Generalized Linear Mixed Models and Practical Applications in R

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Rose-Hulman Undergraduate Mathematics Conference

Overview

Generalized linear mixed models can overcome limitations of other modeling techniques

- Response distribution
- Correlated data

Parallel computing can increase the speed of computation

Linear Models

Assumptions:

- Independent responses
- Normally distributed responses
- Responses have equal variances

Linear Model Issues

What if our responses are not normally distributed?

- Log odds of your favorite sports team winning a game (binomial)
- Log mean number of students per class at your university (Poisson or negative binomial)

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Use a linear mixed model (LMM)!

- Differences between "grouped" data are "fixed" effects
- Differences within "grouped" data are "random" effects

Linear Mixed Models

What are "random" effects?

Random variables, usually normally distributed with a mean 0

Random effects are non-observable, but can be estimated by parameters

Variance component(s): variance(s) of random effects

Parameters for linear mixed models:

- Fixed effects
- Variance components

Generalized Linear Mixed Models

What if responses are not normally distributed and correlated?

Use a generalized linear mixed model (GLMM)!

- Combination of GLM and LMM
- Include random effects to account for correlation
- Model log odds or log mean

Salamander Example

Salamanders: single species from 2 different locations

- Rough Butt (R)
- White Side (W)

Do salamanders prefer to mate with others from same location?

Salamander Example

Correlated: each salamander was mated with multiple others

Unmeasurable: salamanders have personalized tendencies to mate

Assumption: each salamander's tendency is independent

Salamander Example

What affects the probability that a pair of salamanders will mate?

- Type of cross (RR, RW, WR, WW)
- The female's mating tendency
- The male's mating tendency

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Fixed effect: type of cross

Random effect: salamander mating tendencies

Salamander Example

Translation to statistical modeling:

- Response: whether or not the pair mated
- Fixed effects: β_{RR}, β_{RW}, β_{WR}, β_{WW} (log odds of mating)
- Random effects: each salamander (independent, normal)
- Variance components: σ_F^2 and σ_M^2

Using glmm

```
> sal <- glmm(Mate ~ 0 + Cross,
    random = list( ~ 0 + Female, ~ 0 + Male ),
    varcomps.names = c( "F" , "M" ),
    data = salamander,  m = 10^4,
    family.glmm = bernoulli.glmm)
```

Using glmm

```
Fixed Effects:
          Estimate Std. Error z value Pr(>|z|)
CrossR/R    1.4629     0.2720   5.378 7.53e-08 ***
CrossR/W    0.3781     0.2527   1.496 0.134612
CrossW/R   -1.7398     0.3157  -5.512 3.55e-08 ***
CrossW/W    1.0345     0.2683   3.857 0.000115 ***
```

Using glmm

We can translate the log odds back to probabilities:

$$\text{P(mating)} = \frac{\exp\left(\hat{\beta}_{RW}\right)}{1 + \exp\left(\hat{\beta}_{RW}\right)}$$

Cross	RR	WW	RW	WR
Probability of mating	0.812	0.798	0.584	0.149

Parallel Computing

1 person doing 12 calculations

versus

4 people doing 3 calculations each

Splitting the computation work of a function amongst the cores available in the computing device

Most computers have 4 or more cores

Leads to increased computation speed

Parallel Computing

How can we do this in R?

The R package `parallel`

- Detects cores
- Divides likelihood approximation calculations amongst cores
- Sums results from each core
- Returns likelihood approximation

Thank you!

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References

Knudson C. (2015). *glmm: Generalized Linear Mixed Models via Monte Carlo Likelihood Approximation.* R package version 1.0.2, URL `http://CRAN.R-project.org/package=glmm`.

Knudson C. (2016). *Monte Carlo Likelihood Approximation for Generalized Linear Mixed Models.* Ph.D. Thesis, University of Minnesota.

Ripley B., Tierney L., Urbanek S. (2017). *Package 'parallel'* R package version 3.3.1, URL `http://stat.ethz.ch/R-manual/R-devel/library/parallel/doc/parallel.pdf`.

Using `glmm`

Notes:

- `0+Cross` produces log odds for each group, without using reference group.
- `0+Female` centers random effects for females at 0, likewise for males.
- Bigger m gives better estimates but takes more time.