Copula Models of Economic Capital for Life Insurance Companies

Sydney Benson, Jessica Mohr, Regina Burroughs and Thomas Vlasak

Advisors: Arkady Shemyakin and Huan Zhang

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Outline

1. Background
   - The Problem
   - The Approach
2. Assets
   - The Variables
   - Modeling
3. Liabilities
4. Product

The Problem

- Economic Capital
  - The amount of **capital** that a firm needs to ensure that the company stays solvent over a certain period with a certain probability
  - $TotalLoss = InvestmentLoss + InsuranceLiabilities - Capital$
- Economic variables are notoriously difficult to forecast
- Forecasting economic variables is essential for insurance companies

We believe copula models will help us better understand dependencies between economic variables.

Reserving

- Collect premium
- Invest premium
- Estimate liabilities
- Pay claims

Problems with Forecasting

- Assumption of independence
- Correlation
- Adequate tails

Top Level Solvency II Correlation Matrix

	1	2	3	4	5
1. Market Risks SCR	1	0.25	0.25	0.25	0.25
2. Credit Risks SCR	0.25	1	0.25	0.25	0.5
3. Life Insurance Risks SCR	0.25	0.25	1	0.25	0
4. Health Insurance SCR	0.25	0.25	0.25	1	0
5. Non-Life Insurance SCR	0.25	0.5	0	0	1

Our Approach

Assets

- Corporate bonds
- Mortgage backed securities
- High yield (junk bonds)

Liabilities

- Whole life product
- Disability product
- Losses from lapse

Bloomberg Barclay Indices

Name	Symbol
US Mortgage Backed Securities Index	LUMSTRUU
US Corporate Bond Index	LUACTRUU
US Corporate High Yield Bond Index	LF98TRUU
Emerging Markets Bond Index	EMUSTRUU
US Commercial Mortgage Backed Securities Index	LUCMTRUU
US Aggregate 3-5 Year	LU35TRUU
US Aggregate 5-7 Year	LU57TRUU
US Aggregate 7-10 Year	LU71TRUU
US 1-5 Year Corporate Bond Index	LU13TRUU

Table: Selected Bloomberg Barclay Indices

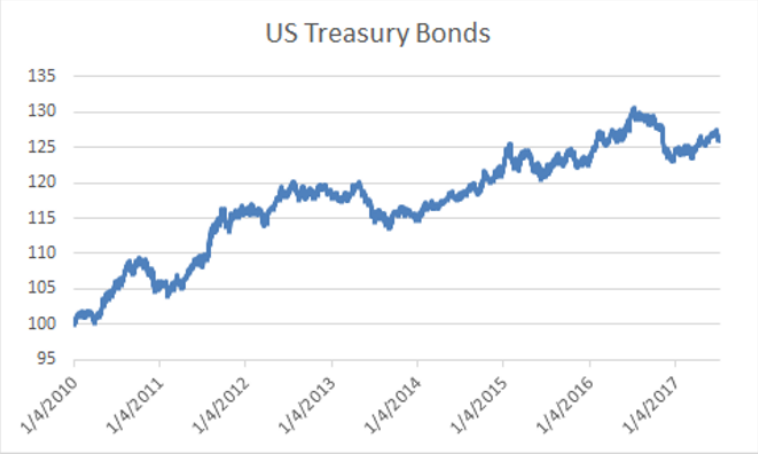
Data Processing and Cleaning

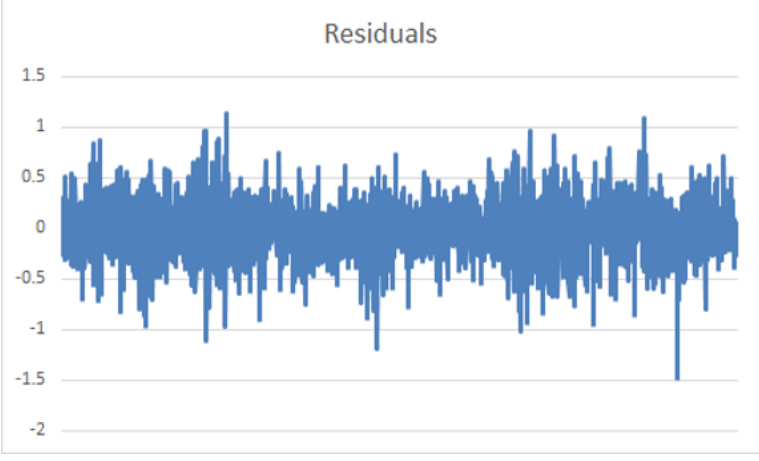
- Convert daily prices to monthly averages
- Log returns
- Deal with trends and autocorrelation

