Section 1.1

Problem 1

a) line
b) plane
c) $\mathbb{R}^3$

Problem 3

Manipulate $\vec{v} - \vec{w} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ to be $\vec{v} = \begin{bmatrix} 1 \\ 5 \end{bmatrix} + \vec{w}$. Then subsitute in $\vec{v}$ within $\vec{v} + \vec{w} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ to yield $\begin{bmatrix} 1 \\ 5 \end{bmatrix} + \vec{w} + \vec{w} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$. Hence, $2\vec{w} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$. Substituting back in yields $\vec{v} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

Problem 5

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$$\vec{w} = -\vec{u} - \vec{v}$$

Since $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$, $\vec{w}$ reaches no part of 3-D space that a linear combination of $\vec{u}$ and $\vec{v}$ (a plane) cannot reach. Hence, the three vectors only span a plane.

Problem 9

The three possible corners arr $(4, 4), (4, 0), (-2, 2)$

Problem 11

The other four corners are $(1, 1, 0), (0, 1, 1), (1, 0, 1), (1, 1, 1)$. The center of the cube is $(0.5, 0.5, 0.5)$. The cube has 12 edges.

Problem 13

a) Since there are 6 pairs of vectors that oppose each other, that is their sum is zero, then the sum of all the vectors must be the origin $(0, 0, 0)$ (the center of the clock).

b) Since the 8:00 no longer has an opposing vector (was 2:00) and all other 5 pairs of vectors still do, 8:00 is the only vector left.

c) Since there are 12 vectors that are evenly spaced within 360 degrees, each has an angle of 30 degrees in between. Let θ be the angle between 2:00 and 3:00. As was just shown, $\theta = 30$ degrees. Hence $\vec{v} = (\cos(30), \sin(30)) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$

Section 1.2

Problem 1

$$\vec{u}\cdot\vec{v} = \begin{bmatrix}-0.6\\0.8\end{bmatrix}\cdot\begin{bmatrix}4\\3\end{bmatrix} = (-0.6)(4)+(0.8)(3) = -2.4+2.4 = 0$$

$$\vec{u}\cdot\vec{w} = \begin{bmatrix}-0.6\\0.8\end{bmatrix}\cdot\begin{bmatrix}1\\2\end{bmatrix} = (-0.6)(1)+(0.8)(2) = -0.6+1.6 = 1$$

$$\vec{u}\cdot(\vec{v}+\vec{w}) = \begin{bmatrix}-0.6\\0.8\end{bmatrix}\cdot(\begin{bmatrix}4\\3\end{bmatrix}+\begin{bmatrix}1\\2\end{bmatrix}) = \begin{bmatrix}-0.6\\0.8\end{bmatrix}\cdot\begin{bmatrix}5\\5\end{bmatrix} = (-0.6)(5)+(0.8)(5) = -3+4 = 1$$

$$\vec{w}\cdot\vec{v} = \begin{bmatrix}4\\3\end{bmatrix}\cdot\begin{bmatrix}1\\2\end{bmatrix} = (4)(1)+(3)(2) = 4+6 = 10$$

Problem 3

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{\vec{v}}{\sqrt{4^2+3^2}} = \frac{\vec{v}}{\sqrt{25}} = \frac{\begin{bmatrix}4\\3\end{bmatrix}}{5} = \begin{bmatrix}0.8\\0.6\end{bmatrix}$$

$$\hat{w} = \frac{\vec{w}}{\|\vec{w}\|} = \frac{\vec{w}}{\sqrt{1^2+2^2}} = \frac{\begin{bmatrix}4\\3\end{bmatrix}}{\sqrt{5}} = \begin{bmatrix}\frac{4}{\sqrt{5}}\\\frac{3}{\sqrt{5}}\end{bmatrix}$$

$$\vec{a} = \begin{bmatrix}2\\4\end{bmatrix}$$

$$\vec{b} = \begin{bmatrix}-2\\1\end{bmatrix}$$

$$\vec{c} = \begin{bmatrix}-3\\-6\end{bmatrix}$$

Problem 5

$$\hat{u}_1 = \frac{\vec{v}}{\|\vec{v}\|} = \frac{\vec{v}}{\sqrt{1^2+3^2}} = \frac{\begin{bmatrix}4\\3\end{bmatrix}}{\sqrt{10}} = \begin{bmatrix}\frac{1}{\sqrt{10}}\\\frac{3}{\sqrt{10}}\end{bmatrix}$$

$$\hat{u}_2 = \frac{\vec{w}}{\|\vec{w}\|} = \frac{\vec{w}}{\sqrt{2^2+1^2+2^2}} = \frac{\begin{bmatrix}2\\1\\2\end{bmatrix}}{\sqrt{9}} = \begin{bmatrix}\frac{2}{9}\\\frac{1}{9}\\\frac{2}{9}\end{bmatrix}$$

$$\hat{U}_1 = \begin{bmatrix}\frac{-3}{\sqrt{10}}\\\frac{1}{\sqrt{10}}\end{bmatrix}\ \hat{U}_2 = \begin{bmatrix}\frac{-2}{9}\\\frac{-1}{9}\\\frac{-2}{9}\end{bmatrix}$$