Section 1.2

Problem 23

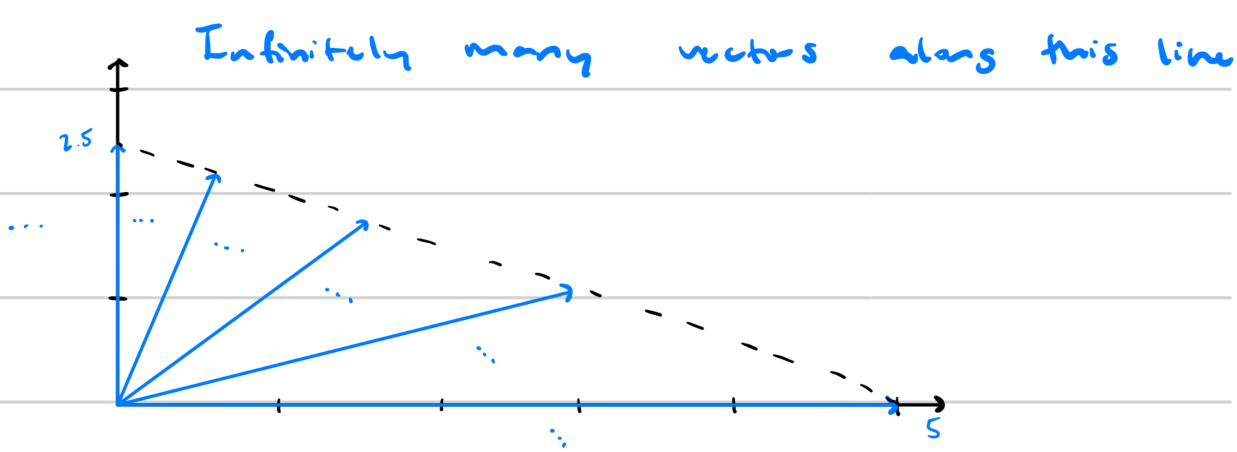
$$\cos(\beta) = \frac{w_1}{||\boldsymbol{w}||}$$

$$\sin(\beta) = \frac{w_2}{||\boldsymbol{w}||}$$

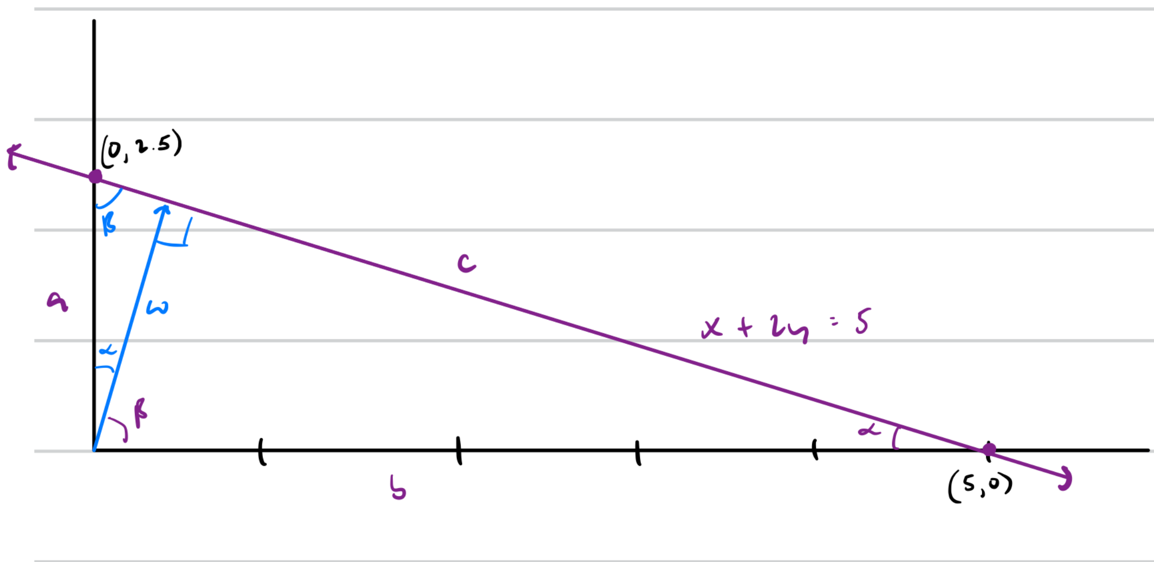
$$\begin{aligned}
\cos(\theta) = \cos(\beta - \alpha) &= \cos(\beta)\cos(\alpha) + \sin(\beta)\sin(\alpha) \\
&= \frac{v_1}{||\boldsymbol{v}||} \cdot \frac{w_1}{||\boldsymbol{w}||} + \frac{v_2}{||\boldsymbol{v}||} \cdot \frac{w_2}{||\boldsymbol{w}||} \\
&= \frac{v_1 \cdot w_1 + v_2 \cdot w_2}{||\boldsymbol{w}||\ ||\boldsymbol{v}||} \\
&= \frac{\boldsymbol{v} \cdot \boldsymbol{w}}{||\boldsymbol{w}||\ ||\boldsymbol{v}||}
\end{aligned}$$

