Woman Pessimality

Woman-pessimal assignment. Each woman receives worst valid partner.

Claim. GS finds woman-pessimal stable matching S*.

Pf.

- Suppose A-Y matched in S*, but Y is not worst valid partner for A.
- There exists stable matching S in which A is paired with a man, say Z, whom she likes less than Y.
- Let B be Y's partner in S.
- Y prefers A to B. ← man-optimality
- Thus, A-Y is unstable in S. ▪

S
Amy-Zeus
Bertha-Yancey
. . .

Deceit: Machiavelli Meets Gale-Shapley

Q. Can there be an incentive to misrepresent your preference profile?

- Assume you know men's propose-and-reject algorithm will be run.
- Assume that you know the preference profiles of all other participants.

Fact. No, for any man yes, for some women. No mechanism can guarantee a stable matching and be cheatproof.

	1st	2nd	3rd
Xavier	A	B	C
Yancey	B	A	C
Zeus	A	B	C

Men's Preference List

	1st	2nd	3rd
Amy	Y	X	Z
Bertha	X	Y	Z
Clare	X	Y	Z

Women's True Preference Profile

	1st	2nd	3rd
Amy	Y	Z	X
Bertha	X	Y	Z
Clare	X	Y	Z

Amy Lies

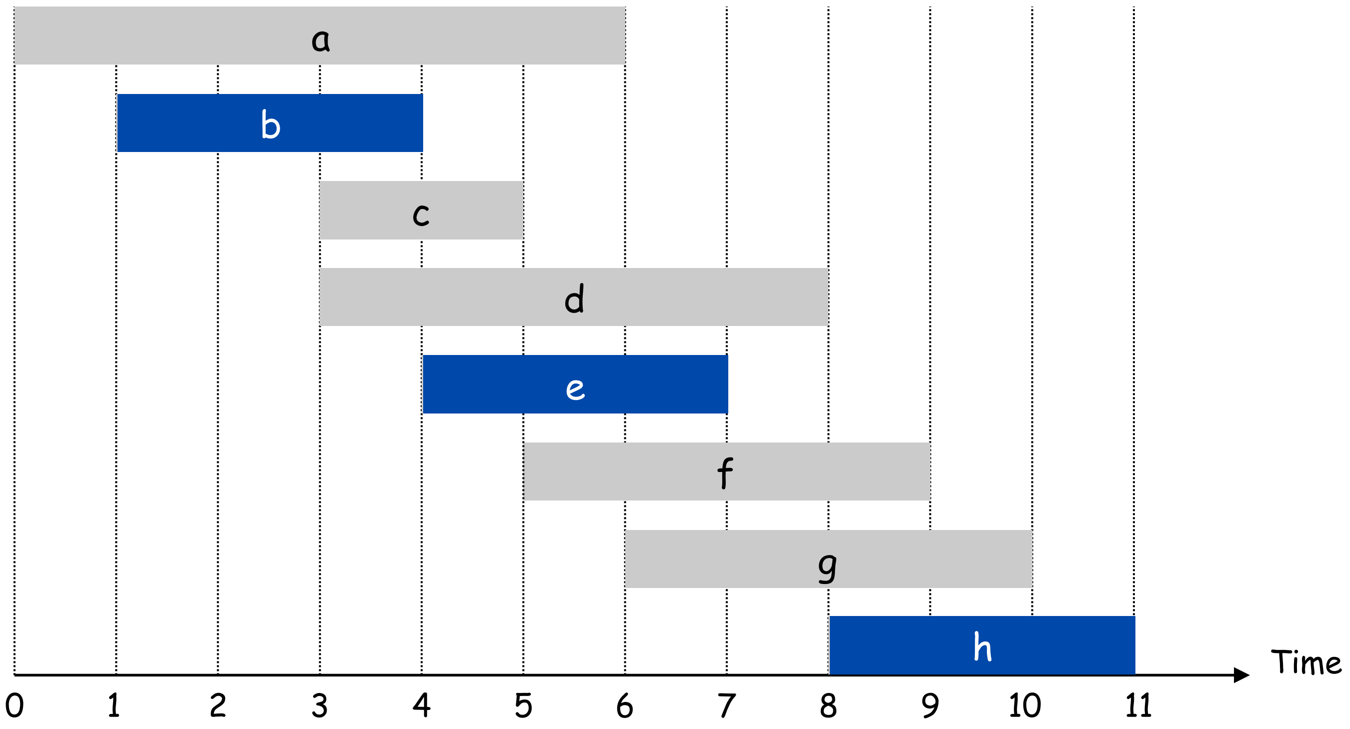
1.2 Five Representative Problems

Interval Scheduling

Input. Set of jobs with start times and finish times.

Goal. Find maximum cardinality subset of mutually compatible jobs.

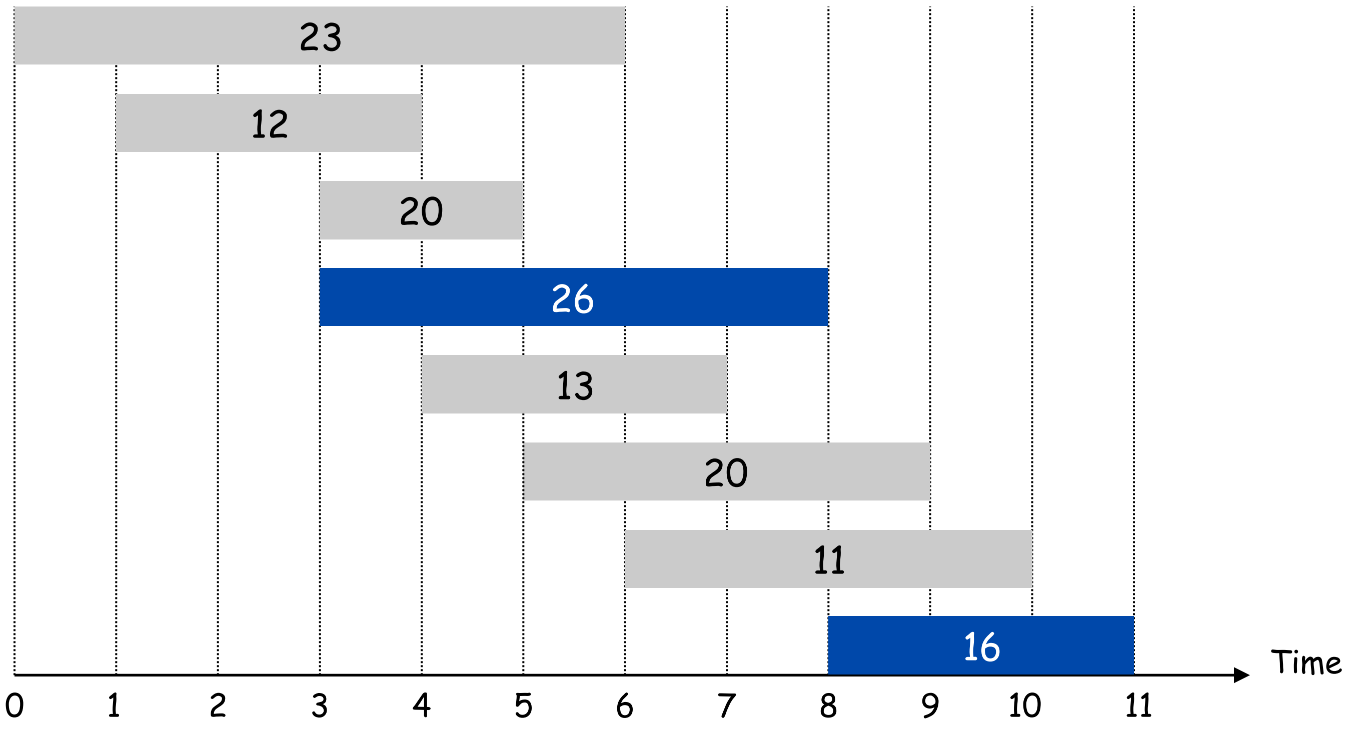
↑ jobs don't overlap



Weighted Interval Scheduling

Input. Set of jobs with start times, finish times, and weights.

