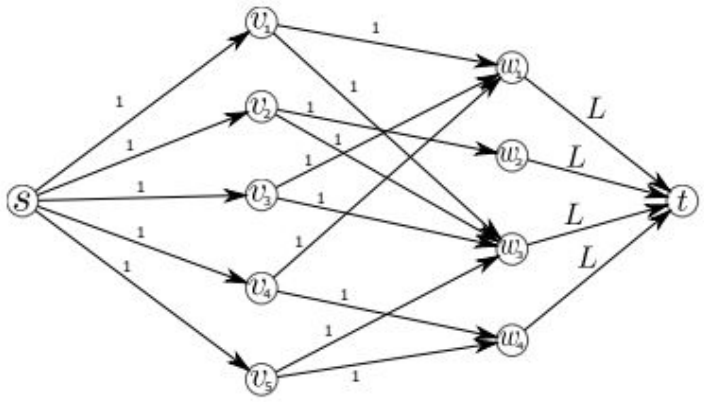
CMPS102 HW4

Jinxuan Jiang

Question 1:

Solution:

We build the following flow network.



Let node Vi denote each restaurant i.

Let node Wj denote each warehouse j.

Let edge (Vi, Wj) of capacity 1 if Distance(Vi, Wj) < m miles.

Then, we connect super-source s to each of the restaurant nodes by an edge of capacity 1 and we connect each of the warehouse nodes to a super-sink t by an edge of capacity L.

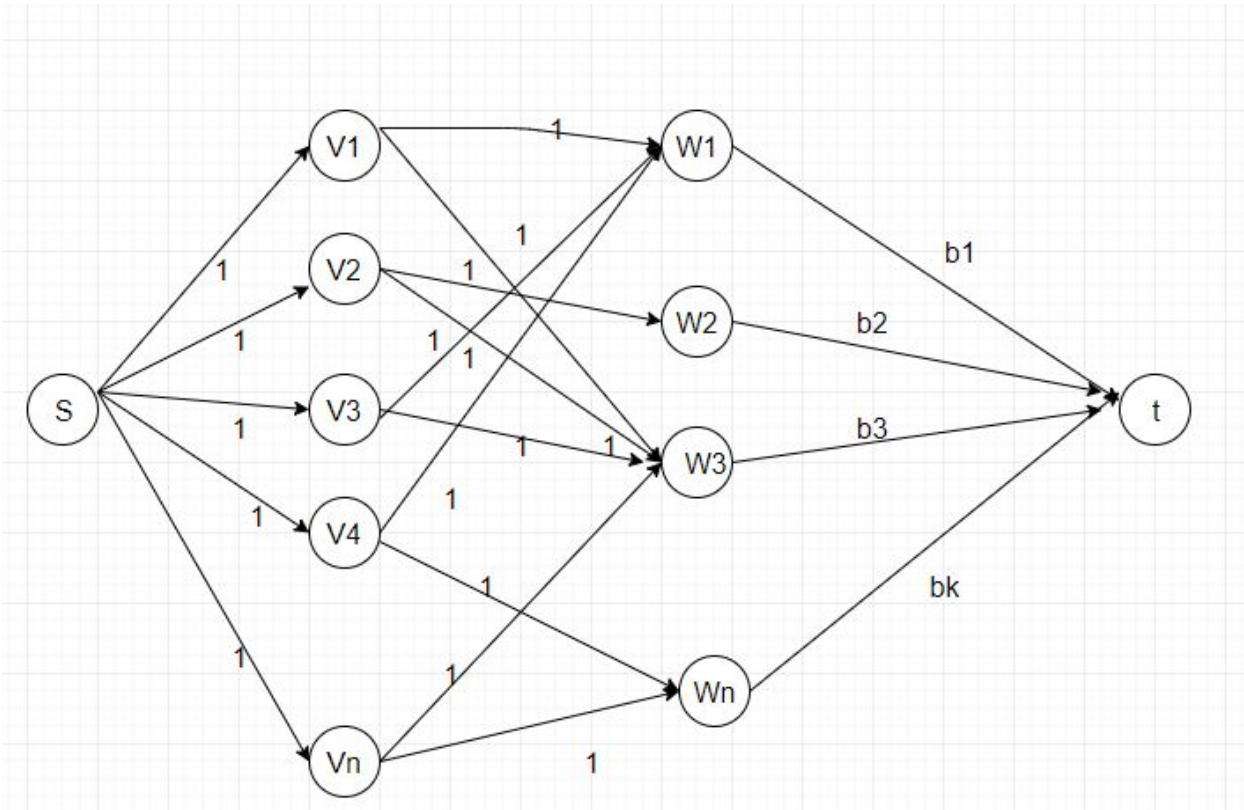
We claim that there is a feasible way to connect all restaurants to warehouse if and only if there is an (s-t) flow of value n. If there is a feasible connection, then we send one unit of flow from s to t along each of paths s, Vi, Wj, t, where restaurant i is supplied by warehouse j. This does not violate the capacity condition in particular on the edges (Wj,

t), due to the load constraints. Conversely, if there is a flow of value n, then by Integrality Theorem, there is one with integer values. We connect restaurant i to warehouse j if edge (Vi, Wj) carries one unit of flow, and we observe that the capacity condition ensures that no warehouse will supply more than c restaurants. The running time is the time required to solve a max-flow problem on a graph with O(n+k) nodes and O(nk) edges.

Question 2:

Solution:

We build the following flow network.



Let node Vi denote each student i.

Let node Wj denote each class j.
Let edge (Vi, Wj) of capacity 1 if student i can attend class j.
Let B denote the number of classes.
Then, we connect super-source s to each of the student nodes Vi by an edge of capacity 1 and connect class nodes Wj to the super-sink t by an edge of capacity bj.

We claim that there is a feasible way to connect all the students to classes if and only if there is an s-t flow of value n. If there is a feasible way, then we send one unit of flow from s to t along each of the paths s, Vi, Wj, t, where student i goes to class j. This does not violate the capacity condition in particular on the edges (Wj ,t), due to the load constraints. Conversely, if there is a flow of value n, then by Integrality Theorem, there is one with integer values. We connect student i to class j if the edge (Vi, Wj) carries one unit of flow, and we observe that the capacity condition ensures that no class is overloaded. The running time is the time required to solve a max-flow problem on a graph with O(n+B) nodes and O(nB) edges.

For Wei-Lin's sauce-squirter class, the solution is exactly the same as above because his deal is the same as Kostas's.

Question 3:
Solution:

Question 4.1
Solution:
Let K denote the flow of arbitrary edge in maximum flow solution.
Then, we reduce the flow of this edge to U = max{ K-X, 0}.

At the same time, the flows on all the relevant edges are reduced to U.
Obviously, the total flow G' = maxflow(G) - (K - U) ≥ maxflow(G) - X.
Therefore, the maxflow(G') ≥ maxflow (G) - X.

Question 4.2

Solution:

If (a) is always true, every minimum s-t cut of G contains e*, then the minimum capacity of the cuts cannot be an integer. As we know, the max-flow min-cut theorem states that in a flow network, the maximum amount of flow passing from the *source* to the *sink* is equal to the total weight of the edges in the minimum cut. Since the minimum capacity of the cuts cannot be an integer, the maximum flow in G cannot be an integer flow value either, which is contradicted to (b).

If (b) is always true (there is a maximum flow in G where every edge in G has an integral flow value), the maximum flow must be an integer and the minimum capacity of cuts must be an integer too.
For the reason that all the edges but e* must have integer values, the minimum s-t cut of G cannot contain e*, which is contradicted to (a).