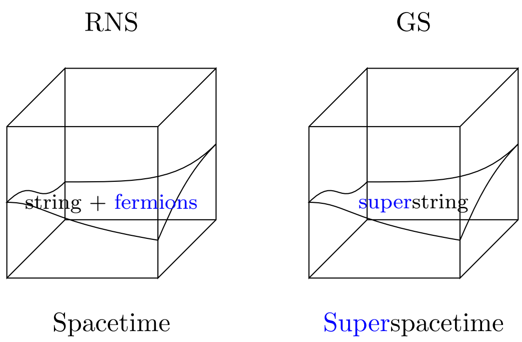
Allow me to summarise what I have learned since the start of the year in this section. I will not aim for a pedagogical approach to the literature and will, instead, aim to classify my notes by topic. Within each topic I will arrange the material in a way which reflects how I learned about them. That is, not a chronological order but rather the order in which the papers appeared to me.

First and foremost, we may give a brief explanations of the various massless excitations in type IIA and type IIB string theory which will pave the way to M-theory [BBS06].
Consider a spacetime to in which propagates a string. We have two ways of effectively describing the supersymmetric degrees of freedom called the Ramond-Neveu-Schwarz and Green-Schwarz formalisms. In the former, a set of fermionic fields is added to the string whereas for the latter we add supersymmetry to the spacetime and consider the string as propagating in this superspace.



Ramond-Neveu-Schwarz

In this formalism we may add to the string action a set of d fermions (one for each spacetime dimension).

$$S = -\frac{1}{4\pi\alpha'}\int d^2\xi(\partial_\alpha X^M \partial^\alpha X_M + i\bar{\psi}^M \gamma_\alpha \partial^\alpha \psi_M) \tag{1}$$

This action is invariant under the global supersymmetric transformations

$$\delta X^M = \bar{\epsilon}\psi^M \quad \delta\psi^M = \gamma_\alpha \partial^\alpha X^M \epsilon\,, \tag{2}$$

which can be made manifest by using the language of superfields. Indeed, in this language we may concatenate the fermionic and bosonic fields into one entity

$$Y^M(\xi,\theta) = X^M(\xi) + \bar{\theta}\psi^M(\xi) + \frac{1}{2}\bar{\theta}\theta B^M(\xi)\,. \tag{3}$$

Together with the covariant derivative $D = \frac{\partial}{\partial\theta} + \rho^\alpha\theta\partial_\alpha$, the action simplifies to

$$S = \frac{i}{8\pi\alpha'}\int d^2\xi d^2\theta \bar{D}Y^M DY_M\,. \tag{4}$$

Imposing the extremal action principle yields the usual equations of motion and boundary conditions for the bosonic sector. The fermionic sector, on the other hand becomes

$$\delta S \sim \int d\tau[(\psi_+\delta\psi_+ - \psi_-\delta\psi_-)|_{\sigma=\pi} - (\psi_+\delta\psi_+ - \psi_-\delta\psi_-)|_{\sigma=0}]\,. \tag{5}$$

There are two different ways of satisfying the above equations for the two different types of strings.

1. Ramond b.c. (R): $\psi_+^M|_{\sigma=\pi} = \psi_-^M|_{\sigma=\pi}$

2. Neveu-Schwarz b.c. (NS): $\psi_+^M|_{\sigma=\pi} = -\psi_-^M|_{\sigma=\pi}$

We can then proceed with canonical quantisation of the theory. As is customary, one can use spurious states to decouple negative-norm states. However, doing so does not remove the tachyonic ground state and the spectrum is manifestly not supersymmetric. The reason behind this is that a further projection must be made, called GSO projection. In performing this projection, two possibilities remain

1. R/L-moving R-sector ground states have opposite chirality: type IIA
2. R/L-moving R-sector ground states have the same chirality: type IIB

We can then analyse the massless spectrum of these two theories.

1. Type IIA:

$$\underbrace{1}_{\phi} \oplus \underbrace{8_V}_{C_{(1)}} \oplus \underbrace{28}_{B_{MN}} \oplus \underbrace{56_t}_{C_{(3)}} \oplus \underbrace{35}_{g_{MN}} \oplus \underbrace{8 \oplus 8' \oplus 56 \oplus 56'}_{\text{gravitinos and dilatinos}} \tag{6}$$

2. Type IIB:

$$\underbrace{1}_{\phi} \oplus \underbrace{1}_{C_{(0)}} \oplus \underbrace{28}_{B_{MN}} \oplus \underbrace{28}_{C_{(2)}} \oplus \underbrace{35}_{g_{MN}} \oplus \underbrace{35_+}_{C_{(4)}} \quad \underbrace{8'^2 \oplus 56^2}_{\text{gravitinos and dilatinos}} \tag{7}$$

Green-Schwarz

One of the apparent problems of the RNS formalism is the need to project out using G-parity and the non-manifest supersymmetry throughout. The GS formalism can solve these by considering the string as an embedding into a superspacetime.

Consider an $\mathcal{N}=2$ superspace and a 10-dim spacetime. Then depending on the theory one studies, we define the action of the Γ_{11} matrix as

$$\begin{aligned} &\text{type IIA} \qquad \Gamma_{11}\theta^A = (-1)^{A+1}\theta^A \\ &\text{type IIB} \qquad \Gamma_{11}\theta^A = \theta^A \end{aligned}$$

The GS action can then be written as

$$S = -\frac{1}{2\pi\alpha'}\int d^2\xi\sqrt{-\det(G_{\alpha\beta})} \qquad G_{\alpha\beta} = \Pi_\alpha^M \Pi_{\beta M} \quad \Pi_\alpha^M = \partial_\alpha X^M - \sum_{A=1}^{\mathcal{N}} \bar{\theta}^A \Gamma^M \partial_\alpha \theta^A \,.$$

Requiring κ-symmetry forces the introduction of a new term which results in the correct number of fermionic degrees of freedom.

$$\begin{aligned} S &= -\frac{1}{2\pi\alpha'}\int d^2\xi\sqrt{-\det(G_{\alpha\beta})} - \frac{1}{\pi\alpha'}\int \Omega_2 \\ \Omega_2 &= \bar{\theta}^1\Gamma_M d\theta^1 - \bar{\theta}^2\Gamma_M d\theta^2 dX^M - \bar{\theta}^1\Gamma_M d\theta^1\bar{\theta}^2\Gamma^M d\theta^2 \end{aligned} \tag{8}$$

Quantisation of such an action is hard but in the light-cone frame the non-linearity of the equations of motions disappear and canonical quantisation follows like the RNS formalism.