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$$

$$u_i - u_{i-1} = u_{i-1} + \phi_1(u_{i-1} + u_{i-2}) + E$$

Data Processing and Cleaning Visual





Copula Models

- Multivariate probability distribution
- $P(X \leq x, Y \leq y) = C_\alpha[F(x), G(y)]$
- Types
  - Gaussian
  - Student T
  - Arichimedean

Copula Models

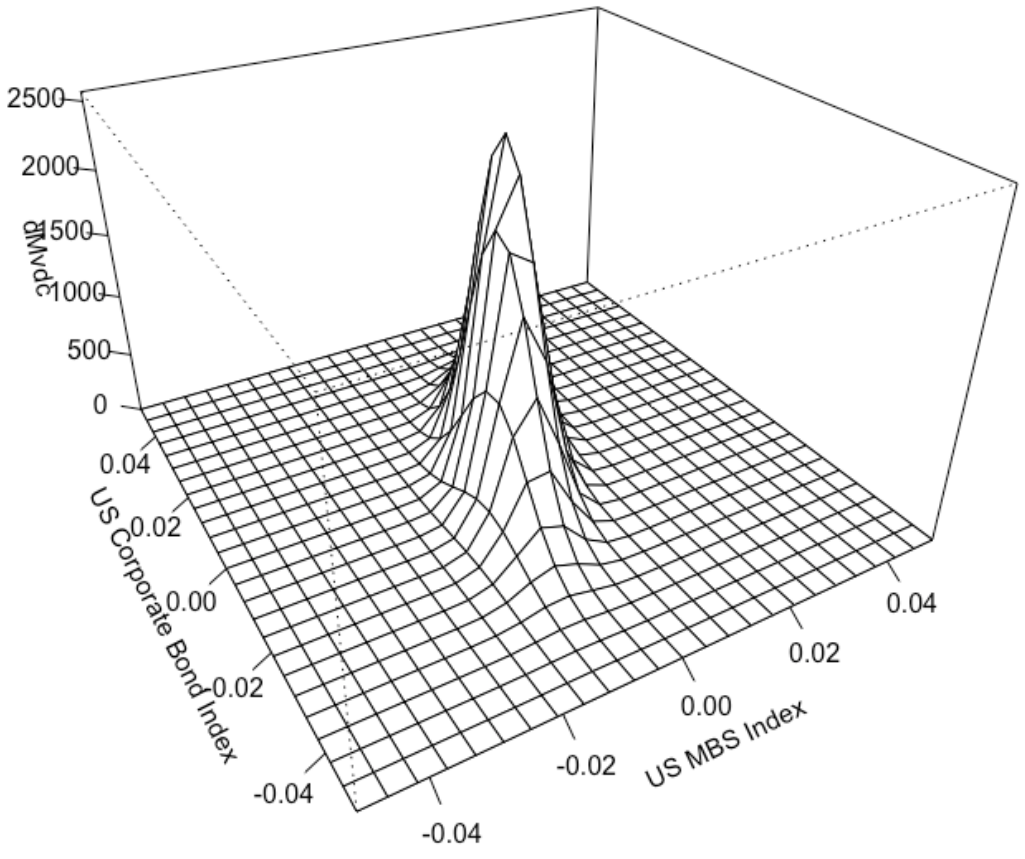


Figure: Perspective Plot: Gaussian Copula, Normal Marginal Distributions

Copula Models

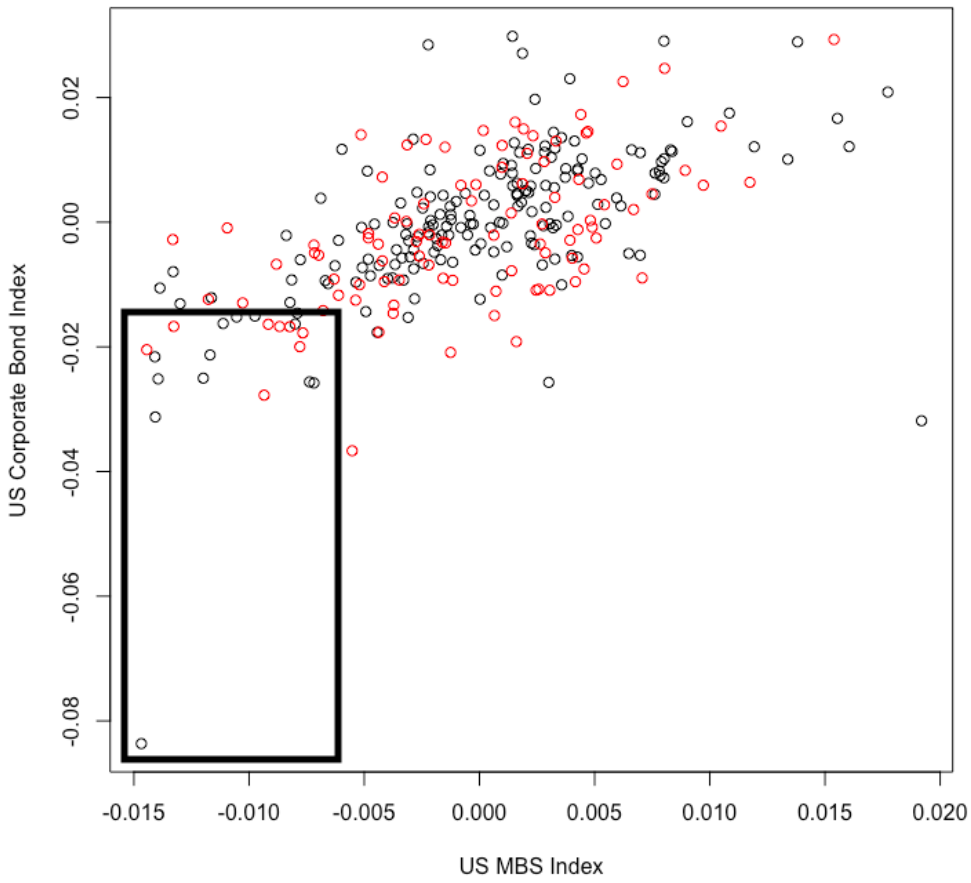


Figure: Simulated Gaussian, Normal (red) vs. Real Data (black)

