)$$

$$and\ \tau_{2o} = R_{0\mu1}C_{\mu1}$$

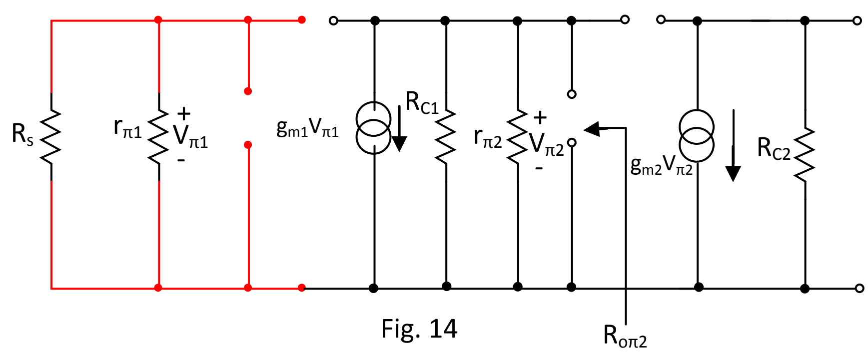

Fig. 14

Open circuit time constant associated with $C_{\pi2}$ is obtained from figure 14. We observe from this figure that

$R_{0\pi2} = R_{C1}//r_{\pi2}$ (since $V_{\pi1} = 0$)

$$\tau_{30} = R_{0\pi2}C_{\pi2} = (R_{C1}//r_{\pi2})C_{\pi2} \ldots (20)$$

Open circuit time constant associated with $C_{\mu 2}$ is obtained from figure 15 and we observe that

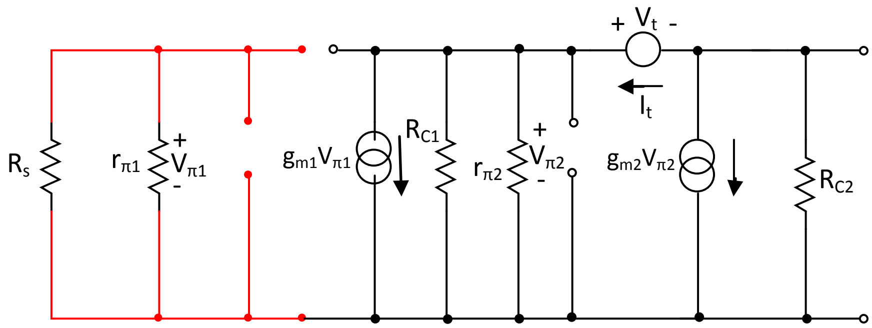

Fig. 15

$$R_{0\mu 2} = \frac{V_t}{I_t} = (R_{C1}//r_{\pi 2})[1 + g_{m2}R_{C2}] + R_{C2} \quad (21)$$

$$and\ \tau_{4o} = R_{0\mu 2}C_{\mu 2}$$

By using equation (17) we can write that

$$\frac{1}{w_h} = -\sum_j \frac{1}{p_j} = \sum_j \tau_{j0}$$

$$= (R_s//r_{\pi 1})C_{\pi 1} + [(R_s//r_{\pi 1})\{1 + g_{m1}(R_{C1}//r_{\pi 2})\} + (R_{C1}//r_{\pi 2})]C_{\mu 1} + (R_{C1}//r_{\pi 2})C_{\pi 2} + [(R_{C1}//r_{\pi 2})\{1 + g_{m2}R_{C2}\} + R_{C2}]C_{\mu 2} \quad . \ . \ . (22)$$

Equation (22) shows that by knowing the various values of the τ_{j0} we can choose components so as to realize the desired upper 3 dB frequency. Note also, however, that the values of the collector resistors are fixed by bias.

EXAMPLE

The parameter values used in the CE-CE cascade indicated in figure 10 are as follows:

$$R_s = 0.6k, R_{C1} = 1.5k, R_{C2} = 0.6k, r_{\pi 1} = 1.2k, g_{m1} = 100\text{ mMhos}, C_{\pi 1} = 24.5pF, C_{\mu 1} = 0.5pF, r_{\pi 2} = 2.4k, g_{m2} = 50\text{ mMhos}, C_{\pi 2} = 19.5pF, \text{and } C_{\mu 2} = 0.5pF$$

(a) Determine the open-circuit constants associated with each capacitance in the circuit.
(b) Determine the approximate value of f_H.

