

Stat 341 – PS 10

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9E1

Number 3 is a requirement. Parameters don't need to be discrete and likelihoods don't have to be gaussian.

9E3

HMC's need to have continuous parameters. The HMC has to glide across parameter space, and it can't do this with a fixed parameter.

9E4

Effective sample size as computed by `precis` gives us the estimated number of independent samples. Usually, the effective sample size is better than the raw sample size because we want independence in the samples. Markov chains are autocorrelated, so usually they don't produce totally independent samples for the entirety of the sample data. We can use ACF plots to determine the independence of parameters in the data.

9E6

Bad trace plot

```
set.seed(34)
y <- rnorm(100)
bad_model9e6 <- ulam(
  alist(
    y ~ dnorm(mu, sigma),
    mu <- a1 + a2,
    a1 ~ dnorm(0, 1000),
    a2 ~ dnorm(0, 1000),
    sigma ~ dexp(14)
  ), data = list(y=y),
  chains = 3
)
```

```
## Trying to compile a simple C file
```

```
## Running /usr/lib/R/bin/R CMD SHLIB foo.c
```

```
## gcc -std=gnu99 -I"/usr/share/R/include" -DNDEBUG -I"/usr/local/lib/R/site-library/Rcpp/include/"
## In file included from /usr/local/lib/R/site-library/RcppEigen/include/Eigen/Core:88:0,
## from /usr/local/lib/R/site-library/RcppEigen/include/Eigen/Dense:1,
## from /usr/local/lib/R/site-library/StanHeaders/include/stan/math/prim/mat/fun/Eigen
```

```

##          from <command-line>:0:
## /usr/local/lib/R/site-library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: error: unknown t
## namespace Eigen {
## ~~~~~
## /usr/local/lib/R/site-library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:17: error: expected
## namespace Eigen {
## ~
## In file included from /usr/local/lib/R/site-library/RcppEigen/include/Eigen/Dense:1:0,
##          from /usr/local/lib/R/site-library/StanHeaders/include/stan/math/prim/mat/fun/Eigen
##          from <command-line>:0:
## /usr/local/lib/R/site-library/RcppEigen/include/Eigen/Core:96:10: fatal error: complex: No such file
## #include <complex>
## ~~~~~
## compilation terminated.
## /usr/lib/R/etc/Makeconf:172: recipe for target 'foo.o' failed
## make: *** [foo.o] Error 1
##
## SAMPLING FOR MODEL 'a5ca083656214e883e45bac762feba8d' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 7e-06 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.07 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:   1 / 1000 [ 0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 0.748099 seconds (Warm-up)
## Chain 1:           0.839086 seconds (Sampling)
## Chain 1:           1.58718 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'a5ca083656214e883e45bac762feba8d' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 3e-06 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.03 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:   1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)

```

```

## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.711036 seconds (Warm-up)
## Chain 2: 0.900816 seconds (Sampling)
## Chain 2: 1.61185 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'a5ca083656214e883e45bac762feba8d' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 4e-06 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.04 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 0.815526 seconds (Warm-up)
## Chain 3: 0.860901 seconds (Sampling)
## Chain 3: 1.67643 seconds (Total)
## Chain 3:

## Warning: There were 1186 transitions after warmup that exceeded the maximum treedepth. Increase max_
## http://mc-stan.org/misc/warnings.html#maximum-treedepth-exceeded

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: The largest R-hat is 2.27, indicating chains have not mixed.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#r-hat

## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#bulk-ess

## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quant.
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess

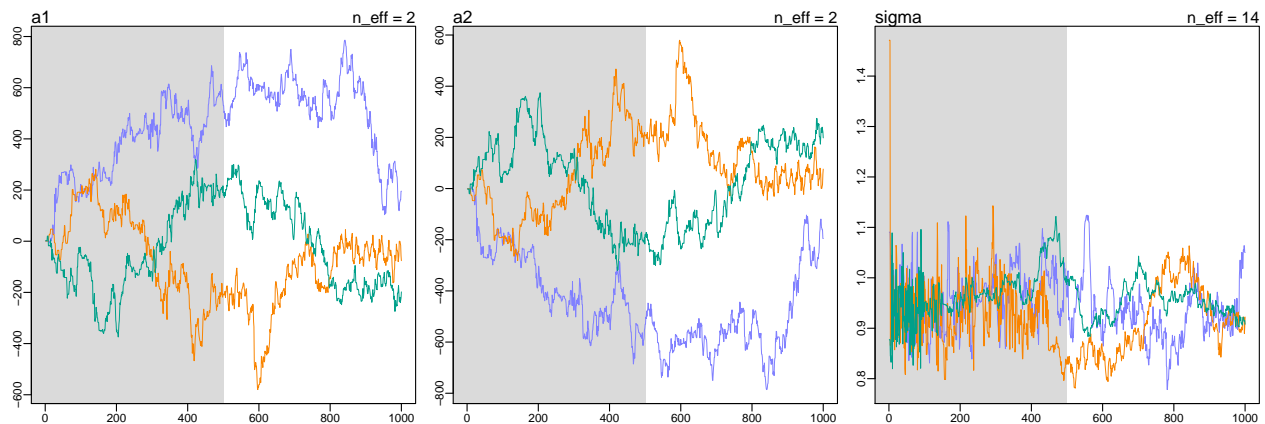
precis(bad_model9e6)

```

```
##           mean          sd        5.5%      94.5%      n_eff      Rhat4
## a1      126.7359641 328.76786240 -253.6857925 653.816297  1.888596 3.269912
## a2     -126.7410276 328.76789053 -653.8474675 253.600360  1.888610 3.269853
## sigma    0.9314076  0.05769606   0.8324593   1.020408 14.066695 1.280899
```

```
traceplot_ulam(bad_model9e6)
```

```
## [1] 1000
## [1] 1
## [1] 1000
```



I followed a pretty similar example to the book for the two models. These trace plots are overall not good and produce a very low `n_eff`. Parameters `a1` and `a2` are both wandering away from the mean of 0 as the step number increases, which is not good.

