Machine Learning HW #3 answer

1.

Mahalanobis distance is quite effective to find outliers for multivariate data. Euclidean distance is also commonly used to find distance between two points in 2 or more than 2 dimensional space. If we use the Euclidean distance to detect, o1 and o2 will be cluster to C1 which is closer to them. But, it doesn't consider about the dispersion of cluster. MD uses a covariance matrix unlike Euclidean. Because of that, MD works well when two or more variables are highly correlated and even if their scales are not the same. But, when two or more variables are not on the same scale, Euclidean distance results might misdirect.

2.

x	0.9	0.7	1.2	2.4	1.8
$g_1(x)$	0.3970	0.3914	0.3910	0.1497	0.2897
$g_2(x)$	0.2179	0.1714	0.2897	0.3683	0.3910
$B_1(x)$	0.6457	0.6900	0.5744	0.2891	0.4256
$B_2(x)$	0.3543	0.3100	0.4256	0.7109	0.5744

$$\alpha_j = \frac{1}{n}\sum_{i=0}^{n} \beta_j(x_i)$$

$$\mu_j = \frac{\sum_{i=0}^{n} \beta_j(x_i) \cdot x_i}{\sum_{i=0}^{n} \beta_j(x_i)}$$

$$\sigma_j{}^2 = \frac{\sum_{i=0}^{n} \beta_j(x_i) \cdot (x_i - \mu_j)^2}{\sum_{i=0}^{n} \beta_j(x_i)}$$

$$\alpha_1 = \frac{1}{5}(0.6457 + 0.6900 + 0.5744 + 0.2891 + 0.4256) = \frac{2.6248}{5} = 0.52496$$

$$\alpha_2 = \frac{1}{5}(0.3543 + 0.3100 + 0.4256 + 0.7109 + 0.5744) = \frac{2.3752}{5} = 0.47504$$

$$\mu_1 = \frac{0.6457 * 0.9 + 0.6900 * 0.7 + 0.5744 * 1.2 + 0.2891 * 2.4 + 0.4256 * 1.8}{2.6248}$$

$$= \frac{3.2133}{2.6248} = 1.224$$

$$\mu_2 = \frac{0.3543 * 0.9 + 0.3100 * 0.7 + 0.4256 * 1.2 + 0.7109 * 2.4 + 0.5744 * 1.8}{2.3752}$$

$$= \frac{3.7867}{2.3752} = 1.594$$

$$\sigma_1{}^2 = \frac{0.6457 * 0.105 + 0.6900 * 0.275 + 0.5744 * 0.0006 + 0.2891 * 1.383 + 0.4256 * 0.332}{2.6248}$$

$$= \frac{1.4092}{2.6248} = 0.5369$$

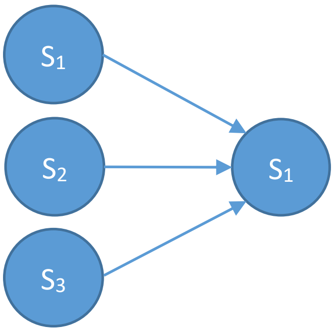
$$\sigma_2{}^2 = \frac{0.3543 * 0.482 + 0.3100 * 0.799 + 0.4256 * 0.155 + 0.7109 * 0.65 + 0.5744 * 0.042}{2.3752}$$

$$= \frac{0.9706}{2.3752} = 0.4087$$

3.

$$S_? \xrightarrow{step1} S_? \xrightarrow{step2} S_?$$

$step1$:



S_{1R}=4/6 1/3 = 2/9

S_{2R}=2/6 1/3 = 1/9

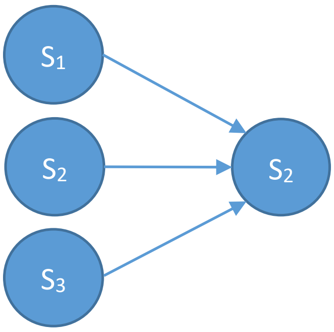
S_{3R}=3/6 1/3 = 1/6

$S_{1R} \rightarrow S_1$ = 2/9 2/3 = 4/27

$S_{2R} \rightarrow S_1$ = 1/9 1/6 = 1/54

$S_{3R} \rightarrow S_1$ = 1/6 1/6 = 1/36

Max: $S_{1R} \rightarrow S_1$ = 4/27

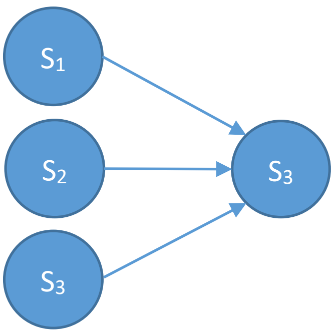


$S_{1R} \rightarrow S_2$ = 2/9 1/6 = 1/27

$S_{2R} \rightarrow S_2$ = 1/9 2/3 = 2/27

$S_{3R} \rightarrow S_2$ = 1/6 1/6 = 1/36

Max: $S_{2R} \rightarrow S_2$ = 2/27



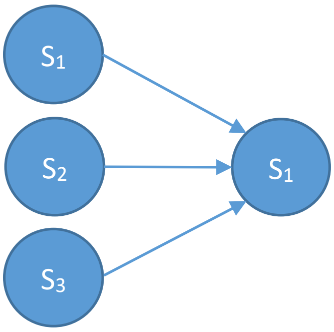
$S_{1R} \rightarrow S_3$ = 2/9 1/6 = 1/27

$S_{2R} \rightarrow S_3$ = 1/9 1/6 = 1/54

$S_{3R} \rightarrow S_3$ = 1/6 2/3 = 1/9

Max: $S_{3R} \rightarrow S_3$ = 1/9

$step2$:



S_{1B}= 1/3 4/27 = 4/81

S_{2B}= 2/3 2/27 = 4/81

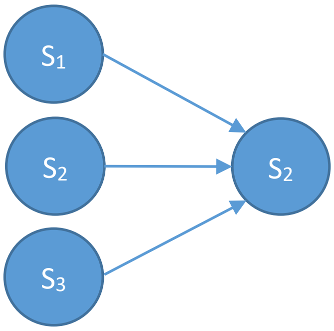
S_{3B}= 1/2 1/9 = 1/18

$S_{1B} \rightarrow S_1$ = 4/81 2/3 = 8/243

$S_{2B} \rightarrow S_1$ = 4/81 1/6 = 2/243

$S_{3B} \rightarrow S_1$ = 1/18 1/6 = 1/108

Max: $S_{1B} \rightarrow S_1$ = 8/243

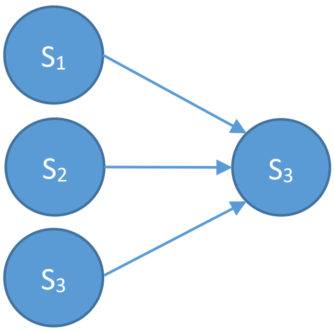


$S_{1B} \rightarrow S_2$ = 4/81 1/6 = 2/243

$S_{2B} \rightarrow S_2$ = 4/81 2/3 = 8/243

$S_{3B} \rightarrow S_2$ = 1/18 1/6 = 1/108

Max: $S_{2B} \rightarrow S_2$ = 8/243



$S_{1B} \rightarrow S_3$ = 4/81 1/6 = 2/243

$S_{2B} \rightarrow S_3$ = 4/81 1/6 = 2/243

$S_{3B} \rightarrow S_3$ = 1/18 2/3 = 1/27

Max: $S_{3B} \rightarrow S_3$ = 1/27

S_{1R}=8/243 4/6 = 16/729

S_{2R}=8/243 2/6 = 8/729

S_{3R}=1/27 3/6 = 1/54

Max: S_{1R}

⇨ $S_{1R} \rightarrow S_{1B} \rightarrow S_{1R}$

4.

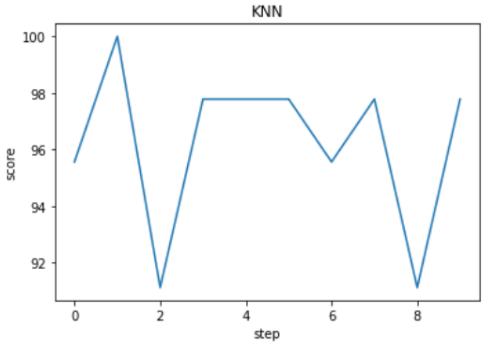
```python
from sklearn import datasets
from sklearn.model_selection import train_test_split
from sklearn.neighbors import KNeighborsClassifier
from sklearn.metrics import accuracy_score
from sklearn.naive_bayes import GaussianNB
from sklearn.mixture import GaussianMixture

import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

iris_dataset = datasets.load_iris()

df_X = iris_dataset.data[:, :4]
df_y = iris_dataset.target
```

