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專題一 結果報告

1. 開發平台:MATLAB
2. 如何執行:執行程式後依序輸入指定參數，如圖一、圖二

```
please enter the joint variable separated by spaces or commas (in degree):
limit: theta1  (-160~160); theta2  (-125~125); theta3  (-135~135);
 theta4  (-140~140) ;theta5  (-100~100); theta6  (-260~260):
20 20 20 20 20 20
```

圖一、順向運動學輸入格式

```
please inuput [nx ox ax px;ny oy ay py;nz oz az pz;0 0 0 1;]
[0.1057541556 -0.6425141383 0.7589411311 0.5776495330;0.7019053129 0.5888588208 0.4007171329 0.3688097239; -0.7043756030 0.4903273101 0.5132583548 0.1968002941;0 0 0 1.0;]
```

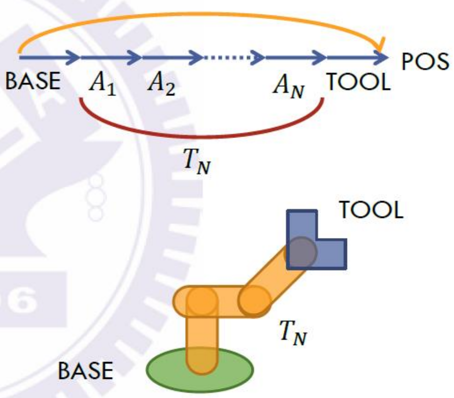
圖二、逆向運動學輸入格式

3. 程式運行流程及核心程式碼說明：

   DH 表示式有兩種，而本次實驗中使用的 DH 表示式為課本的方式，如圖 A。

$$A_n = Rot(z,\theta_n) * Trans(0,0,d_n) * Trans(a_n,0,0) * Rot(x,\alpha_n)$$

$$= \begin{pmatrix} c\theta_n & -s\theta_n c\alpha_n & s\theta_n s\alpha_n & a_n c\theta_n \\ s\theta_n & c\theta_n c\alpha_n & -c\theta_n s\alpha_n & a_n s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$POS = BASE * T_N * TOOL$$

$$T_N = A_1 * A_2 * \cdots * A_N$$

圖 A、DH 表達式定義

3.1 Forward Kinematics:

一開始會要求使用者輸入參數並加上範圍限制，如圖三

```
theta=input("please enter the joint variable separated by spaces or commas (in degree) 可參考附件 FK_input.txt 的格式:" + ...
    "\nlimit: theta1  (-160~160); theta2  (-125~125); theta3  (-135~135);\n theta4  (-140~140) ;theta5  (-100~100); theta6  (-260~260):\n",'s');
theta=str2num(theta);

%判斷是否超出範圍
if(abs(theta(1))>160)
    fprintf("theta1 is out of range!! \n");
end
if(abs(theta(2))>125)
    fprintf("theta2 is out of range!! \n");
end
if(abs(theta(3))>135)
    fprintf("theta3 is out of range!! \n");
end
if(abs(theta(4))>140)
    fprintf("theta4 is out of range!! \n");
end

if(abs(theta(5))>100)
    fprintf("theta5 is out of range!! \n");
end

if(abs(theta(6))>260)
    fprintf("theta6 is out of range!! \n");
end
```

圖三、 若有輸入超出範圍會提示

根據DH表來建立矩陣，將矩陣透過基本平移與旋轉(圖四)來表達(圖五)

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```
function Rot_matrix=Rot(axis,angle)
    angle=angle*pi/180;
    if axis=='x'
        Rot_matrix= [ 1       0           0           0;
                      0       cos(angle)  -sin(angle)  0;
                      0       sin(angle)  cos(angle)   0;
                      0       0           0            1];
    elseif axis=='y'
        Rot_matrix= [ cos(angle)   0   sin(angle)   0;
                      0            1   0            0;
                      -sin(angle)  0   cos(angle)   0;
                      0            0   0            1];
    elseif axis=='z'
        Rot_matrix= [ cos(angle)  -sin(angle)   0   0;
                      sin(angle)  cos(angle)    0   0;
                      0           0             1   0;
                      0           0             0   1];
    end
end
```