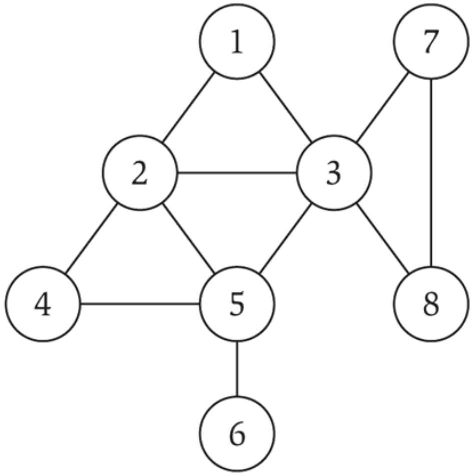
Goal. Find maximum weight subset of mutually compatible jobs.



Undirected Graphs

Undirected graph. G = (V, E)

- V = nodes.
- E = edges between pairs of nodes.
- Captures pairwise relationship between objects.
- Graph size parameters: n = |V|, m = |E|.



V = { 1, 2, 3, 4, 5, 6, 7, 8 }

E = { 1-2, 1-3, 2-3, 2-4, 2-5, 3-5, 3-7, 3-8, 4-5, 5-6 }

n = 8

m = 11

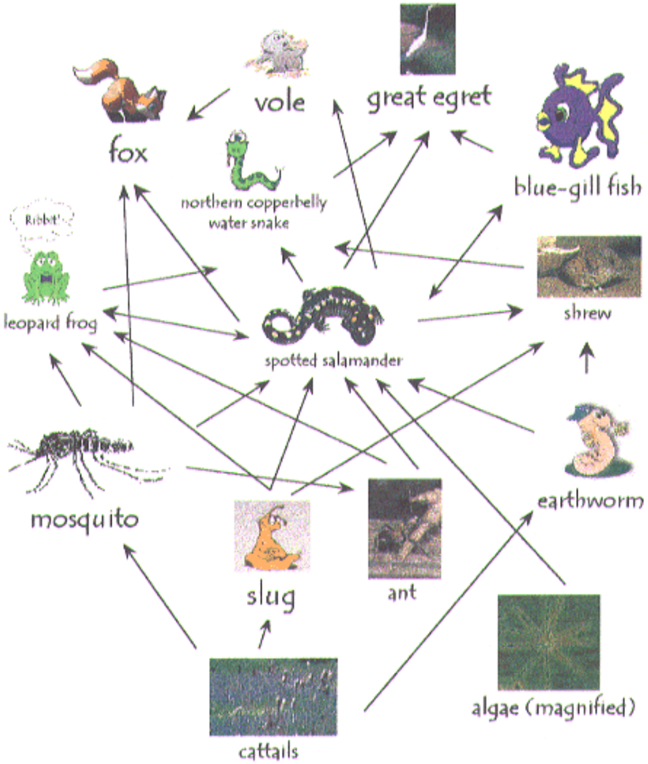
Some Graph Applications

Graph	*Nodes*	*Edges*
transportation	street intersections	highways
communication	computers	fiber optic cables
World Wide Web	web pages	hyperlinks
social	people	relationships
food web	species	predator-prey
software systems	functions	function calls
scheduling	tasks	precedence constraints
circuits	gates	wires

Ecological Food Web

Food web graph.

- Node = species.
- Edge = from prey to predator.

