COT 3100 Homework #8: Counting
Due Date: Friday, March 28th, in recitation

Note: Please _justify_ your answers and why you use each formula.

1) Consider an ant that is walking on a Cartesian grid, starting at (0,0) and ending at (10, 15). The ant always chooses to walk exactly one unit either up or to the right (towards his destination) whenever he arrives at a Lattice point. (A Lattice point is a point with integer coordinates.) Thus, from (0,0) he either walks to (1, 0) or (0, 1). How many different paths can he take on his walk?

2) This question considered permutations of "ENGINEERING".

   a) How many permutations are there total?

   b) How many permutations start and end with vowels?

   c) How many permutations do NOT have consecutive vowels in them?

   d) How many permutations are the letters in alphabetical order?

   e) How many permutations contain the substring "RING"?

3) A class contains 22 girls and 18 boys. For all parts of this question, each boy and girl are distinguishable from one another. Answer the following questions:

   a) In how many ways can a committee of one boy and one girl be chosen?

   b) In how many ways can a committee of five students be chosen?

   c) In how many ways can a committee of two girls and three boys be chosen?

   d) In how many ways can a committee of five students be chosen such that all the students on the committee are the same sex?

   e) In how many ways can the girls and boys form a line where no two boys are standing next to one another?

   f) How many committees of five students contain at least two girls?

4) How many solutions does the equation $a + b + c + d + e + f = 20$ have if each variable must be a non-negative integer and $a < 3$, $b < 5$ and $d > 4$?

5) How many solutions does the equation $a + b + c + d + e + f + g \le 50$ have if each variable must be a non-negative integer?

6) There are N users and M servers with $M \ge N$. Each user can send a request to any of the servers. Determine the number of situations in which at least one collision occurs, i.e., there is at least one pair of users that send the request to the same server.

7) How many integers in between 1 and 10^7 are divisible by 3, 5 or 7?