SectionPy Module Testing Notebook

Jupyter Notebook created by **Ben Fisher** for testing and development of the StaticsPy library and for examples and explanation of the functions

Imports

In [1]:
```
import matplotlib.patches as mpatches
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import os, sys
from math import sqrt
module_path = os.path.abspath(os.path.join("C:\\Users\\benst\\Documents\\_0 Workspa
if module_path not in sys.path:
    sys.path.append(module_path)
import section as sc
sc.__version__
```

Out[1]:
```
'0.0.1'
```

Create Arbitrary Section

Create a numpy array of ordered pairs that describes an arbitrary, simple, closed, convext polygon of *n* verticies and sides.

Note: for visualization purposes, we need to append the first coordinate to the end of the array in order to show a closed polygon in Matplotlib.

In [2]:
```
vertices = np.array([[-1,-2],[4,-1],[6,5],[3,3],[-3,5],[1,1]])
vertices = np.vstack((vertices, vertices[0]))
vertices
```

Out[2]:
```
array([[-1, -2],
       [ 4, -1],
       [ 6,  5],
       [ 3,  3],
       [-3,  5],
       [ 1,  1],
       [-1, -2]])
```

We can check the shape of the array to make sure its n x 2:

In [3]:
```
vertices.shape
```

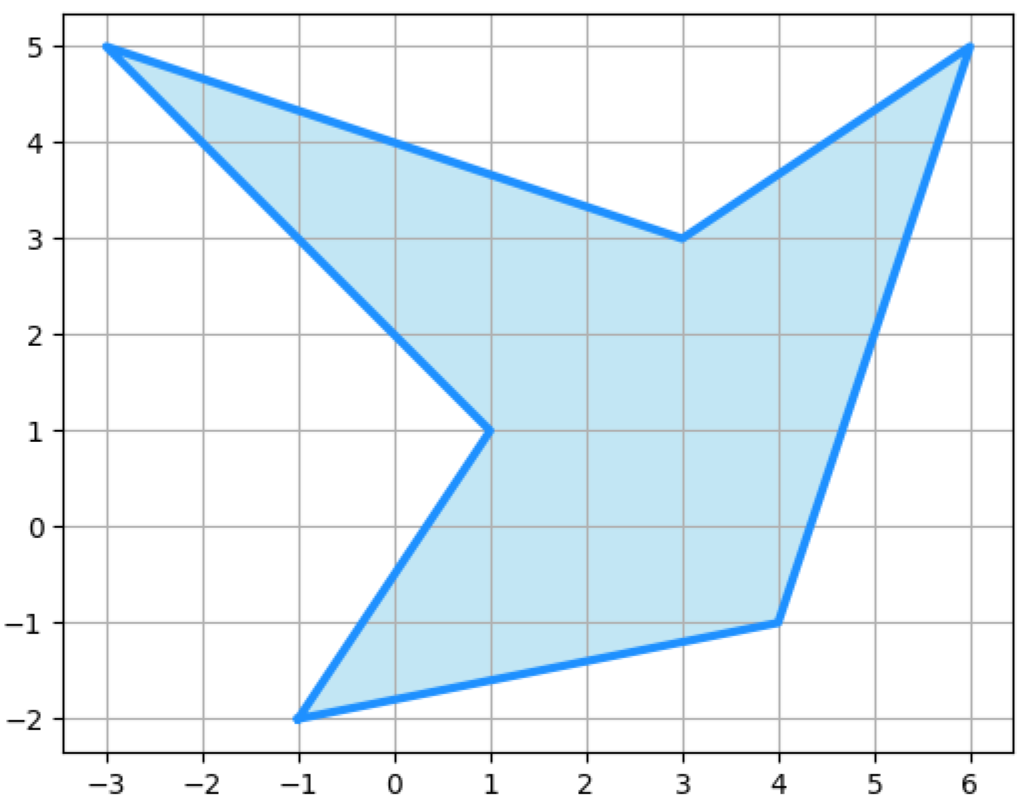
Out[3]:
```
(7, 2)
```

Let's slice this array and visualize it using Matplotlib.

