Let $x^k = \alpha + \varepsilon_k$.

Then, $\alpha + \varepsilon_{k+1} = \alpha + \varepsilon_k - \dfrac{f(\alpha+\varepsilon_k)}{f'(\alpha+\varepsilon_k)} - \dfrac{f\left(\alpha+\varepsilon_k - \frac{\alpha+\varepsilon_k}{f'(\alpha+\varepsilon_k)}\right)}{f'(\alpha+\varepsilon_k)}$ ... ①

$\Rightarrow f(\alpha+\varepsilon_k) = f(\alpha) + \dfrac{\varepsilon_k}{1!} f'(\alpha) + \dfrac{\varepsilon_k}{2!} f'(\alpha)$

$f'(\alpha+\varepsilon_k) = f'(\alpha) + \dfrac{\varepsilon_k}{1!} f''(\alpha) + \cdots$

$\Rightarrow \varepsilon_{k+1} = \varepsilon_k - \dfrac{\varepsilon_k f'(\alpha) + \frac{\varepsilon_k^2}{2!} f''(\alpha) + \cdots}{f'(\alpha) + \frac{\varepsilon_k}{1!} f(\alpha) + \cdots} - \dfrac{f(\alpha+\psi)}{f'(\alpha+\varepsilon_k)}$ ... ②

$\psi = \varepsilon_k f'(\alpha) + \dfrac{\varepsilon_k^2}{1!} f'(\alpha) + \dfrac{\varepsilon_k^3}{2!} f'(\alpha) + \cdots - \varepsilon_k f'(\alpha) + \varepsilon_k^2 f'(\alpha) + \dfrac{\varepsilon_k^3}{2!} f'(\alpha) + \cdots$

$\Rightarrow f'(\alpha) + \dfrac{\varepsilon_k}{1!} f''(\alpha) + \cdots$

then $\dfrac{\frac{\varepsilon_k^2}{2} f''(\alpha) + \frac{\varepsilon_k^3}{3} f'''(\alpha) + \cdots}{f'(\alpha)\left(1 + \frac{\varepsilon_k}{1!} f''(\alpha) + \cdots\cdots\right)}$

$= \left(\dfrac{\varepsilon_k^2}{2}\dfrac{f''(\alpha)}{f'(\alpha)} + \dfrac{\varepsilon_k^3}{3}\dfrac{f'''(\alpha)}{f'(\alpha)} + \cdots\right)\left(1 - \dfrac{\varepsilon_k}{1!} f''(\alpha) + \cdots\right)$

$= \varepsilon_k^2 \dfrac{f''(\alpha)}{f'(\alpha)} + \varepsilon_k^3\left(\dfrac{f'''(\alpha)}{3f'(\alpha)} - \dfrac{1}{2}\dfrac{(f''(\alpha))^2}{f'(\alpha)}\right) + \cdots$

$\Rightarrow \varepsilon_{k+1} = \left(\dfrac{\varepsilon_k^2}{2}\dfrac{f''(\alpha)}{f'(\alpha)} + \dfrac{\varepsilon_k^3}{3}\dfrac{f'''(\alpha)}{f'(\alpha)} - \dfrac{\varepsilon_k^3}{2}\dfrac{(f''(\alpha))^2}{f'(\alpha)}\right) - \dfrac{f\left[\alpha + \frac{\varepsilon_k^2}{2}\frac{f''(\alpha)}{f'(\alpha)} + \frac{\varepsilon_k^3}{3}\frac{f'''(\alpha)}{f'(\alpha)} - \frac{\varepsilon_k^3}{2}\frac{f''(\alpha)^2}{f'(\alpha)}\right]}{f'(\alpha)\left(1 + \frac{\varepsilon_k}{1!} f''(\alpha) + \cdots\right)}$

$= \dfrac{\varepsilon_k^2}{2}\dfrac{f''(\alpha)}{f'(\alpha)} + \dfrac{\varepsilon_k^3}{3}\dfrac{f'''(\alpha)}{f'(\alpha)} - \dfrac{\varepsilon_k^3}{2}\dfrac{(f''(\alpha))^2}{f'(\alpha)} - \dfrac{f(\alpha) + \frac{\varepsilon_k^2}{2}\frac{f''(\alpha)}{f'(\alpha)} + \varepsilon_k^3\left(\frac{f'''}{3f'} - \frac{f''^2}{2f'}\right)}{f'(\alpha)\left(1 + \frac{\varepsilon_k}{1!} f'' + \cdots\right)}$

$= \dfrac{\varepsilon_k^2}{2}\dfrac{f''}{f'} + \dfrac{\varepsilon_k^3}{3}\dfrac{f'''}{f'} - \dfrac{\varepsilon_k^3 (f'')^2}{2f'} - \dfrac{\varepsilon_k^2 f''}{2f'} - \dfrac{\varepsilon_k^3}{3}\left(\dfrac{f'''}{3f'} - \dfrac{(f'')^2}{2f'}\right) + \dfrac{\varepsilon_k^3}{1!}\dfrac{f''(\alpha)}{f'(\alpha)} f'' + \cdots$

$\Rightarrow \varepsilon_{k+1} = \varepsilon_k^3 \dfrac{(f''(\alpha))^2}{f'(\alpha)} + \cdots$

Therefore, the convergence order of the sequence is 3.