```
function Trans_matrix=Trans(X,Y,Z)
    Trans_matrix= [ 1  0  0  X;
                    0  1  0  Y;
                    0  0  1  Z;
                    0  0  0  1];
end
```

圖四、旋轉矩陣(左)與平移矩陣(右)

```
function transf = DH( Theta,D,A,Alpha)

    transf=Trans(0,0,D)*Rot('z',Theta)*Trans(A,0,0)*Rot('x',Alpha);


end
```

圖五、DH法的公式

計算A1~A6 矩陣並得到T6，如圖六

```
%執行
An=zeros([4 4 6]);
T=eye(4);
for i=1:6
    An(:,:,i)=DH(theta(i),d(i),a(i),alpha(i));
    T=T*An(:,:,i);
end
%
```

圖六、計算T6矩陣

取得T6矩陣後計算其以ZYZ表示的RPY，如圖七

```
%計算RPY==>ZYZ
function [X,Y,Z,Phi ,Theta, Psi]= noap2RPY(T)
    %RPY==>ZYX
    %Phi=atan2(T(2,1),T(1,1))/pi*180;
    %Theta=asin(-1*T(3,1))/pi*180;
    %Psi=atan2(T(3,2),T(3,3))/pi*180;

    %RPY==>ZYZ
    X=T(1,4);
    Y=T(2,4);
    Z=T(3,4);

    if (abs(T(3,3))~=1)
        Psi= atan2(T(2,3),T(1,3));
        Theta=atan2(T(1,3)/cos(Psi),T(3,3))/pi*180;

        Psi=Psi/pi*180;
        Phi=atan2(T(3,2),-T(3,1))/pi*180;
    else
        Theta=0;
        Phi=0;
        Psi=atan2(-1*T(1,2),T(1,1))/pi*180;
    end

end
```

圖7、計算RPY

最後顯示T6矩陣及末端執行器的空間位置(XYZ)及姿態(RPY)，如圖八

```
%結果
disp(" ")
%disp("input is :")
%disp(theta);
disp("[n o a p ]:")
disp(T);
[X, Y, Z ,Phi ,Theta, Psi]= noap2RPY(T);
fprintf('\n[X  Y  Z   φ  θ  ψ ]:\n');
disp([X,Y,Z,Phi,Theta,Psi])
%
```

圖八、顯示T6矩陣及末端執行器的空間位置及姿態

輸出結果，圖九，以輸入為[20 20 20 20 20 20] 為例:

```
 joint       d         a       alpha    theta
 _____     _____     _____     _____    _____

   1           0         0       -90       0
   2           0     0.432         0       0
   3       0.149     -0.02        90       0
   4       0.433         0       -90       0
   5           0         0        90       0
   6           0         0         0       0

please enter the joint variable separated by spaces or commas (in degree) 可參考附件 FK_input.txt 的格式:
limit: theta1  (-160~160); theta2  (-125~125); theta3  (-135~135);
 theta4  (-140~140) ;theta5  (-100~100); theta6  (-260~260):
[20 20 20 20 20 20]

[n o a p ]:
    0.1058   -0.6425    0.7589    0.5776
    0.7019    0.5889    0.4007    0.3688
   -0.7044    0.4903    0.5133    0.1968
         0         0         0    1.0000


[X  Y  Z   ϕ  θ  ψ ]:
    0.5776    0.3688    0.1968   34.8424   59.1189   27.8338
```

圖九、程式執行結果

3.2 Inverse Kinematics:

一開始會要求使用者輸入參數[n o a p]，然後計算θ1，θ1有righty及lefty兩組解，如圖十。

```
theta=zeros([8,6]);
T=input("please inuput [nx ox ax px;ny oy ay py;nz oz az pz;0 0 0 1;]  可參考附件 IK_input.txt 的格式:\n",'s');
T= str2num(T);