SOLUTION

$$R_s//r_{\pi 1} = 0.6//1.2 = 0.4k;\ R_{C1}//r_{\pi 2} = 1.5//2.4 = 0.923\text{ k}$$
$$R_{0\pi 1} = R_s//r_{\pi 1} = 0.4k;$$

$$R_{0\mu1} = 0.4[1 + 100 \times 0.923] + 0.923 = 38.2\text{k}$$
$$R_{0\pi2} = 0.923\ \text{k}$$
$$R_{0\mu2} = 0.923[1 + 50 \times 0.6] + 0.6 = 29.2\ \text{k}$$
$$\therefore \frac{1}{w_H} = \sum_j \tau_{jo} = 0.4 \times 24.5 + 38.2 \times 0.5 + 0.923 \times 19.5 + 29.2 \times 0.5 = 61.5\ ns$$
$$f_H = \frac{1}{2\pi \times 61.5 \times 10^{-9}} = 2.59\ MHz$$

Home Assignment

In the network shown in figure 16, the two transistors are said to be in **CASCODE** with the collector of Q_1 connected in series with the emitter of Q_2. The stage Q_1 is a common-emitter amplifier while the stage Q_2 is a common-base. If the Q_2 stage is omitted from the cascode connection with R_L directly connected to the collector of Q_1, the voltage gain of the resultant single-stage common-emitter amplifier would be the same as that of the whole cascode. **Show that this is true**. The cascode connection is, however, superior to the single CE stage because it has a higher bandwidth.

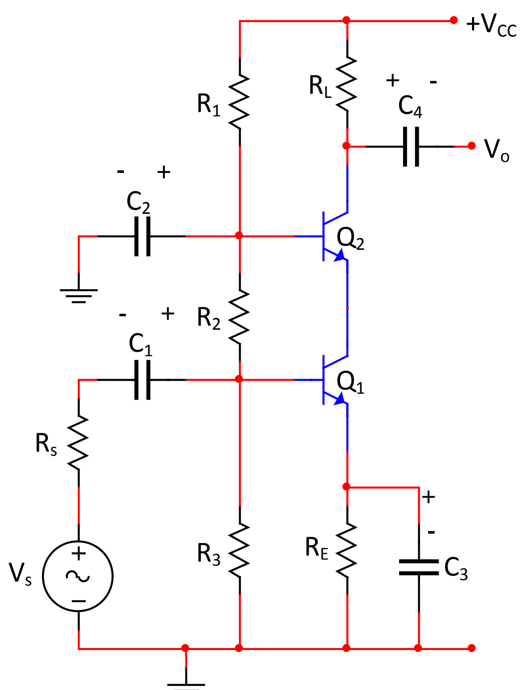

Fig. 16

The bias parameters for the cascode amplifier are given as $\mathrm{R_1 = 11k}$, $\mathrm{R_2 = 9.5k}$, $\mathrm{R_3 = 4k}$, $\mathrm{R_E = 0.2k}$, $\mathrm{R_L = 0.4k}$, $\mathrm{V_{CC} = 12V}$. In addition $\mathrm{R_s = 0.6k}$ and $\beta_0 = \beta_F = 100, g_m = 200\ \mathrm{mMhos}$, $r_\pi = 0.5\mathrm{k}$ for each transistor.

(a) The capacitors C_1, C_2 and C_3 of the cascode amplifier are to be selected to give a lower $3dB$ frequency of $100Hz$. Determine C_1, C_2 and C_3 in μF, with these capacitances selected to give identical short-circuit time constants for C_1 and C_2 and a time constant for C_3 that is 10% of each of the others.

(b) Assume that the transistors used in the cascode amplifier are identical and that the following parameters apply: $\beta_0 = 100, g_m = 200\ \mathrm{mMhos}, C_\mu = 4\rho\mathrm{F}, C_\pi = 100\rho\mathrm{F}$. Calculate the open circuit time constants in nano seconds associated with $C_{\pi1}$, $C_{\mu1}$, $C_{\pi2}$, and $C_{\mu2}$. From these values estimate the upper $3dB$ frequency of the circuit.

(c) If the Q_2 stage is omitted from the cascode connection with R_L directly connected to the collector of Q_1 and with the parameters of the transistor given in part (b), calculate the open circuit time constants in nano seconds associated with $C_{\pi1}$ and $C_{\mu1}$. Estimate the upper $3dB$ frequency of this single stage and compare the result with that of part (b).

(d) With the aid of Multisim software simulate the circuit designed in part (a). Measure the frequency response with the aid of the **Bode Plotter** that is integrated in Multisim. Obtain the Lower and upper $3dB$ points and compare the simulated results with your computed values in parts (a), (b) and (c).