Supergravity

In the pages above we described two approaches to the massless spectrum of type IIA and type IIB string theory. This spectrum is important for supergravity as it is precisely what remains when considering the low energy effective action. In other words, taking the small string coupling limit forces all massive excitations to decouple and we are 'left with massles modes only'. Of course an exact statement would be to integrate out the massive modes and be left with an actions that contains an potential infinity amount of terms. The small string coupling limit allows us to truncate said action and only keep the massless modes.
It is worth noting that the supergravity solutions to be described below are also a consequence of forcing the golbal supersymmetry into a local one. The bosonic actions of type IIB supergravity is as follows:

$$\begin{aligned} S_{\rm IIB} &= \frac{1}{\tilde{\kappa}_{10}^2}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}(R+4\partial_M\phi\partial^M\phi-\frac{1}{2}|H_{(3)}|^2)-\frac{1}{2}|F_{(1)}|^2-\frac{1}{2}|\tilde{F}_{(3)}|^2-\frac{1}{4}|\tilde{F}_{(5)}|^2\right] \\ &\quad -\frac{1}{4\tilde{\kappa}_{10}^2}\int C_{(4)}\wedge H_{(3)}\wedge F_{(3)} \\ *\tilde{F}_{(5)} &= \tilde{F}_{(5)}\,, \end{aligned} \tag{9}$$

where

$$\begin{aligned} &F_{(p)} = dC_{(p)} \quad H_{(3)} = dB_{(2)} \quad \tilde{F}_{(3)} = F_{(3)} - C_{(0)}H_{(3)} \\ &\tilde{F}_{(5)} = F_{(5)} - \frac{1}{2}C_{(2)}\wedge H_{(3)} + \frac{1}{2}B_{(2)}\wedge F_{(3)}\,. \end{aligned}$$

Similarly for type IIA,

$$\begin{aligned} S_{\rm IIA} &= \frac{1}{\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}(R+4\partial_M\phi\partial^M\phi-\frac{1}{2}|H_{(3)}|^2)-\frac{1}{2}|F_{(2)}|^2-\frac{1}{2}|\tilde{F}_{(4)}|^2\right] \\ &\quad -\frac{1}{4\kappa_{10}^2}\int B_{(2)}\wedge F_{(4)}\wedge F_{(4)}\,, \end{aligned} \tag{10}$$

where

$$\tilde{F}_{(4)} = dC_{(3)} - C_{(1)}\wedge F_{(3)}\,.$$

1 Branes

The previous section listed the various massless excitations of type IIA and type IIB string theory as well as the corresponding low energy effective actions of supergravity. We may notice that the various Ramond-Ramond fields exhibit similarities with standard electrodynamics and one may postulate the existence of the objects they are charged under. Indeed, a RR three-form would couple naturally to an extended object of dimension 3 (similarly for higher forms). Furthermore, the GS action (8) generalises effortlessly by extending the range for α to $0 \leq \alpha \leq p$. This too could suggest the existence of higher-dimensional extended objects – branes. Let us motivate the existence branes in M-theory following a set of notes by Townsend [Tow96; Tow99].

M-theory is believed to exhibit supersymmetry in such a way that upon compactification we can recover our perturbative strings theories described before. The super-Poincaré algebra, in the Majorana representation of the Dirac matrices (for which $C = \Gamma^0$), contains the anticommutator

$$\{Q_\alpha, Q_\beta\} = (\Gamma^0\Gamma^M)_{\alpha\beta}P_M\,. \tag{11}$$

In the presence of extended objects, this algebra gets modified through the addition of 'central charges'.

$$\{Q_\alpha, Q_\beta\} = (C\Gamma^M)_{\alpha\beta}P_M + \frac{1}{2}(C\Gamma_{MN})_{\alpha\beta}Z^{MN} + \frac{1}{5!}(C\Gamma_{MNPQR})_{\alpha\beta}Y^{MNPQR} \tag{12}$$

The terms Z^{MN} and Y^{MNPQR} are not central charges of the full super-Poincaré algebra as they don't commute with spacetime translations for example. Surprisingly, these extensions are the only ones compatible with the supersymmetry algebra. On top of the usual particle-like solutions, these charges describe M2 and M5 branes which all break half of supersymmetry.

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It is possible to describe these solutions as 11-dim supergravity solutions. For example, the M2 brane solution has the form

$$\begin{aligned} ds^2 &= H^{-2/3} ds^2(\mathbb{M}^{2,1}) + H^{1/3} ds^2(\mathbb{E}^8) \\ F &= vol(\mathbb{M}^{2,1}) \wedge dH^{-1} \\ H &= 1 + \sum_{s=1}^{n} \frac{|q_s|}{\|x - x_s\|} \, . \end{aligned} \tag{13}$$

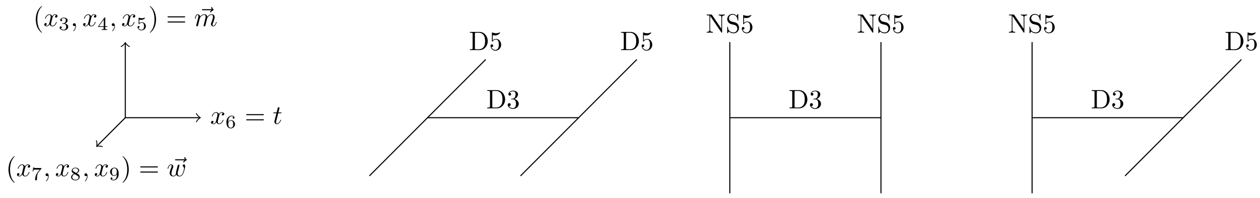
There are, of course, many more details to go through in order to fully appreciate how much information can be extracted from the algebra alone. However, in the interest of time, we will leave this discuss to this point.