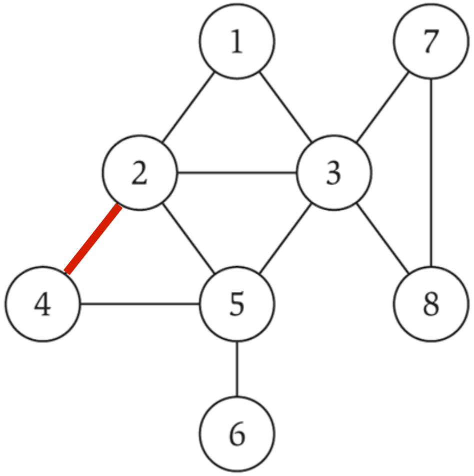


Reference: http://www.twingroves.district96.k12.il.us/Wetlands/Salamander/SalGraphics/salfoodweb.giff

Graph Representation: Adjacency Matrix

Adjacency matrix. n-by-n matrix with A_{uv} = 1 if (u, v) is an edge.

- Two representations of each edge.
- Space proportional to n^2.
- Checking if (u, v) is an edge takes $\Theta(1)$ time.
- Identifying all edges takes $\Theta(n^2)$ time.



	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	0	0
2	1	0	1	1	1	0	0	0
3	1	1	0	0	1	0	1	1
4	0	1	0	0	1	0	0	0
5	0	1	1	1	0	1	0	0
6	0	0	0	0	1	0	0	0
7	0	0	1	0	0	0	0	1
8	0	0	1	0	0	0	1	0

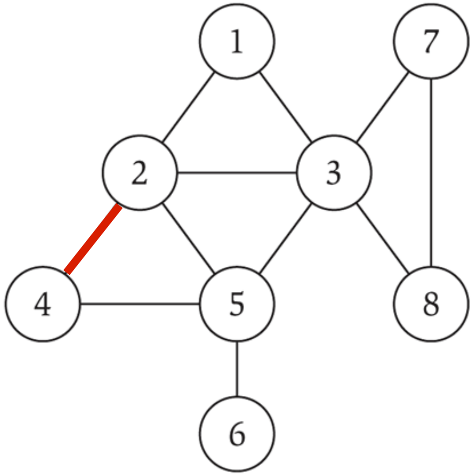
Graph Representation: Adjacency List

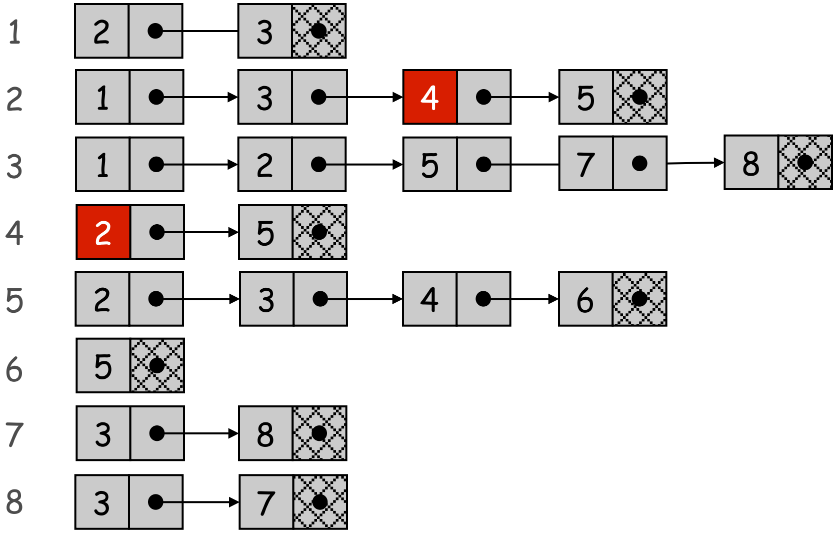
Adjacency list. Node indexed array of lists.

- Two representations of each edge.
- Space proportional to m + n.

degree = number of neighbors of u

- Checking if (u, v) is an edge takes O(deg(u)) time.
- Identifying all edges takes Θ(m + n) time.