%%%%theta1
%1~4 righty
theta(1:4,1)=atan2(T(2,4),T(1,4))-atan2(d(3),-sqrt(T(2,4)^2+T(1,4)^2-d(3)^2));
%5~8 lefty
theta(5:8,1)=atan2(T(2,4),T(1,4))-atan2(d(3),+sqrt(T(2,4)^2+T(1,4)^2-d(3)^2));

%%%%theta1
```

圖十、輸入參數並計算θ1

每組θ1會有對應的Above 或 Below 的 θ3的解，如圖十一

```
%%%%theta3
M=(T(3,4)^2+T(2,4)^2+T(1,4)^2-a(2)^2-a(3)^2-d(3)^2-d(4)^2)/(2*a(2));
%1 2 r_a
theta(1:2,3)=atan2(M,-sqrt(a(3)^2+d(4)^2-M^2)) -atan2(a(3),d(4));
%3 4 r_b
theta(3:4,3)=atan2(M,sqrt(a(3)^2+d(4)^2-M^2)) -atan2(a(3),d(4));
%5 6 l_a
theta(5:6,3)=atan2(M,sqrt(a(3)^2+d(4)^2-M^2)) -atan2(a(3),d(4));
%7 8 l_b
theta(7:8,3)=atan2(M,-sqrt(a(3)^2+d(4)^2-M^2)) -atan2(a(3),d(4));
%%%%theta3
```

圖十一、 θ3 Above 或 Below 計算

再以手臂為 Righty-Above、lefty-Above、Righty-Below、lefty- Below為分類計算θ2，如圖十二

```
%%%%theta2
%%%R
delta_r=(cos(theta(1,1))*T(1,4) +sin(theta(1,1))*T(2,4))^2+T(3,4)^2;

C23_ra=( (cos(theta(1,1))*T(1,4) +sin(theta(1,1))*T(2,4)) *(a(3)+a(2)*cos(theta(1,3)))   +T(3,4)*(d(4)+a(2)*sin(theta(1,3)))) /delta_r;
S23_ra=( (cos(theta(1,1))*T(1,4) +sin(theta(1,1))*T(2,4)) *(d(4)+a(2)*sin(theta(1,3)))   -T(3,4)*(a(3)+a(2)*cos(theta(1,3)))) /delta_r;
theta23_ra=atan2(S23_ra,C23_ra);
theta(1:2,2)=theta23_ra-theta(1,3);

C23_rb=( (cos(theta(1,1))*T(1,4) +sin(theta(1,1))*T(2,4)) *(a(3)+a(2)*cos(theta(3,3)))   +T(3,4)*(d(4)+a(2)*sin(theta(3,3)))) /delta_r;
S23_rb=( (cos(theta(1,1))*T(1,4) +sin(theta(1,1))*T(2,4)) *(d(4)+a(2)*sin(theta(3,3)))   -T(3,4)*(a(3)+a(2)*cos(theta(3,3)))) /delta_r;
theta23_rb=atan2(S23_rb,C23_rb);
theta(3:4,2)=theta23_rb-theta(3,3);
%%%r

%%%L
delta_l=(cos(theta(5,1))*T(1,4) +sin(theta(5,1))*T(2,4))^2+T(3,4)^2;

C23_la=( (cos(theta(5,1))*T(1,4) +sin(theta(5,1))*T(2,4)) *(a(3)+a(2)*cos(theta(5,3)))   +T(3,4)*(d(4)+a(2)*sin(theta(5,3)))) /delta_l;
S23_la=( (cos(theta(5,1))*T(1,4) +sin(theta(5,1))*T(2,4)) *(d(4)+a(2)*sin(theta(5,3)))   -T(3,4)*(a(3)+a(2)*cos(theta(5,3)))) /delta_l;
theta23_la=atan2(S23_la,C23_la);
theta(5:6,2)=theta23_la-theta(5,3);

