P1.1

We begin by massaging this complex exponential into a different form using Euler's formula for ease of use in the questions that follow

$$\frac{1}{2}e^{\frac{j\pi}{4}} = \frac{1}{2}(cos(\frac{\pi}{4}) + jsin(\frac{\pi}{4})) = \frac{1}{2}(\frac{\sqrt{2}}{2}) + j\frac{1}{2}(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4}$$

(a)

$$\boxed{Re\{z\} = \frac{\sqrt{2}}{4}}$$

(b)

$$\boxed{Im\{z\} = \frac{\sqrt{2}}{4}}$$

(c)

$$|z| = \sqrt{(\frac{\sqrt{2}}{4})^2 + (\frac{\sqrt{2}}{4})^2} = \sqrt{\frac{2}{16} + \frac{2}{16}} = \sqrt{\frac{4}{16}} = \frac{2}{4} = \frac{1}{2}$$

$$\boxed{|z| = \frac{1}{2}}$$

As a side note on this problem, any complex exponential without a coefficient (e^{jx} for some x) has a magnitude of 1. Thus, the magnitude simply becomes the leading coefficient, which is $\frac{1}{2}$ in this case.

(d)

$$\boxed{\measuredangle z = \frac{\pi}{4}}$$

(e)

$$\boxed{z^* = \frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}}$$

(f)

$$z + z^* = (\frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4}) + (\frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}) = \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

$$\boxed{z + z^* = \frac{\sqrt{2}}{2}}$$

P1.2

(a)

$$Re\{z\} = \frac{z + z^*}{2} = \frac{(Re\{z\} + jIm\{z\}) + (Re\{z\} - jIm\{z\})}{2} = \frac{2Re\{z\}}{2} = Re\{z\}$$

(b)

$$jIm\{z\} = \frac{z - z^*}{2} = \frac{(Re\{z\} + jIm\{z\}) - (Re\{z\} - jIm\{z\})}{2} = \frac{2jIm\{z\}}{2} = jIm\{z\}$$

P1.3

(a) According to Euler's formula, $Re\{e^{j\theta}\} = cos(\theta)$. Therefore, by P1.2(a),

$$cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

For clarity you can also prove this directly like so

$$\frac{e^{j\theta} + e^{-j\theta}}{2} = \frac{(cos(\theta) + jsin(\theta)) + (cos(\theta) - jsin(\theta))}{2} = \frac{2cos(\theta)}{2} = cos(\theta)$$

(b) According to Euler's formula, $Im\{e^{j\theta}\} = sin(\theta)$. Therefore, by P1.2(b),

$$jsin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2}$$

$$sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{j2}$$

For clarity you can also prove this directly like so

$$\frac{e^{j\theta} - e^{-j\theta}}{2j} = \frac{(cos(\theta) + jsin(\theta)) - (cos(\theta) - jsin(\theta))}{2j} = \frac{2jsin(\theta)}{2j} = sin(\theta)$$

P1.4

(a)(i)

$$\boxed{z^* = re^{-j\theta}}$$

(a)(ii)

$$z^2 = (re^{j\theta})^2 = r^2 e^{j2\theta}$$

$$\boxed{z^2 = r^2 e^{j2\theta}}$$

(a)(iii)

$$jz = jre^{j\theta} = e^{\frac{\pi}{2}} re^{j\theta} = re^{j\theta + \frac{\pi}{2}}$$

$$\boxed{jz = re^{j\theta + \frac{\pi}{2}}}$$

(a)(iv)

$$zz^* = re^{j\theta} re^{-j\theta} = r^2 e^{j\theta - j\theta} = r^2 e^0 = r^2$$

$$\boxed{zz^* = r^2}$$

(a)(v)

$$\frac{z}{z^*} = z\frac{1}{z^*} = re^{j\theta} re^{-j\theta} = r^2 e^{j\theta - j\theta} = r^2 e^0 = r^2$$

$$\boxed{zz^* = r^2}$$