1 2 3

2 1 3 4 5

3 1 2 5 7 8

4 2 5

5 2 3 4 6

6 5

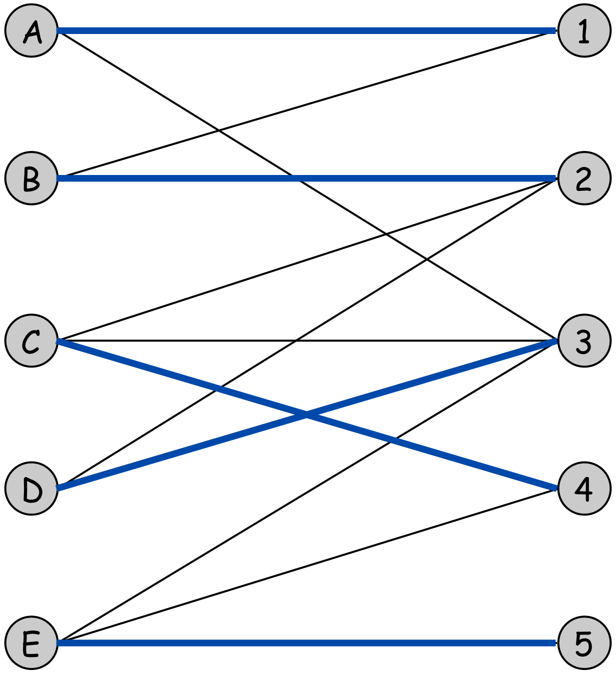
7 3 8

8 3 7

Bipartite Matching

Input. Bipartite graph.

Goal. Find maximum cardinality matching.

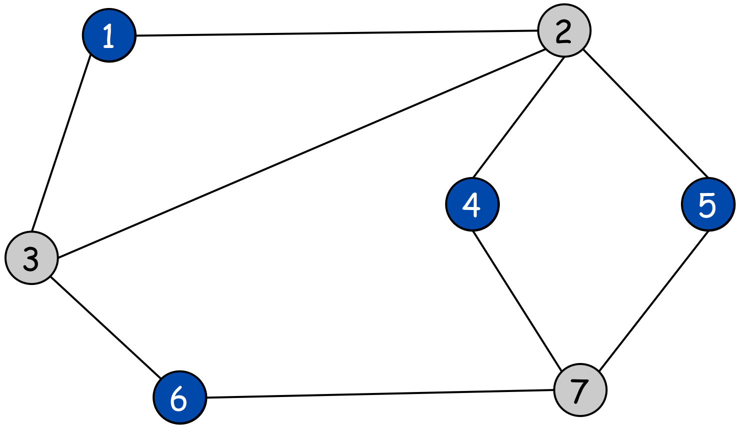


Independent Set

Input. Graph.

Goal. Find maximum cardinality independent set.

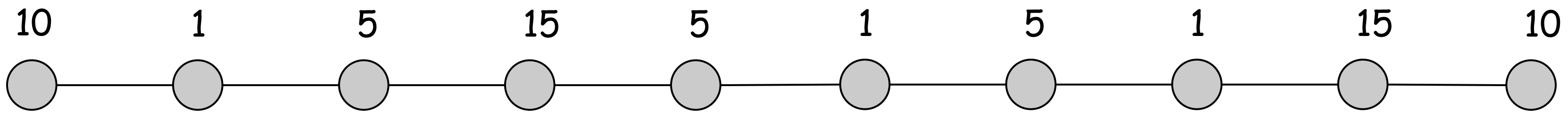
subset of nodes such that no two joined by an edge



Competitive Facility Location

Input. Graph with weight on each each node.
Game. Two competing players alternate in selecting nodes. Not allowed to select a node if any of its neighbors have been selected.

Goal. Select a maximum weight subset of nodes.



Second player can guarantee 20, but not 25.

Five Representative Problems

Variations on a theme: independent set.

Interval scheduling: n log n greedy algorithm.

Weighted interval scheduling: n log n dynamic programming algorithm.

Bipartite matching: n^k max-flow based algorithm.

Independent set: NP-complete.

Competitive facility location: PSPACE-complete.