Subsequent compactifications of the 11-dim spacetime leads to various supergravity descriptions of branes in type IIA/B supergravities. Since these branes appear directly in the supersymmetry algebra of M-theory, and given that a certain amount of supersymmetry remains in the type IIA/B string theories, it is worthwhile to extend the concept of brane to the latter. Doing so allows us to coincide the perturbative string theory definition of branes, D-branes, with our understanding of M-theory branes, or M-branes. Indeed, in perturbative string theory branes are defined as objects on which strings can end, and indeed, a consistent description can be given in terms of 2d BCFTs (see later sections).
This correspondence, allows us to effectively describe one object (M-brane) in two different limits. On one side we are dealing with perturbative string theory where $g_s \ll 1$ yet the number of branes is sufficiently small to allow for no back-reaction on the geometry, $g_s N \ll 1$. On the other side of things, we are dealing with a large number of branes which affect the geometry of the ambient spacetime. In that setting, while $g_s \ll 1$ as before (as it is a low energy effective theory stemming from string theory), we now have $g_s N \gg 1$ [GK99]. It might be important to note that since both these settings are defined perturbatively from the original string picture, we are also looking at an α' expansion. It is indeed possible to add to supergravity further α' corrections.
If we stick to the D-brane picture, where the number of branes remains small compared to the string coupling, we can make some interesting remarks about M-theory. Indeed, being a small energy limit of M-theory, various brane configurations in string theory allow us to describe some low energy effective gauge theories. For example, the low energy limit of the world-volume theory of the M3 brane is $\mathcal{N} = 4$ SYM.
We did not discuss any dualities in string theory, however, these do affect our considerations of branes. Indeed, using various string dualities one can deduce that branes may also end on other branes. From there, we are able to build brane constructions whose low energy effective theory describes interesting supersymmetric gauge theories.

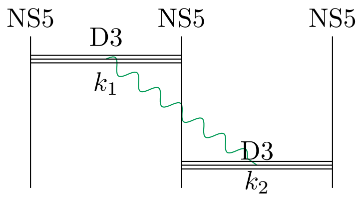
A notable construction was proposed by Hanany and Witten in their paper [HW97]. In this paper, the following brane constuction was considered

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D5	—	—	—	.	.	.	.	—	—	—
NS5	—	—	—	—	—	—	.	.	.	.
D3	—	—	—	.	.	.	—	.	.	.

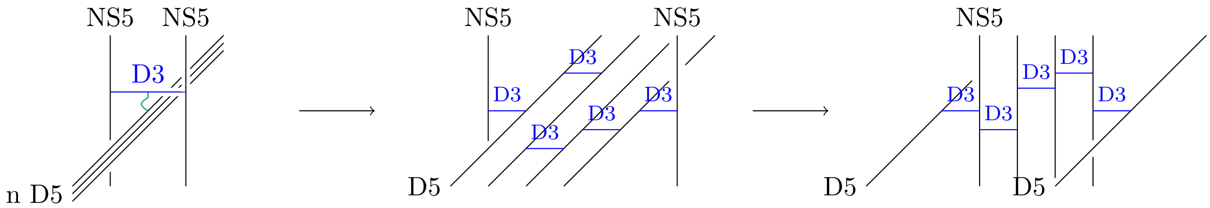
where three brane intersections are possible.



We see that in the first configuration, the D3 brane is free to move along the $\vec{w}$ directions, parametrising the hypermultiplet of the theory: the Higgs branch. In the second configuration, it is free to move along the $\vec{m}$ directions, parametrising the vector multiplet: the Coulomb branch.
Any strings propagating between the branes correspond to a fundamental multiplet. For example



describes a 3-dim $\mathcal{N} = 4$ $U(k_1) \times U(k_2)$ gauge theory.
Let us now provide a sketch of the important contribution from this paper. That is, a brane construction proof of mirror symmetry. Take a stack of D5 branes sandwhiched between two NS5 branes, which are them selves linked by a D3. When the D3 is coincident with the stack of D5's then the theory is sitting in its Higgs branch. If we consider splitting the D5's along the D3 and perform an R-S duality transformation, we recover a set of D3's connecting up two D5 through a set of NS5's. When the D3's are aligned, this corresponds to the Coulomb branch of a new theory.



As a closing remark for this subsection, let me present the work from Aharony et al. [Aha+08] which paves the way towards our research interest. In this paper, the low energy effective field theory of N M2 branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity is constructed. This is particularly interesting as such an effective field theory was not known before. Knowing both this and the, yet unknown, low energy limit of the M5 brane could shed a lot of light on the underlying structure of M-theory. This theory of the M2 brane is realised as two Chern-Simons theories with $SU(N)_k \times SU(N)_{-k}$ gauge group. This particular choice of level allows for parity transformations to remain a symmetry. The manifest $\mathcal{N} = 3$ supersymmetry is enhanced to $\mathcal{N} = 6$ through the intervention of the R-symmetry. More on this in a later chapter.

[De +89] [Wit96] [Ste98] [Pol96] [Pol95] [Mah96] [Joh06] [DKL95] [Ber+99] [Gau+97] [EGK97] [DEG07] [DHo+08] [GY07] [Wal99] [Dun99] [Wit89].

Need to add more details about other SUGRA papers

2 CFT and BCFT

In this section, we will give a brief overview of basic concepts in CFT. To do so, let us first introduce some formal definitions which may be found in [Sch08].

Definition 1 (Conformal Transformation) *Let $(\mathcal{M}, g)$ and $(\mathcal{M}', g')$ be two semi-Riemannian manifolds of the same dimension d and let $U \subset \mathcal{M}$, $V \subset \mathcal{M}'$ be open subsets of $\mathcal{M}$ and $\mathcal{M}'$, respectively. A smooth mapping $\varphi : U \longrightarrow V$ of maximal rank is called a conformal transformation, or conformal map, if there is a smooth function $\Omega : U \longrightarrow \mathbb{R}_+$ such that*

$$\varphi^* g' = \Omega^2 g\,.$$

Heuristically, we can see these transformations as a map that stretches our space at every point a different amount. Note that the conformal factor Ω can sometimes be replaced by another factor $\sigma : U \longrightarrow \mathbb{R}$, where $\Omega^2 = e^{2\sigma}$.
The importance of this conformal map is the scale change it produces, locally. Indeed, a theory which is defined to be invariant with respect to these transformations becomes invariant under various scales (including energy). This is particularly interesting as we could theoretically compute correlation functions for any energy scale of the theory (more on that later).