C23_lb=( (cos(theta(5,1))*T(1,4) +sin(theta(5,1))*T(2,4)) *(a(3)+a(2)*cos(theta(7,3)))   +T(3,4)*(d(4)+a(2)*sin(theta(7,3)))) /delta_l;
S23_lb=( (cos(theta(5,1))*T(1,4) +sin(theta(5,1))*T(2,4)) *(d(4)+a(2)*sin(theta(7,3)))   -T(3,4)*(a(3)+a(2)*cos(theta(7,3)))) /delta_l;
theta23_lb=atan2(S23_lb,C23_lb);
theta(7:8,2)=theta23_lb-theta(7,3);
%%%l
%%%%theta2
```

圖十二、計算θ23及θ2

求得 θ1、θ2、θ3 後移除前三軸的效果(計算出T3 的反矩陣 乘回T6 求得 A4*A5*A6 來計算後三軸)，並計算θ4，如圖十三

```
%%%T456
T3_inv=zeros([4,4,4]);
T456=zeros([4,4,4]);
% 算奇數  偶數另外調整即可
for i =1:2:7
    T3_inv(:,:, ceil(i/2))=T3_inverse(theta(i,1), theta(i,2) ,theta(i,3),a(2),a(3),d(3));
    T456(:,:,ceil(i/2))=T3_inv(:,:,ceil(i/2))*T;
end
%%%T456



%%%%theta4
for i=1:2:7
    theta(i,4)=atan2(T456(2,3,ceil(i/2)),T456(1,3,ceil(i/2)));
    theta(i+1,4)=theta(i,4)+pi;
end
%%%%theta4
```

圖十三、計算θ4

求得 θ4 後移除第四軸的效果(求得 A5*A6 來計算後兩軸)，並計算θ5、θ6(若θ5=0，那剩下的角度就會直接由θ6分擔，因此不做多餘的運算)，如圖十四

```
T56=zeros([4,4,4]);
for i=1:4
    T56(:,:,i)=A4_inverse(theta(i*2-1,4),d(4))*T456(:,:,i);
end


for i=1:2:7
    theta(i,5)=atan2(T56(1,3,ceil(i/2)),-1*T56(2,3,ceil(i/2)));
    theta(i+1,5)=-1*theta(i,5);

    theta(i,6)=atan2(T56(3,1,ceil(i/2)),T56(3,2,ceil(i/2)));
    theta(i+1,6)=theta(i,6)+pi;
end

theta_deg= theta/pi*180;
```

圖十四、計算θ5、θ6

最後依序輸出八組解，並確認是否有超出各軸的工作範圍，如圖十五

```
for i=1:8
    fprintf("================== No.%d's ANS =================================\n",i);
    disp("corresponding variable (theta1, theta2, theta3, theta4, theta5, theta6)")
    if(abs(theta_deg(i,1))>160)
        fprintf("theta1 is out of range!! \n");
    end
    if(abs(theta_deg(i,2))>125)
        fprintf("theta2 is out of range!! \n");
    end
    if(abs(theta_deg(i,3))>135)
        fprintf("theta3 is out of range!! \n");
    end
    if(abs(theta_deg(i,4))>140)
        fprintf("theta4 is out of range!! \n");
    end
    if(abs(theta_deg(i,5))>100)
    fprintf("theta5 is out of range!! \n");
    end
    if(abs(theta_deg(i,6))>260)
    fprintf("theta6 is out of range!! \n");
    end

    disp(theta_deg(i,:))

end
disp(" ")
disp("===========================END===================================")
```

圖十五、顯示各組解

以在順向運動學輸入:[20 20 20 20 20 20]，所取得的[n o a p]來當輸入，如圖十六、十七。

```
 joint      d         a       alpha    theta
 _____    _____    _____    _____    _____