Problem 28

The vectors lie along a line since the dot product is a linear equation. Thus, all solutions lie along a line that satisfies, $x + 2y = 5$.



The shortest $\boldsymbol{w}$ is the vector that is perpendicular to this line. You may be able to intuit this answer to be $\boldsymbol{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, but you can explicity solve for it. From the below diagram, you should be able to prove to yourself (through highschool geometry) that the smaller triangle formed by $\boldsymbol{w}$ has the same angles as the larger triangle formed by the line satisfying $x + 2y = 5$. We can use this relationship to solve for $\boldsymbol{w}$. (Remember that $\boldsymbol{w} = \begin{bmatrix} x \\ y \end{bmatrix}$)



$$a = \frac{5}{2} \qquad b = 5 \qquad c = \sqrt{a^2 + b^2} = \sqrt{\frac{5}{2}^2 + 5^2} = \sqrt{\frac{25}{4} + 25} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2}$$

$$\cos(\alpha) = \frac{b}{c} = \frac{5}{\frac{5\sqrt{5}}{2}} = \frac{2}{\sqrt{5}}$$

$$\sin(\alpha) = \frac{a}{c} = \frac{\frac{5}{2}}{\frac{5\sqrt{5}}{2}} = \frac{1}{\sqrt{5}}$$

$$||\boldsymbol{w}_{min}|| = \frac{5}{2}\cos(\alpha) = \frac{5}{2} \cdot \frac{2}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

$$x = ||\boldsymbol{w}_{min}|| \sin(\alpha) = \frac{1}{\sqrt{5}}\sqrt{5} = 1$$

$$y = ||\boldsymbol{w}_{min}|| \cos(\alpha) = \frac{2}{\sqrt{5}}\sqrt{5} = 2$$

Section 1.3

Problem 4

From the vectors we have the following 3 equations since $x_1 = 1$.

$$1 + 4x_2 + 7x_3 = 0 \qquad 2 + 5x_2 + 8x_3 = 0 \qquad 3 + 6x_2 + 9x_3 = 0$$

We begin solving this sytem by adding $3 + 6x_2 + 9x_3 = 0$ and $\frac{-3}{2}(1 + 4x_2 + 7x_3 = 0)$ to find that $\frac{3}{2} - \frac{3}{2}x_3 = 0$. Thus, $x_3 = 1$. Substituting x_3 back into one of the original equations yields $3 + 6x_2 + 9(1) = 0$. Thus, $x_2 = -2$. Hence, the final solution is $x_1 = 1, x_2 = -2, x_3 = 1$.

