ON A SEARCH FOR SINGULARITIES IN THE GENERALIZED CONSTANTIN–LAX–MAJDA EQUATION

ON A SEARCH FOR SINGULARITIES IN THE GENERALIZED CONSTANTIN–LAX–MAJDA EQUATION

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Abstract

For my term project I will compute and analyse solutions of the Generalized Constantine-Lax-Majda equation. The Constantine-Lax-Majda (CLM) equation was proposed as a one-dimensional model for the three-dimensional vorticity equation. The CLM equation was later expanded on by De Gregorio to include the convection term $u\omega_x$. The De Gregorio equation was generalized by Okamoto et al. to include the real parameter a which determines the relative strength of the convection term. The result is the Generalized Constantin-Lax-Majda equation (GCLM).

The CLM equation was proposed in 1985 and has been shown to exhibit finite time blow-up. While the De Gregorio equation was proposed shortly after in 1989, questions still remain as to whether or not the equation permits finite time blow-up. The goal of this analysis is to examine the affect the parameter a has on the finite time blow-up of the GCLM equation on the periodic domain $x \in [-\pi, \pi]$. I will explore solutions of the GCLM for values of $a \in [-1, 1]$.

The analysis found RESULTS RESULTS RESULTS. CONCLUSION CONCLUSION CONCLUSION.

Contents

1 Introduction

One of the

2 The Generalized Constantin–Lax–Majda Equation

explore this by studying simpler one-dimensional models.

2.1 The CLM Equation

The Constantin-Lax-Majda equation was proposed as a one-dimensional model of the three-dimensional vorticity equation for an incompressible fluid [1].

2.2 The De Gregorio Equation

The De Gregorio equation was proposed as a one-dimensional model that improves on the CLM equation by ... [2]

2.3 Generalization of the CLM Equation

The De Gregorio equation was later generalized to the GCLM [3].

3 Blow-up Criteria

how do we define blow up

4 Numerical Methods

what numerical methods are used

4.1 Domain and Grid

a

4.2 Time Stepping

b

4.3 Initial Conditions

c

5 Results

these are the results

6 Conclusion

this is the conclusion

References

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[3] Hisashi Okamoto, Takashi Sakajo, and Marcus Wunsch. On a generalization of the Constantin–Lax–Majda equation. *Nonlinearity*, 21(10):2447, September 2008.