   1          0         0      -90        0
   2          0     0.432        0        0
   3      0.149     -0.02       90        0
   4      0.433         0      -90        0
   5          0         0       90        0
   6          0         0        0        0

please inuput [nx ox ax px;ny oy ay py;nz oz az pz;0 0 0 1;]  可參考附件 IK_input.txt 的格式:
[0.105754155679965 -0.642514138372515 0.758941131147761 0.577649533099654;0.701905312986400 0.588858820882606  0.4007171329881 0.368809723984954;-0.704375603039942 0.490327310130867 0.513
================== No.1's ANS ==================================
corresponding variable (theta1, theta2, theta3, theta4, theta5, theta6)
theta2 is out of range!!
theta3 is out of range!!
theta4 is out of range!!
 -134.8863 -200.0000  165.2892  146.2403   27.3062   56.4786

================== No.2's ANS ==================================
corresponding variable (theta1, theta2, theta3, theta4, theta5, theta6)
theta2 is out of range!!
theta3 is out of range!!
theta4 is out of range!!
 -134.8863 -200.0000  165.2892  326.2403  -27.3062  236.4786

================== No.3's ANS ==================================
corresponding variable (theta1, theta2, theta3, theta4, theta5, theta6)
theta2 is out of range!!
 -134.8863 -127.2131   20.0000   19.1824   50.8830 -166.6085
```

圖十六、輸入及前三組解

```
================== No.4's ANS ==================================
corresponding variable (theta1, theta2, theta3, theta4, theta5, theta6)
theta2 is out of range!!
theta4 is out of range!!
 -134.8863 -127.2131   20.0000  199.1824  -50.8830   13.3915

================== No.5's ANS ==================================
corresponding variable (theta1, theta2, theta3, theta4, theta5, theta6)
   20.0000   20.0000   20.0000   20.0000   20.0000   20.0000

================== No.6's ANS ==================================
corresponding variable (theta1, theta2, theta3, theta4, theta5, theta6)
theta4 is out of range!!
   20.0000   20.0000   20.0000  200.0000  -20.0000  200.0000

================== No.7's ANS ==================================
corresponding variable (theta1, theta2, theta3, theta4, theta5, theta6)
theta3 is out of range!!
theta4 is out of range!!
   20.0000  -52.7869  165.2892  171.6767   53.9098 -136.1928

================== No.8's ANS ==================================
corresponding variable (theta1, theta2, theta3, theta4, theta5, theta6)
theta3 is out of range!!
theta4 is out of range!!
   20.0000  -52.7869  165.2892  351.6767  -53.9098   43.8072