In [4]:
```
xs = vertices[:,0]
ys = vertices[:,1]
```

In [5]:
```
# First, let's create a plot
fig, ax = plt.subplots()

# Set the grid aspect ratio at 1:1 for square grids
ax.set_aspect('equal')

# Make the scale the same along both axes: 1
ax.xaxis.set_major_locator(plt.MultipleLocator(1))
ax.yaxis.set_major_locator(plt.MultipleLocator(1))

# Show the grid
plt.grid()

# Let's put some fill inside the polygon
ax.fill(xs, ys, color="skyblue",alpha=0.5)

# Finally, let's plot the polygon
plt.plot(xs,ys,color="dodgerblue",linewidth=3)
```

Out[5]: `[<matplotlib.lines.Line2D at 0x2609e98b8b0>]`



Assuming the vertices array is n x 2. Note that this double counts the first point, but that's ok, we'll operate to n-1

In [6]:
```
n = vertices.shape[0]
```

Calculate the Perimeter

Determine the perimeter of the polygon based on the following formula:

$ P = \displaystyle\sum_{i=0}^{n-1}\sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2} $

In [7]:
```
P = 0
for i in range(0,n-1):
    P += sqrt((xs[i+1]-xs[i])**2+(ys[i+1]-ys[i])**2)

# Print P (the answer should be 30.616087)
P
```

Out[7]: 30.616086954686658

In [8]:
```
sc.perimeter(vertices)
```

Out[8]: 30.616086954686658

Calculate the Area

Determine area based on the **shoelace algorithm**, use the *Trapezoid Formula* or *Triangle Formula* as follows (*both formula give the same answer*):

$ A_{trapezoid}=\dfrac{1}{2}\displaystyle\sum_{i=0}^{n-1}(y_i+y_{i+1})(x_i-x_{i+1}) $

$ A_{triangle}=\dfrac{1}{2}\displaystyle\sum_{i=0}^{n-1}(x_iy_{i+1}-x_{i+1}y_i) $

In [9]:
```
A = 0
for i in range(0,n-1):
    # A += (ys[i]+ys[i+1])*(xs[i]-xs[i+1])  # Trapezoid Formula
    A += (xs[i]*ys[i+1]-xs[i+1]*ys[i])    # Triangle Formula
A *= 0.5

# Print the area A (should be 26.5)
A
```

Out[9]: 26.5

In [10]:
```
sc.area(vertices)
```

Out[10]: 26.5

TODO: This only works if the polygon is *simple*, that means it doesn't self-intersect. Determine if need a function to perform Shamos-Hoey algorithm or Bentley-Ottmann algorithm, to determine if self-intersecting.

Calculate the Centroids

Using an extension of the *Trapezoid Formula*, we can calculate the centroids Cx and Cy, using the area A calculated previously:

$ C_x = \displaystyle\frac{1}{6A}\sum_{i=0}^{n-1}(x_i+x_{i+1})(x_iy_{i+1}-x_{i+1}y_i) $

$ C_y = \displaystyle\frac{1}{6A}\sum_{i=0}^{n-1}(y_i+y_{i+1})(x_iy_{i+1}-x_{i+1}y_i) $

In [11]:
```
Cx = Cy = 0
for i in range(0,n-1):
    Cx += (xs[i]+xs[i+1])*(xs[i]*ys[i+1]-xs[i+1]*ys[i])
    Cy += (ys[i]+ys[i+1])*(xs[i]*ys[i+1]-xs[i+1]*ys[i])
Cx /= (6*A)
Cy /= (6*A)
```

In [12]:
```
# Print Cx and Cy (should be 2.075472, 1.54717, respectively)
Cx, Cy
```

