COT 3100 Homework #10: Functions and Relations
Due Date: Friday, April 18th, in recitation

1) Let A, B be sets with |B| = 5. If there are 2^{30} relations from A to B, what is |A|?

2) a) Give an example of a relation that is irreflexive and transitive, but not symmetric.
b) Let R be a non-empty relation on a set A. Prove that if R satisfies any of the two following properties – irreflexive, symmetric, transitive – then it can not satisfy the third.

3) Let |A| = 12. Determine the number of binary relations on AxA that satisfy the following properties:

a) reflexive
b) neither reflexive nor irreflexive
c) symmetric
d) reflexive and symmetric
e) irreflexive and anti-symmetric

4) With proof, determine if the following relations are equivalence relations, partial ordering relations, or neither.

a) $\{ (a, b) \mid a \in Z^+, b \in Z^+, a > 2b \text{ or } b > 2a \}$
b) $\{ (a, b) \mid a \in Z^+, b \in Z^+, a \equiv 0 \bmod b \text{ or } b \equiv 0 \bmod a \}$
c) $\{ (a, b) \mid a \in Z^+, b \in Z^+, \text{ and } \exists c \in Z^+ \text{ such that } b = ac\}$

5) Let g: A $\rightarrow$ A be a bijection. For $n \geq 2$, define $g^n = g \circ g \circ \ldots \circ g$, where g is composed with itself n times. Prove that for $n \geq 1$, that $(g^n)^{-1} = (g^{-1})^n$, by using induction on n.

6) Prove that following function is a bijection from the open interval (0,5) to the positive real numbers:

$$f(x) = \frac{5-x}{5x}$$

7) Determine the inverse of the function defined in question #6.

8) Let f : A $\rightarrow$ B and g: B $\rightarrow$ C denote two functions. Prove or disprove that if $g \circ f : A \rightarrow C$ is injective, then *f* is injective.

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