============================END=====================================
```

圖十七、後五組解

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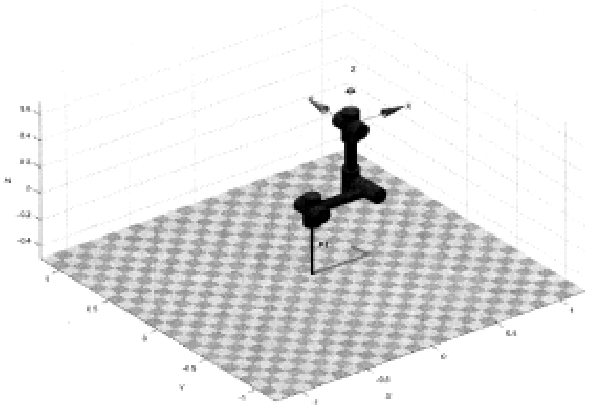
4. 公式推導:



joint	d	a	alpha	theta
1	0	0	-90	0
2	0	0.432	0	0
3	0.149	-0.02	90	0
4	0.433	0	-90	0
5	0	0	90	0
6	0	0	0	0

$$T_6 = \begin{bmatrix} | & | & | & | \\ n & o & a & p \\ | & | & | & | \end{bmatrix}$$

$$A^{-1} T_6 = \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} | & | & | & P_x \\ n & o & a & P_y \\ | & | & | & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1T_6 = \begin{bmatrix} \vdots & \vdots & \vdots & S_{23}d_4 + C_{23}a_3 + a_2C_2 \\ & & & -C_{23}d_4 + S_{23}a_3 + a_2S_2 \\ & & & d_3 \\ & & & 1 \end{bmatrix}$$

$\Rightarrow -S_1P_x + C_1P_y = d_3$, Let $P_x = \rho \cdot \cos\phi$, $P_y = \rho \cdot \sin\phi$ $\Rightarrow \rho = \sqrt{P_x^2 + P_y^2}$, $\phi = \mathrm{Atan2}(P_y, P_x)$

$\Rightarrow C_1 S\phi - S_1 C\phi = \frac{d_3}{\rho} \Rightarrow \sin(\phi - \theta_1) = \frac{d_3}{\rho}$, $\therefore \cos(\phi - \theta_1) = \pm\sqrt{1 - \frac{d_3^2}{\rho^2}}$

兩組解 (-): R (+): L

$\Rightarrow \phi - \theta_1 = \mathrm{atan2}\left(\frac{d_3}{\rho}, \pm\sqrt{1 - \frac{d_3^2}{\rho^2}}\right) \Rightarrow \theta_1 = \mathrm{atan2}(P_y, P_x) - \mathrm{atan2}\left(d_3, \boxed{\pm}\sqrt{P_x^2 + P_y^2 - d_3^2}\right)$

且 $C_1P_x + S_1P_y = S_{23}d_4 + C_{23}a_3 + a_2C_2$ -①

$-P_z = -C_{23}d_4 + S_{23}a_3 + a_2S_2$ -②

$-S_1P_x + C_1P_y = d_3$ -③

$\xRightarrow{①^2+②^2+③^2} P_x^2 + P_y^2 + P_z^2 = d_4^2 + a_3^2 + a_2^2 + 2a_2(d_4S_3 + a_3C_3) + d_3^2$

$\Rightarrow a_3C_3 + d_4S_3 = \frac{P_x^2 + P_y^2 + P_z^2 - d_4^2 - a_3^2 - a_2^2 - d_3^2}{2a_2} = M$

令 $d_4 = \rho\cos\psi$

$a_3 = \rho\sin\psi$

$\Rightarrow \frac{M}{\rho} = C_3S\psi + S_3C\psi = \sin(\psi + \theta_3)$, where $\psi = \mathrm{Atan2}(a_3, d_4)$

$\rho^2 = a_3^2 + d_4^2$

then $\cos(\psi + \theta_3) = \pm\sqrt{1 - (\frac{M}{\rho})^2}$

$\Rightarrow \theta_3 = \mathrm{atan2}\left(\frac{M}{\rho}, \pm\sqrt{1 - (\frac{M}{\rho})^2}\right) - \psi$

$\Rightarrow \theta_3 = \mathrm{atan2}\left(M, \boxed{\pm}\sqrt{a_3^2 + d_4^2 - M^2}\right) - \mathrm{Atan2}(a_3, d_4)$

兩組解 (-) R, a L, b (+) R, b L, a

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$A_1A_2A_3=T_3 \Rightarrow T_3^{-1}T_6={}^3T_6=A_4A_5A_6 \Rightarrow$