Similarly to the case of isomorphisms, we can look for Killing vectors which describe the conformal transformation of our manifold. In that case, a vector field X is a conformal Killing field if there exists a smooth function $\sigma : \mathcal{M} \longrightarrow \mathbb{R}$ such that

$$\partial_\mu X_\nu + \partial_\nu X_\mu = 2\sigma g_{\mu\nu} \tag{14}$$

By rearranging the above equation, we can find one that describes the conformal Killing factor σ alone. In that case, we have

$$(d-2)\partial_\mu \partial_\nu \sigma + g_{\mu\nu} \Delta_g \sigma = 0\,, \tag{15}$$

where $\Delta_g = g^{\mu\nu}\partial_\mu\partial_\nu$ is the Laplce-Beltrami operator associated to g.

Using the previous equation, we are able to classify conformal transformations depending on the number of dimensions involved. In the case where $d > 2$ the solutions to the equation are

1. $\varphi(q) = c + \Lambda q$: Poincarré transformations (translations + rotations),
2. $\varphi(q) = e^{2\sigma} q$: Dilatations,
3. $\varphi(q) = \frac{q - \langle q,q \rangle b}{1 - 2\langle b,q \rangle + \langle q,q \rangle \langle b,b \rangle}$: Special conformal transformations.

Together, these four transformations form group called the conformal group $\mathrm{Conf}(\mathbb{R}^{p,q})$.

Of course, a subtlety arises in when dealing with the group structure of these transformations. Indeed, special conformal transformations have singularities and, thus, should not close into a group under composition. To remidy this, we must consider a slightly more relaxed definition for the conformal group.

Definition 2 (Conformal Group) *The conformal group Conf($\mathbb{R}^{','}$) is the connected component containing the identity in the group of conformal diffeomorphisms of the conformal compactification of $\mathbb{R}^{p,q}$.*

As we can see, this definition allows for the special conformal transformations to close under composition as any point at infinity would be included in the conformal compactification of the manifold.
With this in mind, we can identify the conformal group in dimensions larger than two as

$$\mathrm{Conf}(\mathbb{R}^{p,q}) \cong SO(p+1, q+1) \quad p+q > 2\,. \tag{16}$$

An interesting point of confusion comes from the case $d = 2$. Indeed, in this setting the conformal group is often described as being 'infinite dimensional' in light of the Witt algebra under which the theory is

invariant. This algebra is indeed infinite dimensional, however, it is not the Lie algebra of any group (infinite dimensional or not). Indeed, the only sub-algebra of the Witt algebra which generates a group is that of $sl(2)$ for which the corresponding group is the Möbius transformations.
Another point of confusion originates in the definition of the conformal group. Indeed, for $d = 2$, two nonequivalent signatures are possible: $\mathbb{R}^{2,0}$ and $\mathbb{R}^{1,1}$. In the former, we are used to the conformal compactification to S^2 which leads to the conformal group $\mathrm{Conf}(\mathbb{R}^{2,0}) \cong \mathrm{Mb} \cong SO(3,1)$. However, the latter is often skimmed over. Indeed, in the physics literature we are used to taking the Wick rotation to consider the Euclidean plane instead of the Minkowski one. The problem with this and our definition of the conformal group is that a conformal compactification to S^2 does not exist for $\mathbb{R}^{1,1}$ as it is not compatible with the Lorentzian structure. Instead we have to consider the conformal compactification $\mathbb{R}^{1,1} \longrightarrow S^1 \times S^1 \subset \mathbb{R}^{2,0} \times \mathbb{R}^{0,2}$. By following our definition of the conformal group, this leads to

$$\mathrm{Conf}(\mathbb{R}^{1,1}) \cong \mathrm{Diff}_+(S) \times \mathrm{Diff}_+(S)\,. \tag{17}$$

In this case, the group is indeed infinite dimensional and its Lie algebra is $\mathrm{Lie}(\mathrm{Diff}_+(S)) = \mathrm{Vect}(S)$, the space of smooth vector fields on S. We can also note that its complexification contains the Witt algebra.

Let us skip ahead and consider a field theory which is invariant under the conformal group. In that case the various fields that constitute this theory may transform in various ways under the conformal transformations φ. It is for that reason that is it worthwhile to distinguish a certain class of fields, which transform in a specific way.

Definition 3 (Conformal Primary Fields) *Let ϕ be a field with scaling dimension Δ. The field ϕ is said to be quasi-primary if, under a global conformal transformation, it transforms as*

$$\phi'(\varphi(q)) = \left|\frac{\partial\varphi(q)}{\partial(q)}\right|^{-\Delta/d} \phi(q) \tag{18}$$

If this property extends to local conformal transformations (i.e. Witt algebra action) then ϕ is called a primary field.

Note that in $p + q > 2$, the conformal group is precisely that of global conformal transformations and the notion of *quasi-primary* and *primary* fields coincide. That being said, it is not true for $d = 2$, where global conformal transformations (Möbius group) only form a finite dimensional subgroup of the full local conformal group ($\mathrm{Diff}_+(S) \times \mathrm{Diff}_+(S)$). Also note that the definition above may be extended to fields with spin.

Let us now see how we can use quasi-primary fields to derive exact correlation functions. If the field theory is described by an action $S[\phi]$ then the 2-point function of spinless quasi-primary fields in the quantum field theory may be written as

$$\langle\phi_1(x_1)\phi_2(x_2)\rangle = \frac{1}{Z}\int \mathcal{D}\phi\phi_1(x_1)\phi_2(x_2)e^{-S[\phi]}\,. \tag{19}$$

Consider now a conformal transformation of the fields (dilatation)

$$\left|\frac{\partial x'}{\partial x}\right| = \lambda^d \quad \langle\phi_1(x_1)\phi_2(x_2)\rangle = \lambda^{\Delta_1+\Delta_2}\langle\phi_1(\lambda x_1)\phi_2(\lambda x_2)\rangle\,. \tag{20}$$

We can then utilise invariance under different transformations to restrict the form of the correlator $\langle\phi_1(x_1)\phi_2(x_2)\rangle = G_{12}(x_1, x_2)$.