Choosing Marginal Distributions

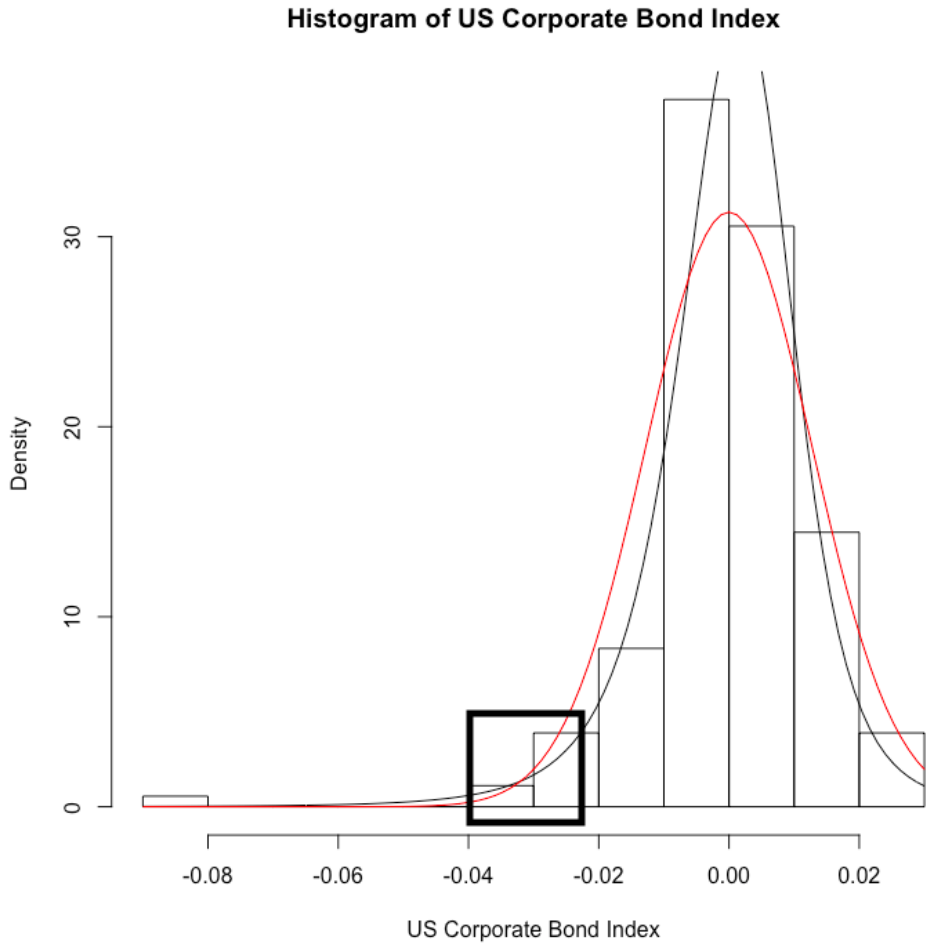


Figure: Normal (red) vs. Skew Student T (black) Marginal Distribution

Other Copula Models

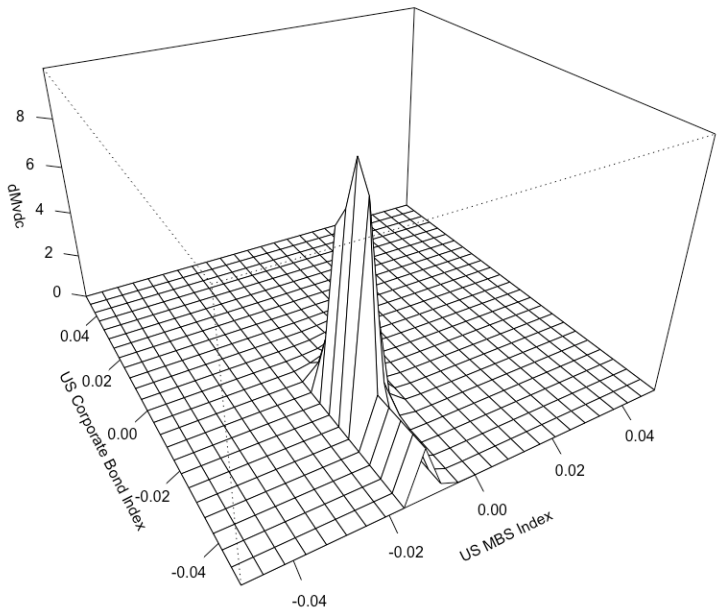


Figure: Clayton Archimedean Copula, Empirical Marginal Distributions

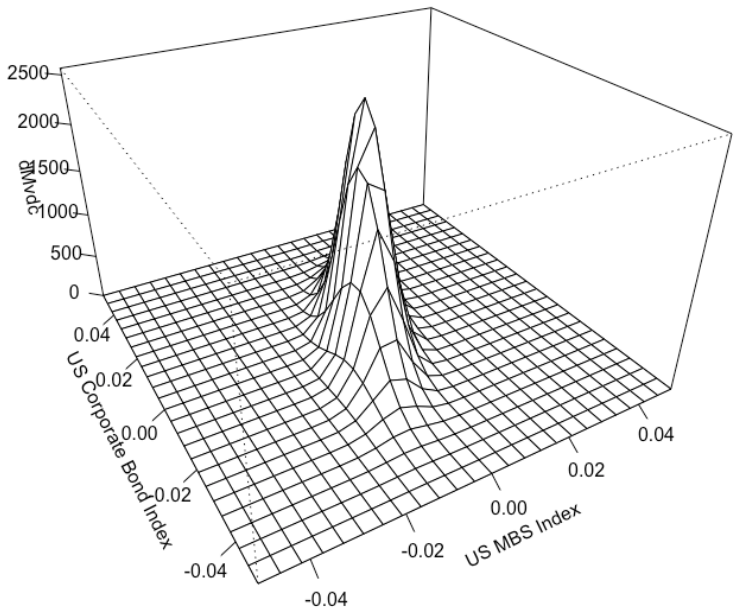