$$\begin{bmatrix} C_1C_{23} & S_1C_{23} & -S_{23} & -a_3-a_2C_3 \\ -S_1 & C_1 & 0 & -d_3 \\ C_1S_{23} & S_1S_{23} & C_{23} & -a_2S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} | & | & | & P_x \\ n & o & a & P_y \\ | & | & | & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \vdots & \vdots & C_4S_5 & 0 \\ \vdots & \vdots & S_4S_5 & 0 \\ \vdots & \vdots & C_5 & d_4 \\ \vdots & \vdots & 0 & 1 \end{bmatrix}$$

$\Rightarrow C_1C_{23}P_x+S_1C_{23}P_y-S_{23}P_z=a_3+a_2C_3$

$C_1S_{23}P_x+S_1S_{23}P_y+C_{23}P_z=d_4+a_2S_3 \Rightarrow$

$C_{23}\cdot\underbrace{(C_1P_x+S_1P_y)}_{A}-S_{23}\cdot\underbrace{P_z}_{B}=a_3+a_2C_3$

$C_{23}\cdot\underbrace{P_z}_{B}+S_{23}\cdot\underbrace{(C_1P_x+S_1P_y)}_{A}=d_4+a_2S_3$

Let $A=C_1P_x+S_1P_y$, $B=P_z$, $\Delta=(C_1P_x+S_1P_y)^2+P_z^2$,

$\Rightarrow C_{23}=\dfrac{\begin{vmatrix} a_3+a_2c_3 & -B \\ d_4+a_2S_3 & A \end{vmatrix}}{\Delta}$, $S_{23}=\dfrac{\begin{vmatrix} A & a_3+a_2C_3 \\ B & d_4+a_2S_3 \end{vmatrix}}{\Delta}$

$\theta_{23}=\operatorname{atan2}(S_{23},C_{23})=\operatorname{atan2}\left(\dfrac{(C_1P_x+S_1P_y)\cdot(d_4+a_2S_3)-P_z(a_3+a_2C_3)}{(C_1P_x+S_1P_y)^2+P_z^2},\ \dfrac{(C_1P_x+S_1P_y)\cdot(a_3+a_2C_3)+P_z(d_4+a_2S_3)}{(C_1P_x+S_1P_y)^2+P_z^2}\right)$

$\theta_2=\theta_{23}-\theta_3$.

$\theta_4=\operatorname{atan2}(-S_1a_x+C_1a_y,\ C_1C_{23}\cdot a_x+S_1C_{23}\cdot a_y-S_{23}a_z)$

$$T_4^{-1}T_6={}^4T_6=\begin{bmatrix} C_5C_6 & -C_5S_6 & S_5 & 0 \\ C_6S_5 & -S_5S_6 & -C_5 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \theta_5=\operatorname{atan2}({}^4T_6(1,3),\ -1\cdot{}^4T_6(2,3))$

$\theta_6=\operatorname{atan2}({}^4T_6(3,1),\ {}^4T_6(3,2))$

$\theta_4,\theta_5,\theta_6$ 存在另一組解 $\theta_4',\theta_5',\theta_6'$

where $\theta_4'=\theta_4+180^\circ$

$\theta_5'=-\theta_5$

$\theta_6'=\theta_6+180^\circ$

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若是 θ5=0:

$$-C_4S_6 - S_4C_6 = -S_{46}$$
$$-S_4S_6 + C_4C_6 = C_{46}$$

Choose the previous θ4, Then θ6=θ64-θ4.(但我們實驗中沒有上一個角度)

但是既然因為關節退化，後三軸可以表示為Z軸直接旋轉(Z-Y-Z)，也可以將其中一個設為任意角度，剩下的角度由另一軸分擔，只要不要超過限制即可(在實驗中所使用的方式為直接計算θ4，若θ5=0，則剩下的角度由θ6分擔)。

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5. 討論:逆向運動學(代數法，幾何法)的優缺點

代數法有計算快的優點因為代數法通常使用代數表達式和矩陣運算，而現在的電腦運算速度可以在短時間內算出解；但也因為透過代數和矩陣表達，因此一旦機器人過於複雜，就會難以直觀理解，且在求解時若軸數過，常需要拆解為許多組三軸來計算(因為坐標系是三維空間)。

而幾何法通常基於機器人的幾何結構，因此更容易直觀理解機器人的運動過程，然而幾何法通常適用於特定的幾何形狀和機械結構，並且在某些複雜或特定情況下可能難以應用，例如球型手腕(無法映射至平面中求解)。

以本次使用的puma 560為例，在求解前三軸時兩種方法皆可使用，但如果有辦法準確地得知手臂規格及知臂末端執行器在卡式坐標系中的座標，則透過幾何法求解會更為直觀且方便，而在後三軸的部分，puma 560 設計為球型軸因此直接使用代數法，直接對比Euler-angle 及後三軸的旋轉矩陣可輕易地得到解。