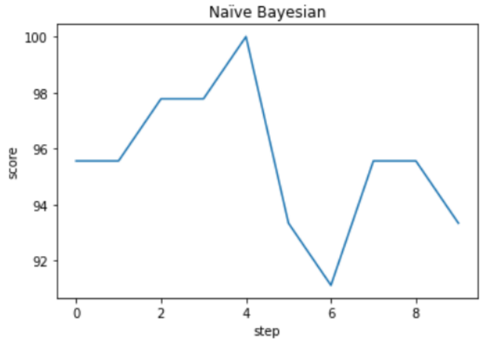
```python
KNN_scores = []
GaussianNB_scores = []
GMM_scores = []
```

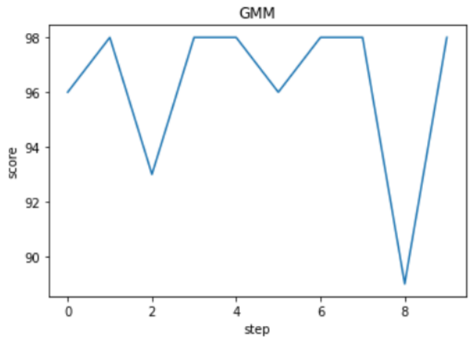
```python
def kNN_classifier(X_train, X_test, y_train, y_test):
    classifier = KNeighborsClassifier(n_neighbors=7)
    classifier.fit(X_train, y_train)
    return round(accuracy_score(classifier.predict(X_test), y_test) * 100, 3)

def GaussianNB_classifier(X_train, X_test, y_train, y_test):
    classifier = GaussianNB()
    classifier.fit(X_train, y_train)
    return round(accuracy_score(classifier.predict(X_test), y_test) * 100, 3)

def GMM_classifier(X_train, X_test, y_train, y_test):
    s=[[], [], []]
    for i in range(3):
        X_train_list = X_train[y_train==i, :]
        gmm = GaussianMixture(n_components=2)
        gmm.fit(X_train_list)
        s[i] = gmm.score_samples(X_test)
    predict_0 = np.logical_and((s[0]>s[1]), (s[0]>s[2]))
    predict_1 = np.logical_and((s[1]>s[0]), (s[1]>s[2]))
    predict_2 = np.logical_and((s[2]>s[1]), (s[2]>s[0]))

    accuracy_0 = np.logical_and(predict_0, y_test==0)
    accuracy_1 = np.logical_and(predict_1, y_test==1)
    accuracy_2 = np.logical_and(predict_2, y_test==2)

    return round((sum(accuracy_0)+sum(accuracy_1)+sum(accuracy_2)) / len(y_test) * 100)
```

```python
for i in range(10):
        X_train, X_test, y_train, y_test = train_test_split(df_X, df_y, test_size=0.3)

        KNN_scores.append(kNN_classifier(X_train, X_test, y_train, y_test))
        GaussianNB_scores.append(GaussianNB_classifier(X_train, X_test, y_train, y_test))
        GMM_scores.append(GMM_classifier(X_train, X_test, y_train, y_test))

plt.plot(KNN_scores)
plt.title('KNN')
plt.xlabel('step')
plt.ylabel('score')
plt.show()

plt.plot(GaussianNB_scores)
plt.title('Naïve Bayesian')
plt.xlabel('step')
plt.ylabel('score')
plt.show()

plt.plot(GMM_scores)
plt.title('GMM')
plt.xlabel('step')
plt.ylabel('score')
plt.show()
```

```python
print(f"KNN Score = ", KNN_scores)
print(f"KNN Score average: {np.mean(KNN_scores)}")
print(f"Naïve Bayesian classifier Score = ", GaussianNB_scores)
print(f"Naïve Bayesian classifier Score average: {np.mean(GaussianNB_scores)}")
print(f"GMM Score = ", GMM_scores)
print(f"GMM Score average: {np.mean(GMM_scores)}")
```








```
KNN Score =  [95.556, 100.0, 91.111, 97.778, 97.778, 97.778, 95.556, 97.778, 91.111, 97.778]
KNN Score average: 96.22240000000001
Naïve Bayesian classifier Score =  [95.556, 95.556, 97.778, 97.778, 100.0, 93.333, 91.111, 95.556, 95.556, 93.333]
Naïve Bayesian classifier Score average: 95.5557
GMM Score =  [96, 98, 93, 98, 98, 96, 98, 98, 89, 98]
GMM Score average: 96.2
```