Figure: Gaussian Copula, Normal Marginal Distributions

Archimedean Copula

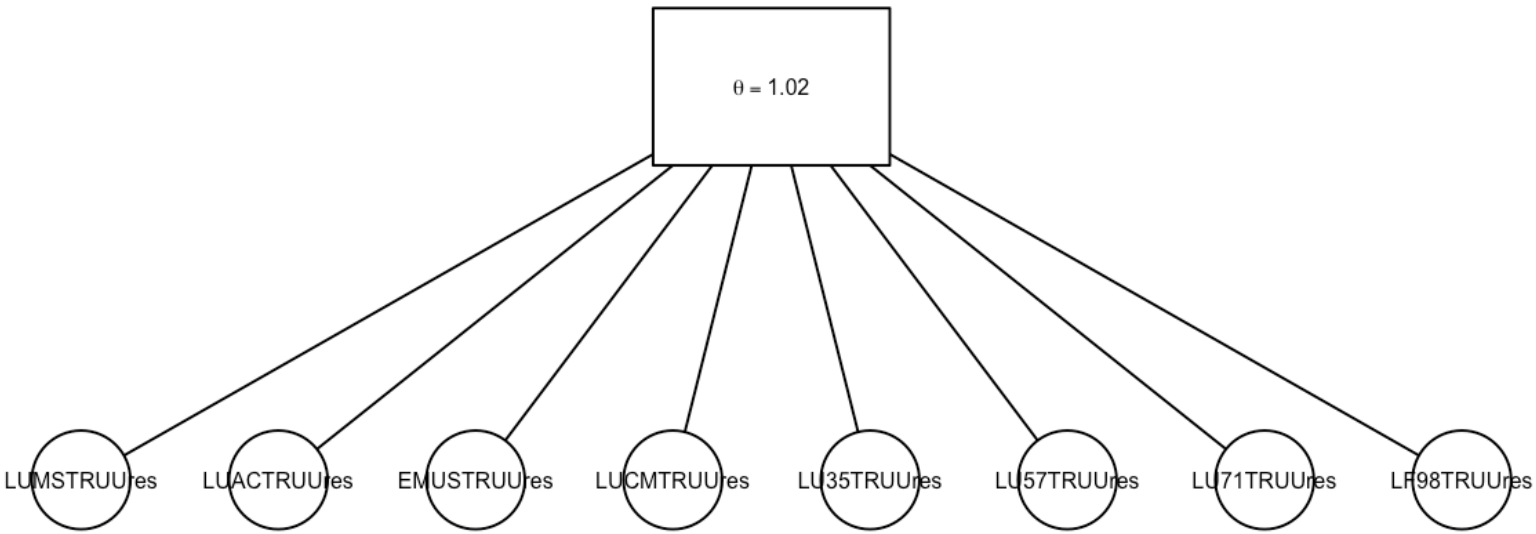


Figure: Clayton Archimedean Copula

Hierarchical Archimedean Copula

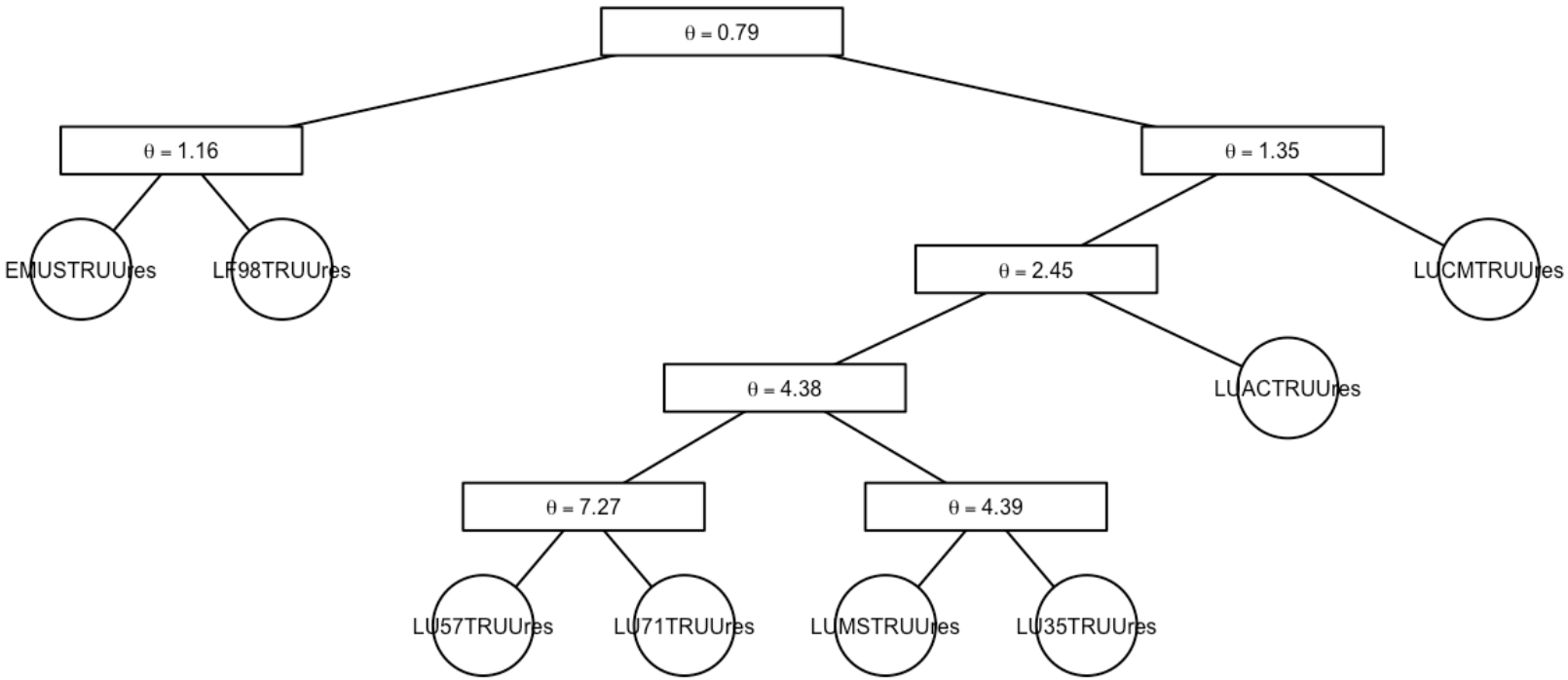


Figure: Clayton Hierarchical Archimedean Copula

Vine Archimedean Copula