Out[12]:
```
(2.0754716981132075, 1.5471698113207548)
```

In [13]:
```
sc.centroids(vertices)
```

Out[13]:
```
array([2.0754717 , 1.54716981])
```

In [14]:
```
sc.Cx(vertices), sc.Cy(vertices)
```

Out[14]:
```
(2.0754716981132075, 1.5471698113207548)
```

Calculate Extrema

Determine the maxima and minima of the polygon:

In [15]:
```
min_x = np.min(xs)
max_x = np.max(xs)
min_x, max_x
```

Out[15]:
```
(-3, 6)
```

In [16]:
```
min_y = np.min(ys)
max_y = np.max(ys)
min_y, max_y
```

Out[16]:
```
(-2, 5)
```

In [17]:
```
sc.extrema(vertices)
```

Out[17]:
```
array([[-3,  6],
       [-2,  5]])
```

Calculate the Second Moment of Area about Own Axis

Continuing to use the *shoelace algorithm* calculate the second moments of area (moments of inertia) of the polygon as follows:

$$I_y=\dfrac{1}{12}\displaystyle\sum_{i=0}^{n-1}(x_iy_{i+1}-x_{i+1}y_i)(x_i^2+x_ix_{i+1}+x_{i+1}^2)$$

$$I_x=\dfrac{1}{12}\displaystyle\sum_{i=0}^{n-1}(x_iy_{i+1}-x_{i+1}y_i)(y_i^2+y_iy_{i+1}+y_{i+1}^2)$$

Below is the formula for the polar moment of inertia

$$I_{xy}=\dfrac{1}{24}\displaystyle\sum_{i=0}^{n-1}(x_iy_{i+1}-x_{i+1}y_i)(x_iy_{i+1}+2x_iy_i+2x_{i+1}y_{i+1}+x_{i+1}y_i)$$

In [18]:
```
Ix = Iy = Ixy = 0
for i in range(0,n-1):
    Ix += (xs[i]*ys[i+1]-xs[i+1]*ys[i])*(ys[i]**2+ys[i]*ys[i+1]+ys[i+1]**2)
    Iy += (xs[i]*ys[i+1]-xs[i+1]*ys[i])*(xs[i]**2+xs[i]*xs[i+1]+xs[i+1]**2)
    Ixy += (xs[i]*ys[i+1]-xs[i+1]*ys[i])*(xs[i]*ys[i+1]+2*xs[i]*ys[i]+2*xs[i+1]*ys[

Ix /= 12
Iy /= 12
Ixy /= 24
```

In [19]:
```
Ix, Iy, Ixy, Ix+Iy
```

Out[19]:
```
(140.08333333333334, 203.41666666666666, 83.79166666666667, 343.5)
```

TODO: There appears to be some sort of error calculating the *polar moment* as it should be Ix + Iy. For now, just use Ix + Iy, then use point formula in later update.

In [20]:
```
sc.inertias(vertices)
```

Out[20]:
```
array([140.08333333, 203.41666667, 343.5       ])
```

In [21]:
```
Ix = sc.Ix(vertices)
Iy = sc.Iy(vertices)
Ixy = sc.Ixy(vertices)
Ix, Iy, Ixy
```