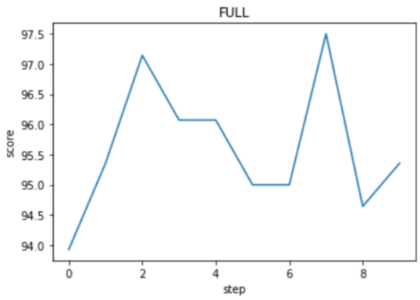
5.

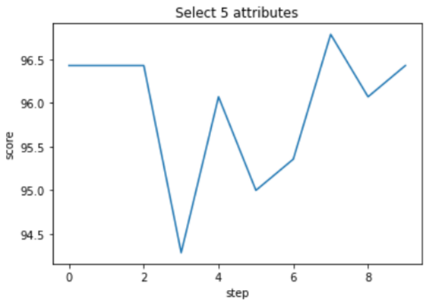
```python
from sklearn.model_selection import train_test_split
from sklearn.metrics import accuracy_score
from sklearn.naive_bayes import GaussianNB
from sklearn.feature_selection import SequentialFeatureSelector
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
```

```python
def get_mean_of_missing_values_attr(df):
    numbers = 0
    total = 0
    for value in df:
        if value != '?':
            total += int(value)
            numbers += 1
    return total / numbers

def GaussianNB_classifier(X_train, X_test, y_train, y_test):
    classifier = GaussianNB()
    classifier.fit(X_train, y_train)
    return round(accuracy_score(classifier.predict(X_test), y_test) * 100, 3)

def select_top5_attributes(X_val, y_val):
    sfs = SequentialFeatureSelector(GaussianNB(), n_features_to_select=5)
    sfs.fit(X_val, y_val)
    return sfs.get_support()
```

```
breast_cancer = pd.read_csv('https://archive.ics.uci.edu/ml/machine-learning-databases/breast-cancer-wisconsin/breast-cancer-wisconsin.data',
breast_cancer.columns = ['Id', 'Clump Thickness', 'Uniformity of Cell Size', 'Uniformity of Cell Shape', 'Marginal Adhesion',
                         'Single Epithelial Cell Size', 'Bare Nuclei', 'Bland Chromatin', 'Normal Nucleoli', 'Mitoses', 'Class']
breast_cancer = breast_cancer.drop(['Id'], axis=1)
breast_cancer.replace(to_replace = '?', value = get_mean_of_missing_values_attr(breast_cancer.iloc[:,5]), inplace = True)
breast_cancer = breast_cancer.astype('int64')

df_X = breast_cancer.iloc[:,:9].values
df_y = breast_cancer.iloc[:,9].values

full_scores = []
select_scores = []

for i in range (10):
    X_train, X_test, y_train, y_test = train_test_split(df_X, df_y, test_size=0.4)
    X_train, X_val, y_train, y_val = train_test_split(X_train, y_train, test_size=0.5)

    full_scores.append(GaussianNB_classifier(X_train, X_test, y_train, y_test))

    selected = select_top5_attributes(X_val, y_val)
    select_scores.append(GaussianNB_classifier(X_train[:, selected], X_test[:, selected], y_train, y_test))

plt.plot(full_scores)
plt.title('FULL')
plt.xlabel('step')
plt.ylabel('score')
plt.show()
```

```
plt.plot(select_scores)
plt.title('Select 5 attributes')
plt.xlabel('step')
plt.ylabel('score')
plt.show()

print(f"Full Score = ", full_scores)
print(f"Full Score average: {np.mean(full_scores)}")
print(f"Select 5 attributes Score = ", select_scores)
print(f"Select 5 attributes Score average: {np.mean(select_scores)}")
```






```
Full Score =  [93.929, 95.357, 97.143, 96.071, 96.071, 95.0, 95.0, 97.5, 94.643, 95.357]
Full Score average: 95.6071
Select 5 attributes Score =  [96.429, 96.429, 96.429, 94.286, 96.071, 95.0, 95.357, 96.786, 96.071, 96.429]
Select 5 attributes Score average: 95.9287
```