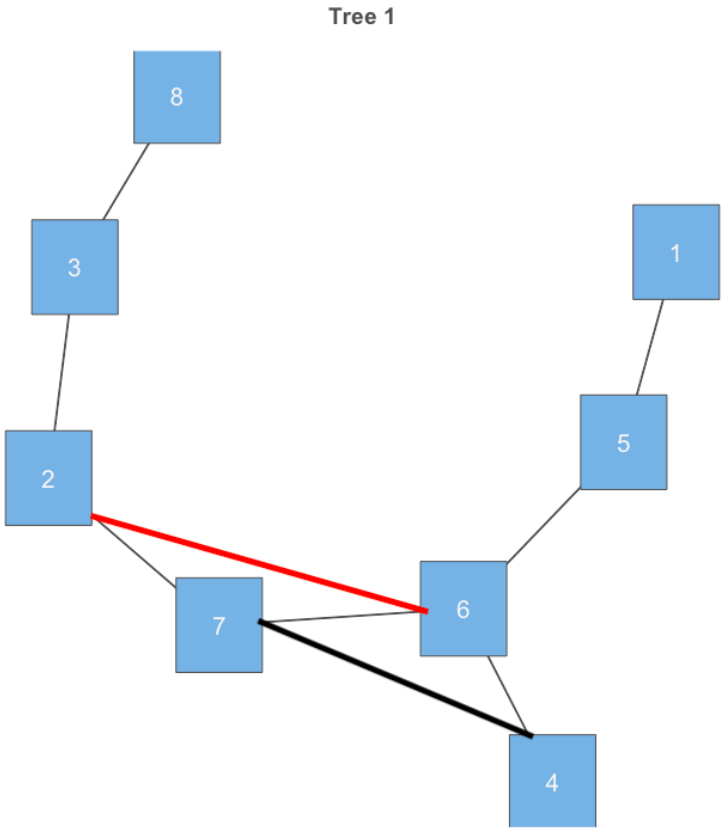


Figure: First Tree from Clayton Vine Copula

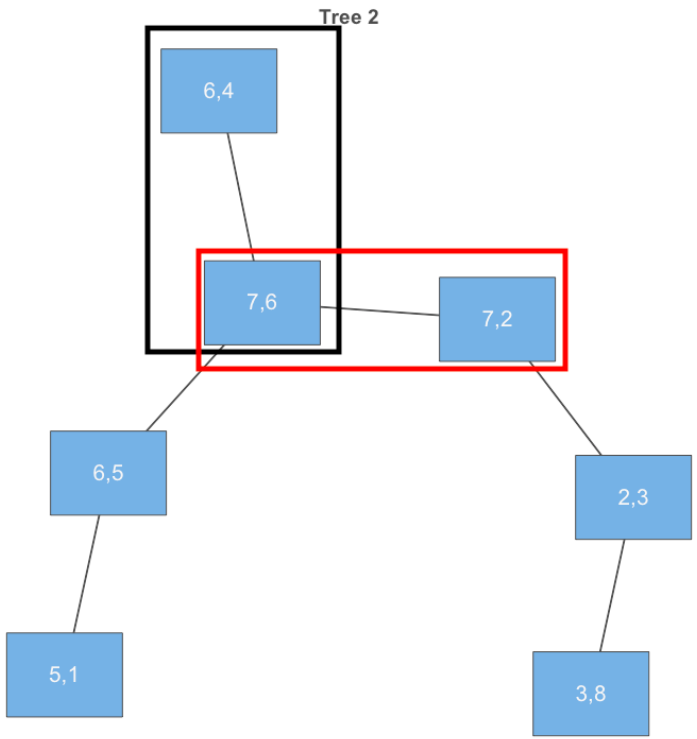


Figure: Second Tree from Clayton Vine Copula

Comparing the Models (All Indices)

Copula	**Marginal Distribution**	**AIC**
Gaussian	Normal	-12775.12
Gaussian	Skew Student T	-12847.88
Student T	Normal	-12381.67
Student T	Skew Student T	-12984.65
Clayton	Normal	-10814.02
Clayton	Skew Student T	-11556.39

Future Work with Copulas

- Expanding R packages for modeling
  - Skew student T copulas
  - Using skew student T marginal distributions with hierarchical archimedean copulas
- Including liabilities in the model