1. Poincarré inv.: $G_{12}(x_1, x_2) = G_{12}(r_{12})$, where $r_{12} = \sqrt{|(x_1 - x_2)^2|}$
2. Dilatation inv.: $G_{12}(x_1, x_2) = \frac{c_{12}}{r_{12}^{\Delta_1+\Delta_2}}$

3. SCT inv.: $\Delta_1 = \Delta_2$ otherwise $G_{12} = 0$

This procedure can be repeated for n-point correlators (with less constraints the larger n gets).

[FMS12] [Gin88] [Car04] [Sax] [He] [RS13] [Bil+16] [Car89] [CL91] [MO93] [Ryc17] [Her] [HS21] [Min98] [MO95] [Zam86] [MS10].

Need to add Chris Herzog lectures on BCFT and Cardy, Zamolodchikov papers

3 Entanglement Entropy

Here are some references related to the Entanglement Entropy part of this research. In the interest of time, I will not write up this section. [Est+19] [CHM11] [Che] [Hea19] [JO13] [NRT09] [RT06a] [RT06b] [Son].

4 AdS/CFT

Being one of the central elements of this research, it is essential for us briefly describe AdS/CFT. Indeed, many of the theories described in a previous section can be described holographically. That is, there exists a duality between the conformal field theory and gravity in a larger dimensional space which is asymptotically AdS.
For a consistent introduction to AdS/CFT, a proper definition of AdS would be required. However, let us assume the reader is already acquainted with the concept.

The power behind the AdS/CFT correspondence originates in our lack of knowledge of the 'AdS' side of the duality. Indeed, we do not yet have a consistent theory of quantum gravity in any dimensions with AdS asymptotics. However, we do know how to get exact results in CFT (see previous section). This means that, through the duality, we should be able to recover new properties about the quantum gravity theory. In practice, we are only able to get classical or semi-classical results on the gravity side, however, these are already enough for many purposes (eg. quantum information and holographic entanglement entropy).

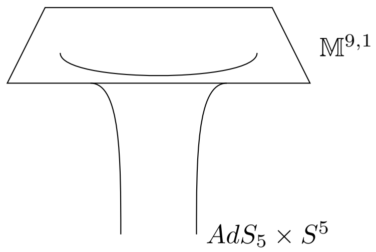
Let us look at a review of the holographic correspondence [Aha+00].
Consider N D3 branes in $\mathbb{M}^{9,1}$ type IIB string theory which are sitting close to each other. In the low energy limit, only massless excitations remain:

- closed strings: type IIB SUGRA
- open strings: $\mathcal{N} = 4$ $U(N)$ SYM

Let us look at the supergravity perspective of this setup. Here, the D3 branes appear as supergravity solutions which act as sources for various supergravity fields. A possible D3 solution is

$$\begin{aligned} ds^2 &= H^{-1/2} ds^2(\mathbb{M}^{3,1}) + H^{1/2} ds^2(\mathbb{E}^6)\,, \\ F_5 &= (1+*)dt\, dx_1 dx_2 dx_3 dH^{-1}\,, \\ H &= 1 + \frac{4\pi g_s \alpha'^2 N}{r^4}\,. \end{aligned} \tag{21}$$

This solution interpolates between $\mathbb{M}^{9,1}$ at infinity and $AdS_5 \times S^5$ near the horizon.



The two kinds of excitations that remain at low energy are:

- massless particles propagating in the "bulk" (at large transverse distance r): type IIB sugra
- any excitations close to the branes ($r = 0$): type IIB strings in the $AdS_5 \times S^5$ throat

We can now reconnect the string perspective of the D3 branes and the supergravity perspective. This allows us to conjecture the following dualities

D-brane picture		type IIB SUGRA picture
type IIB SUGRA in bulk	$=$	type IIB SUGRA in bulk
$SU(N)$ SYM on $\mathbb{M}^{3,1}$	$\xleftrightarrow{\text{dual}}$	type IIB strings on $AdS_5 \times S^5$

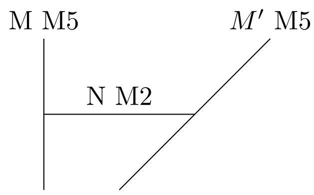
[DF02]

Need to add further details, including other papers that use the correspondence (eg. Ryu-Takayanagi, Myers and Sinha, Casini Huerta Myers, holographic RG flow, ...)

5 Future Research

At this stage, only a brief understanding of each topic was achieved. It goes without saying that throughout the PhD, I will come back to the previously mentioned references and dig deeper into each topic.

At the current time we would like to focus on our supervisor's paper [Est+19] which computes a central charge holographically. Indeed, we wish to consider the following setup.



In this brane configuration, the M2 brane ending on the M5 brane introduces a defect in the low energy effective field theory on the M5. Indeed, this will be a Wilson surface whose non-abelian expression is not known as of yet (but is for the abelian case). We will restrict ourselves to Wilson surfaces extended along the $\mathbb{R}^{1,1}$ subspace of the flat M5. By symmetry considerations, we know that the M5 isometries break down in the presence of the flat Wilson surface

$$SO(6,2) \times SO(5)_R \longrightarrow SO(2,2) \times SO(4)_R \times SO(4)_R \,. \tag{22}$$

This remnant bosonic symmetry is actually the bosonic part of the super-group $D(2,1;-\frac{1}{2}) \times D(2,1;-\frac{1}{2})$ and describes the isometry of the holographically dual space $AdS_3 \times S^3 \times S^3 \times \Sigma$.