Problem 13

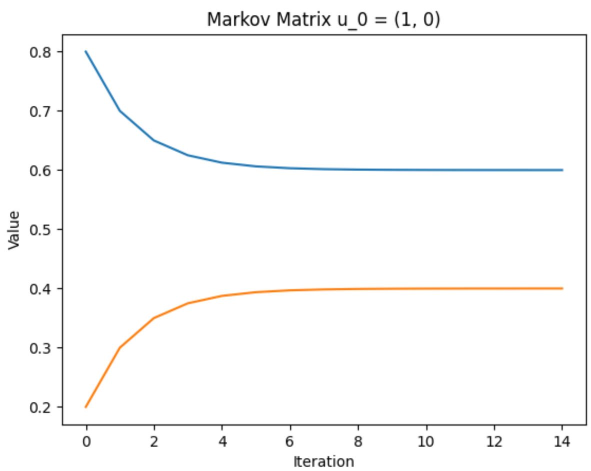
Section 2.1

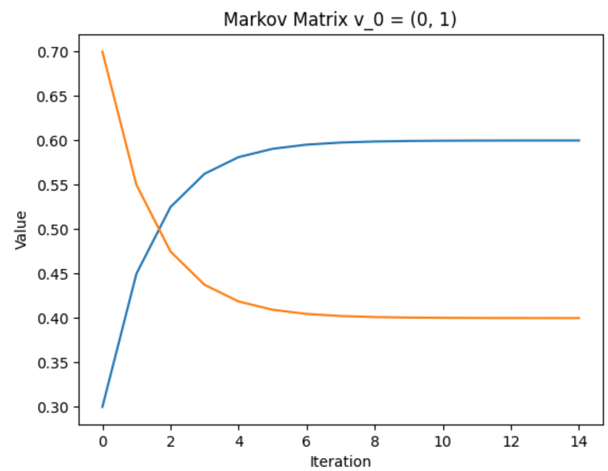
Problem 29

$$\boldsymbol{u}_2 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} .8 \\ .2 \end{bmatrix} = \begin{bmatrix} .8 \cdot .8 + .3 \cdot .2 \\ .8 \cdot .2 + .2 \cdot .7 \end{bmatrix} = \begin{bmatrix} .64 + .06 \\ .16 + .14 \end{bmatrix} = \begin{bmatrix} .7 \\ .3 \end{bmatrix}$$
$$\boldsymbol{u}_3 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} .7 \\ .3 \end{bmatrix} = \begin{bmatrix} .7 \cdot .8 + .3 \cdot .3 \\ .7 \cdot .2 + .3 \cdot .7 \end{bmatrix} = \begin{bmatrix} .56 + .09 \\ .14 + .21 \end{bmatrix} = \begin{bmatrix} .65 \\ .35 \end{bmatrix}$$

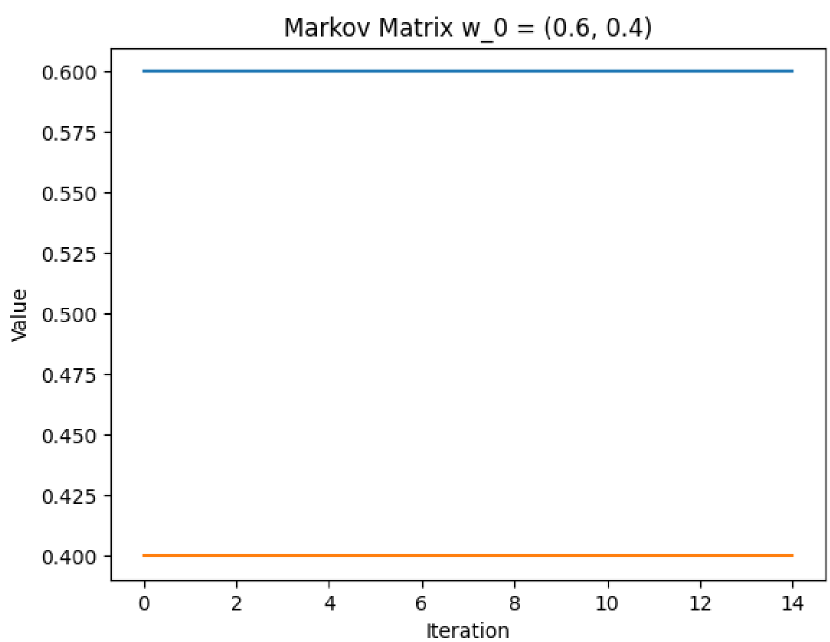
Problem 30

You can see that both vectors converge to $\begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$.





We can prove this by showing that $\begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$ does not change when multiplied by our Markov Matrix.



Section 2.2

*Three planes can fail to have an intersection point, even if no planes are parallel. The system is singular if row 3 of A is a **linear combination** of the first two rows.*

Section 2.3

Section 2.4

Section 2.5