Liabilities

- Best Estimate Data
- By Product and Cause

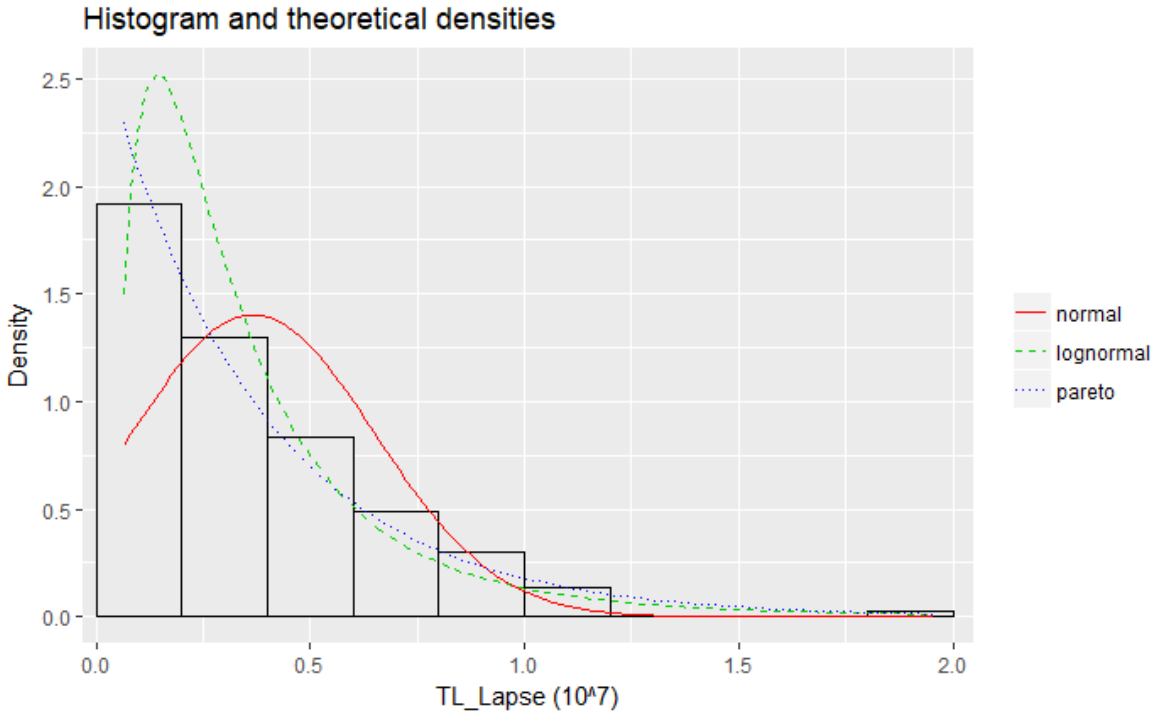
Products

- Whole life
- Term life
- Disability

Causes

- Mortality
- Morbidity
- Lapse

Distribution Fitting

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Goodness-of-fit criteria
                                  normal lognormal    pareto
Akaike's Information Criterion 63.75188 -35.11644 -30.02098
Bayesian Information Criterion 70.19259 -28.67573 -23.58026
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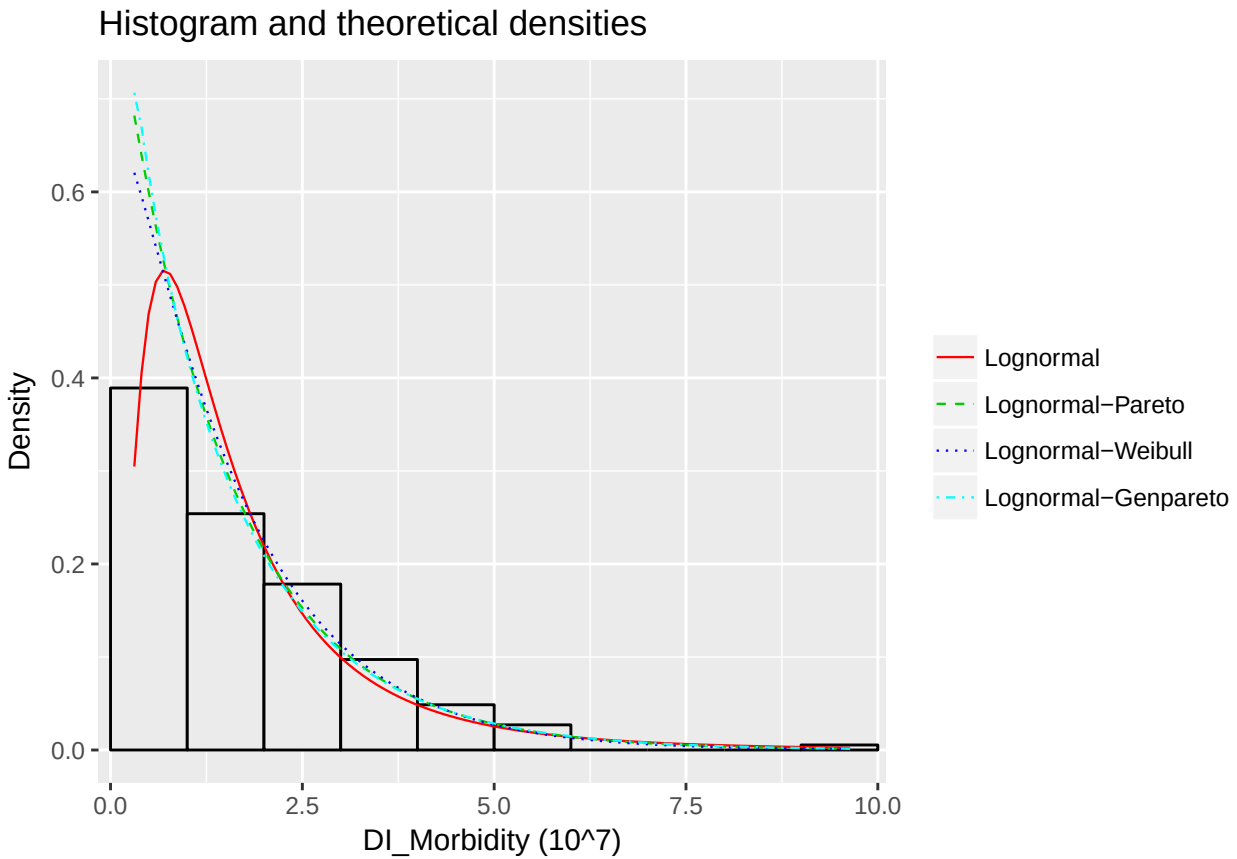
Composite Models



