Poon's Textbook Summaries

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October 9, 2018

1 Chapter 1: Wave optics

1.1 Maxwell's equations and the wave equation

Here are the Maxwell's equations:

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J = J_c + \frac{\partial D}{\partial t}$$

E is the electric field strength in (V/m), D is the electric flux density (C/m^2), H is the magnetic field strength (A/m), and B is the magnetic flux density (Wb/m^2). J_c is the current dnesity (A/m^2), and ρ_v is the electric charge density (C/m^3). The D and E, B and H are related in the following fashion.

$$D = \epsilon E$$

$$B = \mu H$$

ϵ is the permittivity of the medium (F/m), and μ is the permeability of the medium (H/m). In linear, homogeneous, isotropic medium, ϵ and μ are scalar constants. Based on these equations, we can derive the three-dimensional scalar wave equation in a source-free medium.

$$\nabla^2 \Psi \ = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

1.2 Plane waves and spherical waves

The plane wave solution of a simple harmonic oscillation at angular frequency w_0 (radian/second) is:

$$\psi(x, y, z, t) = A \exp[j(w_o t - k_0 \cdot R)]$$

where $j = \sqrt{-1}$, k_0 is the propagation vector, and R is the position vector. Magnitude of k_0 is the wave number, A is the amplitude of the plane wave. If wave is travelling along positive z-direction, this equation becomes:

$$\psi(z, t) = A \exp[j(w_0 t - k_0 z)$$

which is a solution to the wave equation

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

The spherical wave solution of the same oscillation is:

$$\frac{\partial^2 (R\psi)}{\partial R^2} = \frac{1}{v^2}\frac{\partial^2 (R\psi)}{\partial t^2}$$

$$\psi(R, t) = \frac{A}{R} \exp[j(w_0 t - k_0 R)]$$

1.3 Scalar diffraction theory

Let's assume some aperture, such as a diffraction screen, has a transparency function t(x,y) that is complex. Then,

$$\psi(x,y,z=0,t) = At(x,y)\exp(j\omega_0 t) = \psi_p(x,y;z=0)\exp(j\omega_0 t) = \psi_{p0}(x,y)exp(j\omega_0 t)$$

ψ_{o0} is called the complex amplitude, the initial condition of the aperture. To find the field distribution at z away from the aperture.

$$\psi(x,y,z,t) = \psi_p(x,y;z)\exp(j\omega_0 t)$$

$$\frac{\partial^2\psi_p}{\partial x^2} + \frac{\partial^2\psi_p}{\partial y^2} + \frac{\partial^2\psi_p}{\partial z^2} + k_0^2\psi_p = 0$$

To solve the above equations, Fourier transform is used and is defined as:

$$\mathcal{F}\{f(x,y)\} = F(k_x,k_y) = \iint\limits_{-\infty}^{+\infty} f(x,y)\exp(jk_x x + jk_y y)\,dx\,dy$$

$$\mathcal{F}^{-1}\{F(k_x,k_y)\} = f(x,y) = \frac{1}{4\pi^2}\iint\limits_{-\infty}^{+\infty} F(k_x,k_y)\exp(-jk_x x - jk_y y)\,dk_x\,dk_y$$

Taking Fourier transform of the wave equation:

$$\mathcal{F}\{\frac{\partial^2\psi_p}{\partial x^2}\} = (-jk_x)^2\Psi_p(k_x,k_y;z)$$

$$\mathcal{F}\{\frac{\partial^2\psi_p}{\partial y^2}\} = (-jk_y)^2\Psi_p(k_x,k_y;z)$$

$$\frac{d^2\Psi_p}{dz^2} + k_0^2(1 - \frac{k_x^2}{k_0^2} - \frac{k_y^2}{k_0^2})\Psi_p = 0$$

$$\Psi_p(k_x,k_y;z) = \Psi_{p0}(k_x,k_y)\exp[-jk_0\sqrt{(1-k_x^2/k_0^2-k_y^2/k_0^2)}z]$$

From these results, the spatial frequency transfer function of propagation through a distance z is defined as:

$$\mathcal{H}(k_x,k_y;z) = \Psi_p(k_x,k_y;z)/\Psi_{p0}(k_x,k_y) = \exp[-jk_0\sqrt{(1-k_x^2/k_0^2-k_y^2/k_0^2)}z]$$

The field distribution that we intended to get in the first place is given by:

$$\psi_p(x,y;z) = \mathcal{F}^-1\{\Psi_{p0}(k_x,k_y)\} = \mathcal{F}^-1\{\Psi_{p0}(k_x,k_y)\mathcal{H}(k_x,k_y;z)\}$$

$$= \frac{1}{4\pi^2}\iint\limits_{-\infty}^{+\infty}\Psi_{p0}(k_x,k_y)\exp[-jk_0\sqrt{(1-k_x^2/k_0^2-k_y^2/k_0^2)}z]\exp(-jk_x x - jk_y y)\,dk_x\,dk_y$$

1.3.1 Fresnel diffraction

When propagating waves make small angles such that $k_x^2 + k_y^2 << k_0^2$, the following approximation can be made:

$$\sqrt{(1-k_x^2/k_0^2-k_y^2/k_0^2)} = 1 - k_x^2/2k_0^2 - k_y^2/2k_0^2$$

$$\psi_p(x,y;z) = \exp(-jk_0 z)\frac{jk_0}{2\pi z}\exp[\frac{-jk_0}{2z}(x^2+y^2)] \times \mathcal{F}\{\psi_{p0}(x,y)\exp[\frac{-jk_0}{2z}(x^2+y^2)]\}_{k_x=\frac{k_0 x}{z},k_y=\frac{k_0 y}{z}}$$

1.3.2 Fraunhofer diffraction

For more distant objects, if the below statement is true, more approximation is possible:

$$\frac{k_0}{2}[(x')^2 + (y')^2]_{max} = \frac{\pi}{\lambda_0}[(x')^2 + (y')^2]_{max} << z$$

$$\psi_p(x, y; z) = \exp(-jk_0 z)\frac{jk_0}{2\pi z}\exp[\frac{-jk_0}{2z}(x^2 + y^2)] \times \mathcal{F}\{\psi_{p0}(x, y)\}_{k_x = \frac{k_0 x}{z}, k_y = \frac{k_0 y}{z}}$$

This approximation is the Fraunhofer approximation or the far field approximation.

1.4 Ideal thin lens as an optical Fourier transformer

An ideal thin lens is a phase object, so its transfer function is given by:

$$t_f(x, y) = exp[\frac{jk_0}{2f}(x^2 + y^2)]$$

1.5 Optical image processing

This is an example section, with MATLAB code. The main idea is given some focal length, if z is replaced with the focal length, the optical system can be described by the equations developed so far.