Good trace plot

```
set.seed(34)
good_model9e6 <- ulam(
  alist(
    y ~ dnorm(mu, sigma),
    mu <- a1 + a2,
    a1 ~ dnorm(0, 10),
    a2 ~ dnorm(0, 10),
    sigma ~ dexp(1)
  ), data = list(y=y),
  chains = 3
)
```

```
## Trying to compile a simple C file
```

```
## Running /usr/lib/R/bin/R CMD SHLIB foo.c
## gcc -std=gnu99 -I"/usr/share/R/include" -DNDEBUG -I"/usr/local/lib/R/site-library/Rcpp/include/"
## In file included from /usr/local/lib/R/site-library/RcppEigen/include/Eigen/Core:88:0,
## from /usr/local/lib/R/site-library/RcppEigen/include/Eigen/Dense:1,
## from /usr/local/lib/R/site-library/StanHeaders/include/stan/math/prim/mat/fun/Eigen
## from <command-line>:0:
## /usr/local/lib/R/site-library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:1: error: unknown t
## namespace Eigen {
## ~~~~~
## /usr/local/lib/R/site-library/RcppEigen/include/Eigen/src/Core/util/Macros.h:628:17: error: expected
```

```

## namespace Eigen {
## ~
## In file included from /usr/local/lib/R/site-library/RcppEigen/include/Eigen/Dense:1:0,
##           from /usr/local/lib/R/site-library/StanHeaders/include/stan/math/prim/mat/fun/Eigen
##           from <command-line>:0:
## /usr/local/lib/R/site-library/RcppEigen/include/Eigen/Core:96:10: fatal error: complex: No such file
## #include <complex>
##           ~~~~~
## compilation terminated.
## /usr/lib/R/etc/Makeconf:172: recipe for target 'foo.o' failed
## make: *** [foo.o] Error 1
##
## SAMPLING FOR MODEL '891f4364bbaa597b05591eddef7a0d2b' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 8e-06 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.08 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:   1 / 1000 [ 0%] (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 0.259801 seconds (Warm-up)
## Chain 1:                  0.294812 seconds (Sampling)
## Chain 1:                  0.554613 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL '891f4364bbaa597b05591eddef7a0d2b' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 4e-06 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.04 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:   1 / 1000 [ 0%] (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%] (Sampling)

```

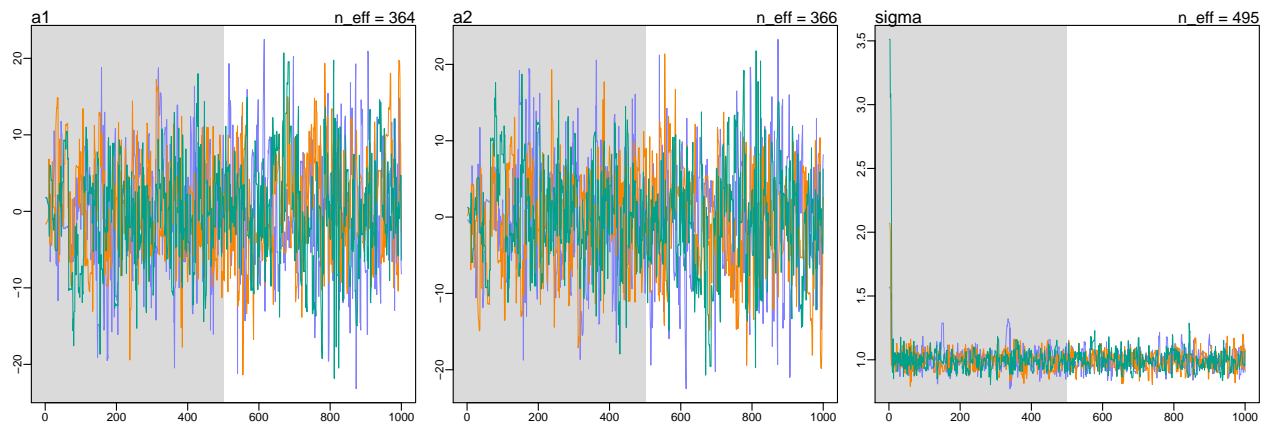
```

## Chain 2: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.253821 seconds (Warm-up)
## Chain 2: 0.26725 seconds (Sampling)
## Chain 2: 0.521071 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL '891f4364bbaa597b05591eddef7a0d2b' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 3e-06 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.03 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration: 1 / 1000 [ 0%] (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%] (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%] (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%] (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%] (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%] (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%] (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%] (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%] (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%] (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%] (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%] (Sampling)
## Chain 3:
## Chain 3: Elapsed Time: 0.256202 seconds (Warm-up)
## Chain 3: 0.276102 seconds (Sampling)
## Chain 3: 0.532304 seconds (Total)
## Chain 3:
precis(good_model9e6)

##          mean          sd      5.5%      94.5%    n_eff    Rhat4
## a1      0.8299587 7.34609968 -10.5077632 12.341384 363.5683 1.014095
## a2     -0.8347821 7.34982488 -12.3901529 10.514923 366.2662 1.014133
## sigma  0.9937337 0.07230584  0.8836093  1.112487 495.0865 1.003679
traceplot_ulam(good_model9e6)

## [1] 1000
## [1] 1
## [1] 1000

```



Neither a_1 , a_2 , or σ are wandering much from their means, so overall this plot is much better. The priors being less wide certainly help this model sample effectively.