## Question 1.4

a)

**Base case:** Let n = 0.

Then $intToBin(0) = [Zero] \rightarrow binToInt(intToBin(0)) = 0$

**Induction:** Let suppose that by strong induction for all k <n – 1 its true then we have to prove:

$$binToInt(intToBin(n)) = n$$

$$(1)\quad intToBin(n) = \begin{cases} One :: intToBin\left(\frac{n-1}{2}\right), if\ n\ mod\ 2 = 1 \\ Zero :: intToBin\left(\frac{n}{2}\right)\ , if\ n\ mod\ 2 = 0 \end{cases}$$

$$(2)\quad binToInt(b) = \begin{cases} 2 * binToInt(b) + 1, if\ One :: b \\ 2 * binToInt(b)\ , if\ Zero :: b \end{cases}$$

$$binToInt(intToBin(n)) = \begin{cases} 2 * binToInt\left(intToBin\left(\frac{n-1}{2}\right)\right) + 1, if\ n\ mod\ 2 = 1 \\ 2 * binToInt\left(intToBin\left(\frac{n}{2}\right)\right)\ , if\ n\ mod\ 2 = 0 \end{cases}$$

But as its true for all k < n-1 specially for n/2

$$binToInt\left(intToBin\left(\frac{n}{2}\right)\right) = \frac{n}{2}$$

Then

$$binToInt(intToBin(n)) = \begin{cases} 2 * \frac{n-1}{2} + 1, if\ n\ mod\ 2 = 1 \\ 2 * \frac{n}{2}\ , if\ n\ mod\ 2 = 0 \end{cases}$$

$$binToInt(intToBin(n)) = \begin{cases} n, if\ n\ mod\ 2 = 1 \\ n\ , if\ n\ mod\ 2 = 0 \end{cases}$$

Thus

$$binToInt(intToBin(n)) = n$$

For all $n \geq 0$

b) Let b = [] then the property is false as

$$intToBin(binToInt([])) = [Zero]$$

And [] $\neq [Zero]$

**Bonus:** Let the property be:

$$intToBin(binToInt(b)) = b\ @\ [Zero]$$

## Question 2.4

The first equation gives us this equality:

$$(1)\ sBinToInt(x :: ws) = sBinToInt(ws) + x$$

The second equation gives us this equality:

$$(2)\ sBinToInt'(x :: ws, acc) = \ sBinToInt'(ws, x + acc)$$

Let's now prove the following equation:

$$sBinToInt(ws) + acc \ = \ sBinToInt(ws, acc)$$

*Induction:*

Base case:

$$sBinToInt([]) + acc = acc = \ sBinToInt([], acc)$$

Step: Let assume for any 'acc'

$$(*)\ sBinToInt(ws) + acc \ = \ sBinToInt(ws, acc)$$

We must prove that:

$$sBinToInt(x :: ws) + acc \ = \ sBinToInt(x :: ws, acc)$$

Then

$$sBinToInt(x :: ws) + acc = sBinToInt(ws) + x + \ acc \ \ \big(By\ equation\ (1)\big)$$

$$sBinToInt(x :: ws) + acc = \ \ sBinToInt'(ws, x + acc)\ \big(By\ equation\ (*)\big)$$

$$sBinToInt(x :: ws) + acc == \ \ sBinToInt'(x :: ws, acc)\ \ \big(By\ equation\ (2)\big)$$

QED