Computational methods and applications (AMS 147)

Homework 5 - Due Sunday, March 4

Please submit to CANVAS a .zip file that includes the following Matlab functions:

`int_midpoint_rule.m`

`int_trapezoidal_rule.m`

`int_Simpson_rule.m`

`test_integration.m`

Exercise 1 Consider the following integral of a function $f(x)$ on a finite interval $[a, b]$

$$I(f) = \int_a^b f(x)dx. \tag{1}$$

Write three Matlab/Octave functions implementing, respectively, the composite midpoint rule, the composite trapezoidal rule, and the composite Simpson rule to compute the numerical approximation of $I(f)$. Such functions should be of the form

`function [I]=int_midpoint_rule(fun,a,b,n)`	(composite midpoint rule)
`function [I]=int_trapezoidal_rule(fun,a,b,n)`	(composite trapezoidal rule)
`function [I]=int_Simpson_rule(fun,a,b,n)`	(composite Simpson rule)

Input:

`fun`: function handle representing $f(x)$

`a,b`: endpoints of the integration interval

`n`: number of evenly-spaced points in $[a, b]$ (including endpoints)

$$x_j = a + (j-1)h, \qquad h = \frac{b-a}{n-1}, \qquad j = 1, ...n.$$

Output:

`I`: numerical approximation of the integral (1).

Exercise 2 Use the functions you coded in Exercise 1 to compute the numerical approximation of the integral

$$I = \int_{-3}^{1} \left[\frac{1}{1+x^2} \cos\left(\frac{3}{2}e^{-x^2}\right) - \frac{x^3}{30} \right] dx. \tag{2}$$

To this end, write a Matlab/Octave function

```
[em,et,es] = function test_integration()
```

that returns the following items:

- `en`, `et`, `es`: row vectors with components the absolute values of the integration errors

$$|I_{\text{ref}} - I_n| \qquad n = 2, 3, ..., 10000 \tag{3}$$

  obtained with the midpont (vector `en`), trapezoidal (vector `et`) and Simpson (vector `es`) rules. Here,

$$I_{\text{ref}} = 1.6851344770476$$

  is the reference value of the integral (2) while I_n is the numerical approximation obtained by using the composite midpoint, trapezoidal, and Simpson rules with n nodes.

- The function `test_integration()` should also return the plot of the integrand function appearing in (2) in `figure(1)`, and the plots of the errors `en`, `et` and `es` versus n in a log-log scale in `figure(2)` (one figure with three plots). (Hint: use the Matlab command `loglog()`).