The objective will then be to compute the central charge associated to the Wilson surface. The apparent difficulty stems from the observation that in the M5 theory, the Wilson surfaces appears as a defect and not a fully fledge 2d CFT. Thus, we cannot recover the central simply by considering the Virasoro algebra as this one is limited to its global sub-algebra only. Instead, we will recreate the authors' results by considering the holographic entanglement entropy of a region centered on the defect. This will allow us to extract the central charge from cutoff independent quantities (as was done in [RT06a]).

Consider the following metric which describes a Riemann surface fibred over $AdS_3 \times S^3 \times S^3$.

$$ds^2 = f_1^2 ds^2_{AdS_3} + f_2^2 ds^2_{S^3} + f_3^2 ds^2_{S^3} + 2\Omega^2 |dw|^2 \tag{23}$$

$$ds^2_{AdS_3} = \frac{1}{u^2}\left(du^2 - dt^2 + dx^2_{||}\right)$$

We wish to find a minimal surface at fixed t which ends on the boundary of AdS_3. To do so, we consider such a surface through its embedding $\mathcal{M}_{RT} \xrightarrow{\phi} AdS_3 \times S^3 \times S^2 \times \Sigma$, where $\phi(\partial\mathcal{M}_{RT}) \subset AdS_3$. Thus, we wish to minimize the action

$$\mathcal{M}_{RT} \xrightarrow{\phi} A[\mathcal{M}_{RT}] = \int_{\mathcal{M}_{RT}} vol(\mathcal{M}_{RT}) \tag{24}$$

where $vol(\mathcal{M}_{RT})$ is the volume form induced by the embedding ϕ, that is $vol(\mathcal{M}_{RT}) = vol_{\phi^* g}$. Since our Ryu-Takayanagi surface exists at fixed time and has codimension 2, we can constrain the form of the embedding.

$$(\phi_1, \phi^i_{S^3}, \phi^j_{S^3}, \phi_w, \phi_{\bar{w}}) \xrightarrow{\phi} (u, x_{||}, \varphi^i_{S^3}, \varphi^j_{S^3}, w, \bar{w})$$

In this general form, by packaging the various coordinates into the symbols ϕ_μ and x_M, the induced metric takes the form

$$(\phi^* g)_{ij} = \sum_{M,N=1}^{10} g_{MN} \left(\frac{\partial x^M}{\partial \phi^\mu}\right)\left(\frac{\partial x^N}{\partial \phi^\nu}\right)$$

As indicated in [Est+19] the embedding ϕ which extremises this surface area is that for which

$$u^2 = l^2 - x^2_{||}$$

Alternatively, we can write

$$(\phi_1, \varphi^i_{S^3}, \varphi^j_{S^3}, w, \bar{w}) \xrightarrow{\phi} (l\sin(\phi_1), l\cos(\phi_1), \varphi^i_{S^3}, \varphi^j_{S^3}, w, \bar{w})$$

Having identified this solution, we are able to compute the components of the induced metric.

$$\begin{aligned}(\phi^* g)_{11} &= g_{11}\left(\frac{\partial u}{\partial \phi_1}\right)^2 + g_{22}\left(\frac{\partial x_{||}}{\partial \phi_1}\right)^2 \\ &= \frac{f_1^2}{u^2} l^2 \\ (\phi^* g)_{ii} &= g_{ii} \quad i > 1\end{aligned}$$

$$\begin{aligned}|\phi^* g| &= f_1^2 \frac{l^2}{u^2} (f_2^2)^{\dim(S^3)} |g_{S^3}| (f_3^2)^{\dim(S^3)} |g_{S^3}| \Omega^4 \\ &= \frac{l^2}{u^2} f_1^2 f_2^6 f_3^6 |g_{S^3}|^2 \Omega^4\end{aligned}$$

Hence, this minimal surface has a volume

$$A[\mathcal{M}_{RT}] = \mathrm{Vol}(S^3)^2 \int dw d\bar{w} \frac{\Omega^2}{f_1^2} f_1^3 f_2^3 f_3^3 \int_0^\pi d\phi_1 \frac{1}{\sin(\phi_1)}$$

The final integral is the above expression can be written in a form which benefits our analysis. Indeed, given the divergence of such an integral, we are encouraged to enforce a cutoff at $\phi_1 = \epsilon_\phi$ and $\phi = \pi - \epsilon_\phi$, which correspond to a cutoff at 'small' u. Indeed, let us perform the change of variable $u = l\sin(\phi_1)$, for which $du = l\cos(\phi_1)d\phi_1$ for both intervals $[\epsilon_\phi, \frac{\pi}{2}]$ and $[\frac{\pi}{2}, \pi - \epsilon_\phi]$.

$$\begin{aligned}\int_{\epsilon_\phi}^{\pi-\epsilon_\phi} d\phi_1 \frac{1}{\sin(\phi_1)} &= \int_{\epsilon_\phi}^{\frac{\pi}{2}} d\phi_1 \frac{1}{\sin(\phi_1)} + \int_{\frac{\pi}{2}}^{\pi-\epsilon_\phi} d\phi_1 \frac{1}{\sin(\phi_1)} \\ &= \int_{\epsilon_u}^{l} du \frac{1}{u\cos(\sin^{-1}(\frac{u}{l}))} + \int_{l}^{\epsilon_u} du \frac{1}{u\cos(\sin^{-1}(\frac{u}{l}))}\end{aligned}$$

We can compute this regulated integral by considering the asymptotic behaviour as ϵ_u tends to zero.

$$\begin{aligned}\int_{\epsilon_u}^{l} \frac{l}{u\sqrt{l^2-u^2}} &= \int_0^{\sqrt{1-\epsilon_u^2/l^2}} dv \frac{1}{1-v^2} \\ &= \int_0^{\tanh^{-1}(\sqrt{1-\epsilon_u^2/l^2})} dx \\ &= \frac{1}{2}\ln\left(1+\sqrt{1-\epsilon_u^2/l^2}\right) - \frac{1}{2}\ln\left(1-\sqrt{1-\epsilon_u^2/l^2}\right) \\ &= \frac{1}{2}\ln\left(2 - \frac{\epsilon_u^2}{2l^2} + o(\epsilon_u^2)\right) - \frac{1}{2}\ln\left(\frac{\epsilon_u^2}{2l^2} + o(\epsilon_u^2)\right) \\ &= \frac{1}{2}\ln\left(\frac{4-\epsilon_u^2/l^2}{\epsilon_u^2/l^2} + o(1)\right) \\ &= \frac{1}{2}\ln\left(\frac{4l^2}{\epsilon_u^2} - 1 + o(1)\right) \\ &= \ln\left(\frac{2l}{\epsilon_u}\right) + o(\epsilon_u)\end{aligned}$$

