

Section 1.2

Problem 23

$$\cos(\beta) = \frac{w_1}{\|\mathbf{w}\|}$$

$$\sin(\beta) = \frac{w_2}{\|\mathbf{w}\|}$$

$$\cos(\theta) = \cos(\beta - \alpha) = \cos(\beta) \cos(\alpha) + \sin(\beta) \sin(\alpha)$$

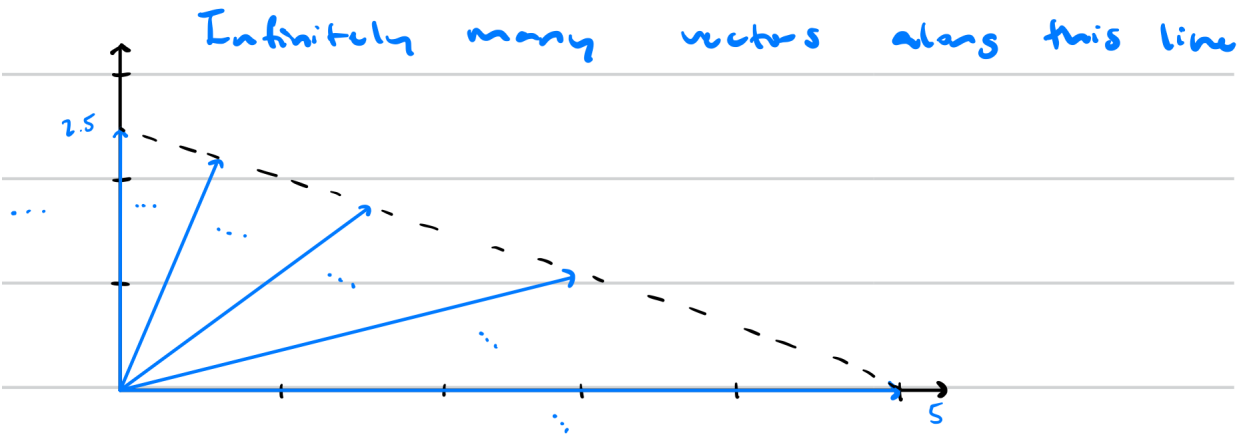
$$= \frac{v_1}{\|\mathbf{v}\|} \cdot \frac{w_1}{\|\mathbf{w}\|} + \frac{v_2}{\|\mathbf{v}\|} \cdot \frac{w_2}{\|\mathbf{w}\|}$$

$$= \frac{v_1 \cdot w_1 + v_2 \cdot w_2}{\|\mathbf{w}\| \|\mathbf{v}\|}$$

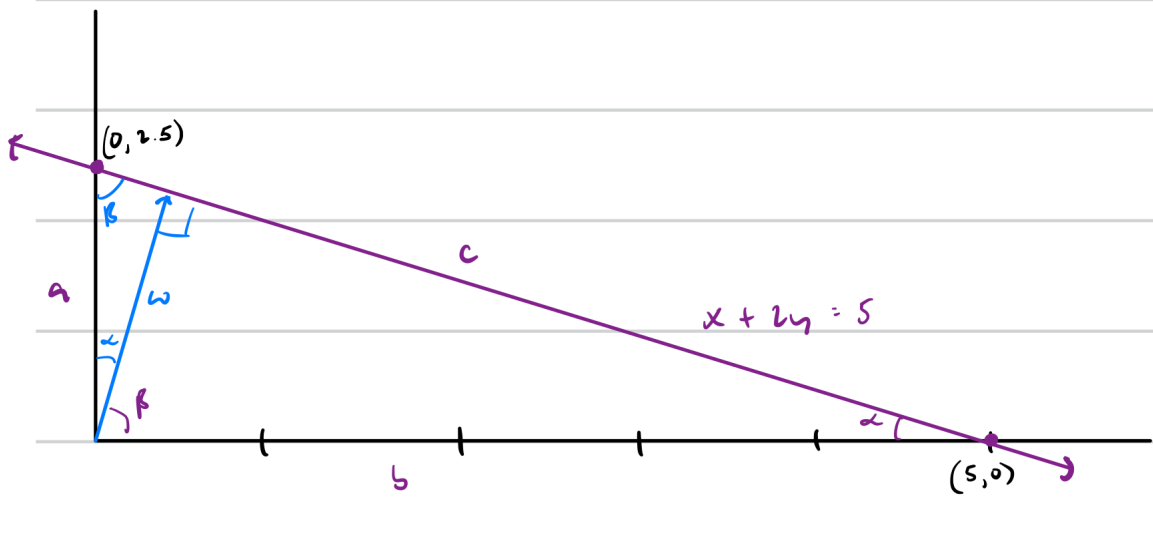
$$= \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{w}\| \|\mathbf{v}\|}$$

Problem 28

The vectors lie along a line since the dot product is a linear equation. Thus, all solutions lie along a line that satisfies, $x + 2y = 5$.



The shortest \mathbf{w} is the vector that is perpendicular to this line. You may be able to intuit this answer to be $\mathbf{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, but you can explicitly solve for it. From the below diagram, you should be able to prove to yourself (through highschool geometry) that the smaller triangle formed by \mathbf{w} has the same angles as the larger triangle formed by the line satisfying $x + 2y = 5$. We can use this relationship to solve for \mathbf{w} . (Remember that $\mathbf{w} = \begin{bmatrix} x \\ y \end{bmatrix}$)



$$a = \frac{5}{2} \quad b = 5 \quad c = \sqrt{a^2 + b^2} = \sqrt{\frac{5^2}{2} + 5^2} = \sqrt{\frac{25}{4} + 25} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2}$$

$$\cos(\alpha) = \frac{b}{c} = \frac{5}{\frac{5\sqrt{5}}{2}} = \frac{2}{\sqrt{5}}$$

$$\sin(\alpha) = \frac{a}{c} = \frac{\frac{5}{2}}{\frac{5\sqrt{5}}{2}} = \frac{1}{\sqrt{5}}$$

$$\|\mathbf{w}_{min}\| = \frac{5}{2} \cos(\alpha) = \frac{5}{2} \cdot \frac{2}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

$$x = \|\mathbf{w}_{min}\| \sin(\alpha) = \frac{1}{\sqrt{5}} \sqrt{5} = 1$$

$$y = \|\mathbf{w}_{min}\| \cos(\alpha) = \frac{2}{\sqrt{5}} \sqrt{5} = 2$$

Section 1.3

Problem 4

Problem 13

Section 2.1

Problem 29

$$\mathbf{u}_2 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} .8 \\ .2 \end{bmatrix} = \begin{bmatrix} .8 \cdot .8 + .3 \cdot .2 \\ .8 \cdot .2 + .7 \cdot .7 \end{bmatrix} = \begin{bmatrix} .64 + .06 \\ .16 + .49 \end{bmatrix} = \begin{bmatrix} .7 \\ .65 \end{bmatrix}$$

$$\mathbf{u}_3 = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} .7 \\ .3 \end{bmatrix} = \begin{bmatrix} .8 \cdot .7 + .3 \cdot .3 \\ .2 \cdot .7 + .7 \cdot .3 \end{bmatrix} = \begin{bmatrix} .56 + .09 \\ .14 + .21 \end{bmatrix} = \begin{bmatrix} .65 \\ .35 \end{bmatrix}$$

Section 2.2

Section 2.3

Section 2.4

Section 2.5