Out[21]:
```
(140.08333333333334, 203.41666666666666, 343.5)
```

Calculate the radii of gyration

$ r = \displaystyle\sqrt{\frac{I}{A}} $

In [22]:
```
rx = sqrt(Ix/A)
ry = sqrt(Iy/A)
rx, ry
```

Out[22]:
```
(2.2991658317773815, 2.7705776706186778)
```

In [24]:
```
radii = sc.radii(vertices)
radii
```

Out[24]:
```
array([2.29916583, 2.77057767])
```

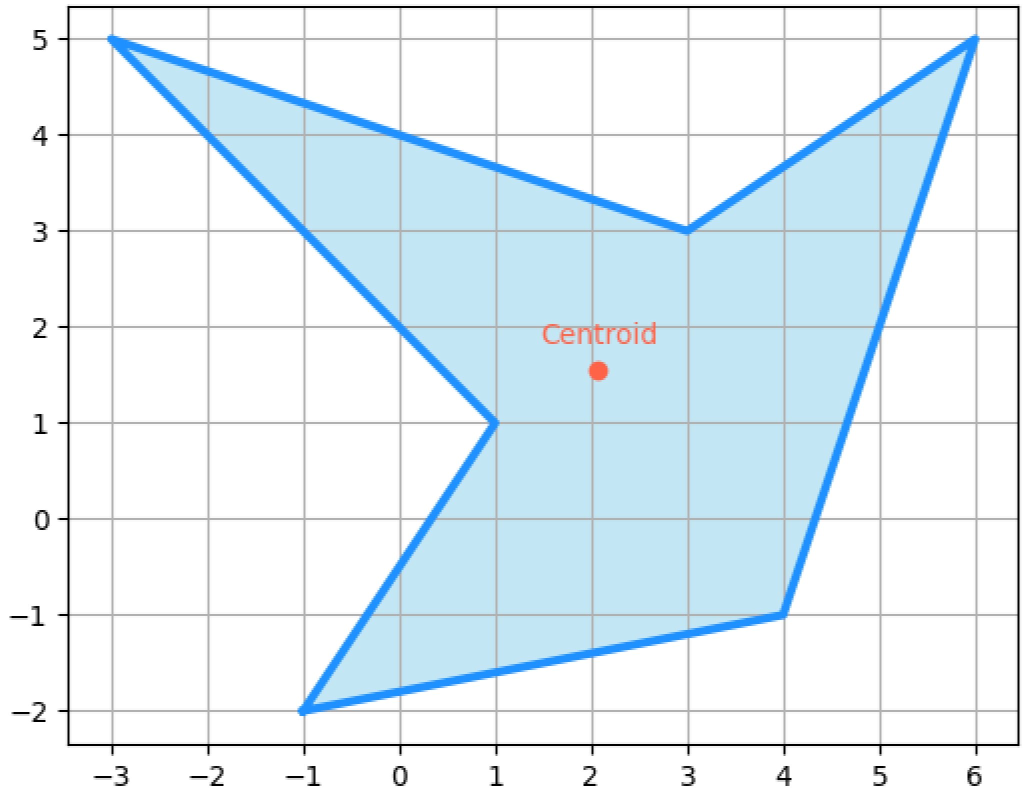
In [30]:
```
sc.rx(vertices), sc.ry(vertices)
```

Out[30]:
```
(2.2991658317773815, 2.7705776706186778)
```

Let's replot the shape, but put plot the centroid on the shape.

In [81]:
```
fig, ax = plt.subplots()
ax.set_aspect('equal')
ax.xaxis.set_major_locator(plt.MultipleLocator(1))
ax.yaxis.set_major_locator(plt.MultipleLocator(1))
plt.grid()
ax.fill(xs, ys, color="skyblue",alpha=0.5)
plt.plot(xs,ys,color="dodgerblue",linewidth=3)

plt.scatter(sc.Cx(vertices), sc.Cy(vertices),color="tomato",marker="o",zorder=2)

plt.annotate("Centroid",                             # the actual label
                 (sc.Cx(vertices), sc.Cy(vertices)), # these are the coordinates to
                 textcoords="offset points",         # how to position the text
                 xytext=(0,10),                      # distance from text to points
                 ha='center',                        # horizontal alignment can be
                 color="tomato")
```

Out[81]:
```
Text(0, 10, 'Centroid')
```



Line Intersection Functions

TODO List

1. Linear interpolation
2. Bezier parameters, *t* and *u*, to test intersection
3. Multilinear interpolation - provide table of numbers and interpolate between the appropriate ordinates
4. Bilinear interpolation