We can then insert this result in the Ryu-Takayanagi formula.

$$S_{EE} = \frac{2\mathrm{Vol}(S^3)^2}{4G_N} \ln\left(\frac{2l}{\epsilon_u}\right) \int dw d\bar{w} \left(\frac{\Omega^2}{f_1^2} f_1^3 f_2^3 f_3^3\right) + o(\epsilon_u) \tag{25}$$

The objective in the following pages will be to compute the above integral. To do so, we will use our cutoffs (REF) together with the parametrisations of the fibration coefficients (REF).
First of all, using (REF), we can simplify the following

$$\frac{\Omega^2}{f_1^2} = \frac{1}{2}\frac{|\partial_w|^2}{c_2 c_3 h^2}(G\bar{G}-1)c_1^2$$

$$f_1^2 f_2^2 f_3^2 = \frac{h^2}{c_1^2 c_2^2 c_3^2}$$

$$\begin{aligned}\mathcal{I} &= \frac{2(2\pi^2)^2}{4G_N} \int dw d\bar{w} \frac{c_1^2|\partial_w h|^2}{c_2 c_3 h^2}(1-G\bar{G})\frac{h^3}{c_1^3 c_2^3 c_3^3} \\ &= \frac{\pi^4}{G_N}\frac{1}{c_1 c_2^4 c_3^4}\int dw d\bar{w} |\partial_w h|^2 h(1-G\bar{G}) \\ &= \frac{2\pi^4}{G_N}\frac{1}{c_1 c_2^4 c_3^4}\int_0^\pi d\theta \sin(\theta) \int_0^{r_c} dr r^2 (1-G\bar{G})\end{aligned}$$

6 Asymptotically Locally $AdS_7 \times S^4$ Solutions

We can use equation (2.6) from [Est+19],

$$G = -i\left(1 + \sum_{j=1}^{2n+2} (-1)^j \frac{w - \xi_j}{|w - \xi_j|}\right) \qquad h = -i(w - \bar{w}) \tag{26}$$

and plug it in the entanglement entropy integral.

$$\begin{aligned}
G\bar{G} &= \left(1 + \sum_{j=1}^{2n+2} (-1)^j \frac{w - \xi_j}{|w - \xi_j|}\right)\left(1 + \sum_{k=1}^{2n+2} (-1)^k \frac{w - \xi_k}{|w - \xi_k|}\right) \\
&= 1 + \sum_{j=1}^{2n+2} (-1)^j \frac{w + \bar{w} - 2\xi_j}{|w - \xi_j|} + \sum_{j,k=1}^{2n+2} (-1)^{j+k} \frac{(w - \xi_j)(\bar{w} - \xi_k)}{|w - \xi_j||w - \xi_k|} \\
&= 1 + \sum_{j=1}^{2n+2} (-1)^j \frac{2r\cos(\theta) - 2\xi_j}{|re^{i\theta} - \xi_j|} + \sum_{j,k=1}^{2n+2} (-1)^{j+k} \frac{(re^{i\theta} - \xi_j)(re^{-i\theta} - \xi_k)}{|re^{i\theta} - \xi_j||re^{i\theta} - \xi_k|}
\end{aligned}$$

Hence,

$$\begin{aligned}
\mathcal{I} &= \frac{2(2\pi^2)^2}{4G_N} \frac{2}{c_1 c_2^4 c_3^4} \int_0^\pi d\theta \sin(\theta) \int_0^{r_c} dr r^2 \int_{j=1}^{2n+2} (-1)^j \frac{2r\cos(\theta) - 2\xi_j}{|re^{i\theta} - \xi_j|} \\
&+ \frac{2(2\pi^2)^2}{4G_N} \frac{2}{c_1 c_2^4 c_3^4} \int_0^\pi d\theta \sin(\theta) \int_0^{r_c} dr r^2 \sum_{j,k=1}^{2n+2} (-1)^{j+k} \frac{(re^{i\theta} - \xi_j)(re^{-i\theta} - \xi_k)}{|re^{i\theta} - \xi_j||re^{i\theta} - \xi_k|}
\end{aligned}$$

Let us rewrite the following expressions

$$\begin{aligned}
&\sum_{j,k=1}^{2n+2} (-1)^{j+k} \frac{(re^{i\theta} - \xi_j)(re^{-i\theta} - \xi_k)}{|re^{i\theta} - \xi_j||re^{i\theta} - \xi_k|} = \sum_{\substack{j,k=1 \\ j<k}}^{2n+2} (\cdots) + \sum_{\substack{j,k=1 \\ j>k}}^{2n+2} (\cdots) + \sum_{\substack{j,k=1 \\ j=k}}^{2n+2} (\cdots) \\
&= 2n + 2 + 2 \sum_{\substack{j,k=1 \\ j<k}}^{2n+2} (-1)^{j+k} \frac{r^2 - r\cos(\theta)(\xi_j + \xi_k) + \xi_j \xi_k}{\sqrt{r^2 - 2r\xi_j\cos(\theta) + \xi_j^2}\sqrt{r^2 - 2r\xi_k\cos(\theta) + \xi_k^2}}
\end{aligned}$$

Upon placing this expression back into the integral, we can define parts of the integral which we can evaluate separately.