Goodness-of-fit criteria

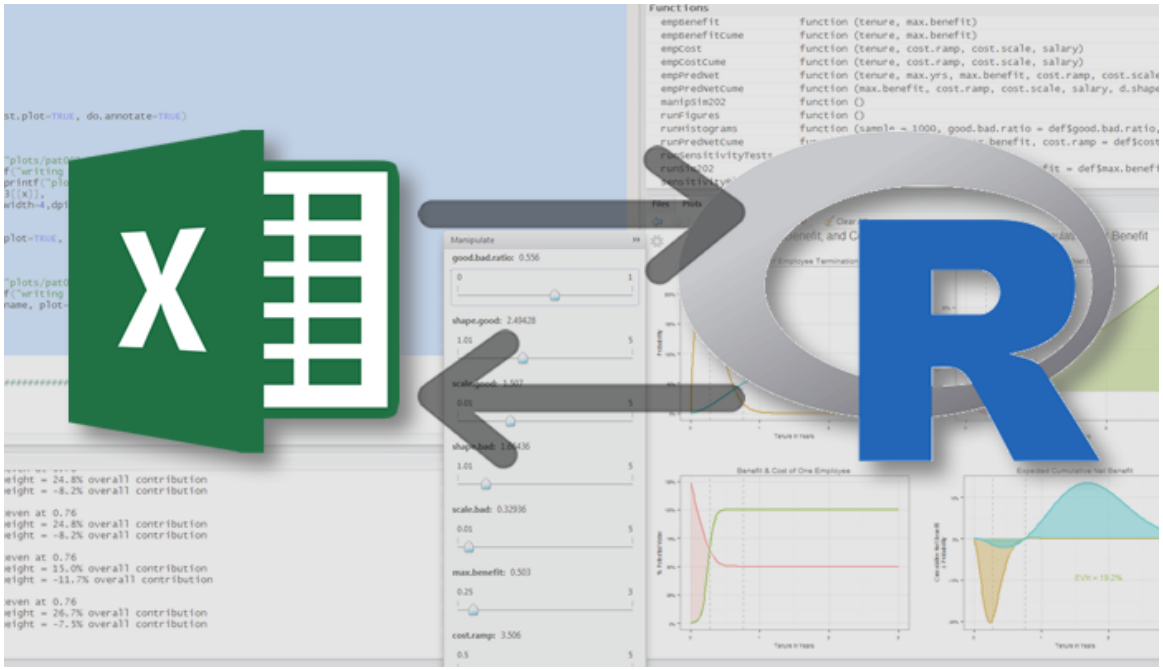
	Lognormal	Lognormal-Pareto	Lognormal-Weibull	Lognormal-Genpareto
Akaike's Information Criterion	-35.11644	-62.20491	-62.90708	-35.54132
Bayesian Information Criterion	-28.67573	-49.32349	-50.02565	-19.43954

Final Product

- Expected customer: insurance company
- User friendly product

Connecting R and Excel

- RExcel
- BERT
- XLConnect
- xlsx package
- User friendly R code (function)



Questions?