$$\mathcal{I} = \frac{2(2\pi^2)^2}{4G_N} \frac{4}{c_1 c_2^4 c_3^4} (J + K_0 + K_1 + K_2 + K_3)$$

$$
\begin{aligned}
J &= \sum_{j=1}^{2n+2} \int_0^\pi d\theta \sin(\theta) \int_0^{r_c} dr(-1)^j \frac{r^2(r\cos(\theta)-\xi_j)}{|re^{i\theta}-\xi_j|} \\
K_0 &= \int_0^\pi d\theta \cos(\theta) \int_0^{r_c} drr^2(n+1) = \int_0^\pi d\theta \sin(\theta) \frac{r_c^3}{3}(n+1) \\
K_1 &= \sum_{\substack{j,k=1 \\ j<k}}^{2n+2} \int_0^\pi d\theta \sin(\theta) \int_{|\xi_k|}^{r_c} drr^2(-1)^{j+k} \frac{r^2 - r\cos(\theta)(\xi_j+\xi_k)+\xi_j\xi_k}{\sqrt{r^2-2r\xi_j\cos(\theta)+\xi_j^2}\sqrt{r^2-2r\xi_k\cos(\theta)+\xi_k^2}} \\
K_2 &= \sum_{\substack{j,k=1 \\ j<k}}^{2n+2} \int_0^\pi d\theta \sin(\theta) \int_{|\xi_j|}^{|\xi_k|} drr^2(-1)^{j+k} \frac{r^2 - r\cos(\theta)(\xi_j+\xi_k)+\xi_j\xi_k}{\sqrt{r^2-2r\xi_j\cos(\theta)+\xi_j^2}\sqrt{r^2-2r\xi_k\cos(\theta)+\xi_k^2}} \\
K_3 &= \sum_{\substack{j,k=1 \\ j<k}}^{2n+2} \int_0^\pi d\theta \sin(\theta) \int_0^{|\xi_j|} drr^2(-1)^{j+k} \frac{r^2 - r\cos(\theta)(\xi_j+\xi_k)+\xi_j\xi_k}{\sqrt{r^2-2r\xi_j\cos(\theta)+\xi_j^2}\sqrt{r^2-2r\xi_k\cos(\theta)+\xi_k^2}}
\end{aligned}
$$

Since we imposed ordering of the poles $j<k \implies \xi_j < \xi_k$, each integral K_i can be expanded in terms in Legendre polynomials. Indeed, for $|\frac{r}{\xi_j}|<1$,

$$
\sum_{l\in\mathbb{N}} P_i(\cos(\theta)) \left(\frac{r}{\xi_j}\right)^i = \frac{1}{\sqrt{\frac{r^2}{\xi_j^2} - \frac{2\cos(\theta)r}{\xi_j} + 1}} = \frac{|\xi_j|}{\sqrt{r^2 - 2\cos(\theta)r\xi_j + \xi_j^2}}
$$

As an aside, note the following Legendre polynomial identities

$$
\int_0^\pi P_l(\cos(\theta))P_m(\cos(\theta))\sin(\theta)d\theta = \int_{-1}^1 P_l(x)P_m(x)dx = \delta_{lm}\frac{2}{2l+1}
$$

$$
xP_l(x) = \frac{l+1}{2l+1}P_{l+1}(x) + \frac{l}{2l_1}P_{l-1}(x)
$$

$$
\begin{aligned}
\int_0^\pi P_l(\cos(\theta))P_m(\cos(\theta))\sin(\theta)\cos(\theta)d\theta &= \int_{-1}^1 P_l(x)P_m(x)xdx \\
&= \frac{l+1}{2l+1}\frac{2}{2l+3}\delta_{m,l+1} + \frac{l}{2l+1}\frac{2}{2l-1}\delta_{m,l-1}
\end{aligned}
$$

6.1 Computing J

6.2 Computing K

Let us our Legendre polynomial identities in the K integrals.

$$
\begin{aligned}
K_1 &= \sum_{\substack{j,k=1\\ j<k}}^{2n+2} \int_0^\pi d\theta \sin(\theta) \int_{|\xi_k|}^{r_c(\theta,\epsilon_v)} drr^2(-1)^{j+k} \frac{r^2 - r\cos(\theta)(\xi_j+\xi_k)+\xi_j\xi_k}{\sqrt{r^2-2r\xi_j\cos(\theta)+\xi_j^2}\sqrt{r^2-2r\xi_k\cos(\theta)+\xi_k^2}} \\
&= \sum_{\substack{j,k=1\\ j<k}}^{2n+2} \int_0^\pi d\theta \sin(\theta) \int_{|\xi_k|}^{r_c(\theta,\epsilon_v)} dr(-1)^{j+k}(r^2 - r\cos(\theta)(\xi_j+\xi_k)+\xi_j\xi_k) \sum_{l,m=0}^{\infty} P_l(\cos(\theta))P_m(\cos(\theta)) \left(\frac{\xi_j}{r}\right)^l \left(\frac{\xi_k}{r}\right)^m \\
&= \sum_{\substack{j,k=1\\ j<k}}^{2n+2} (-1)^{j+k} \sum_{l,m=0}^{\infty} \xi_j^l \xi_k^m \int_{-1}^{1} dx P_l(x)P_m(x) \int_{|\xi_k|}^{r_c(x,\epsilon_v)} dr(r^{2-l-m} - r^{1-l-m}x(\xi_j+\xi_k) + r^{-l-m}\xi_j\xi_k) \\
&= \sum_{\substack{j,k=1\\ j<k}}^{2n+2} (-1)^{j+k} \sum_{l,m=0}^{\infty} \xi_j^l \xi_k^m \int_{-1}^{1} dx P_l(x)P_m(x) \left[\left.\frac{r^{3-l-m}}{3-l-m}\right|_{3-l-m\neq 0} - \left.\frac{r^{2-l-m}}{2-l-m}\right|_{2-l-m\neq 0} x(\xi_j+\xi_k) \right. \\
&\qquad \left. + \left.\frac{r^{1-l-m}}{1-l-m}\right|_{1-l-m\neq 0} \xi_j\xi_k + \ln(r)\left(\delta_{3-l-m,0} - x(\xi_j+\xi_k)\delta_{2-l-m,0} + \xi_j\xi_k\delta_{1-l-m,0}\right) \right]_{|\xi_k|}^{r_c(x,\epsilon_v)} \\
&= K_1^{\text{upper}} + K_1^{\text{lower}}
\end{aligned}
$$

$$K_1^{\text{upper}} = ...$$

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