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HW2
10/8/15

1) a) $(t^2-2t+1)u(t)$ ← shifts to $e^{-t_0 s}$

$\mathcal{L}(t^2-2t+1)$ $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$ => $\left(\frac{2}{s^3} - 2\frac{1}{s^2} + \frac{1}{s}\right)$ => $\mathcal{L}((t^2-2t+1)u(t))$
$= 1\left[\frac{2}{s^3} - 2\frac{1}{s^2} + \frac{1}{s}\right]$
$\boxed{= \frac{s^2-2s+2}{s^3}}$

b) $\mathcal{L}[(t+1)^2 u(t-1)]$

$\mathcal{L}(t+1)^2 = \mathcal{L}(t^2+2t+1) = \frac{2}{s^3} + 2\frac{1}{s^2} + \frac{1}{s}$

$\boxed{\mathcal{L}[(t+1)^2 u(t-1)] = e^{-s}\left[\frac{2}{s^3} + \frac{2}{s} + \frac{1}{s}\right]}$

c) $(te^{-2t}u(t-8))$

$\mathcal{L}(te^{-2t}) = \frac{1}{(s+2)^2}$ → $\boxed{\frac{e^{-8s}}{(s+2)^2}}$

d) $te^{-2t}\cos(2t-1)u(t-1)$

$\mathcal{L}[e^{at}f(t)](s) = F(s-a)$ $\quad e^{it} = \cos(t) + i\sin t$

$\mathcal{L}[\cos(2t-1)]$ $\quad \boxed{\mathcal{L}(\cos(at+b)) = \frac{s\cos b - a\sin b}{s^2+a^2}}$ $\quad Re[\mathcal{L}[e^{it}]] = \mathcal{L}(\cos t)$

$= \frac{s\cos(-1) - 2\sin(-1)}{s^2+2^2}$

$\mathcal{L}[\cos(2t-1)\,t]$

$\mathcal{L}[[\cos(2t-1)te^{-2t}]] = e^{-1s} \cdot \frac{d}{ds} \frac{(s+2)\cos(1) + 2\sin(1)}{(s+2)^2+2^2}$

$\frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$\frac{(s+2)\cos(1) + 2\sin(1)}{s^2+4s+8}$

ugh

$\frac{(s^2+4s+8)(\cos(1)) - ((s+2)\cos(1) + 2\cos(1) + 2\sin(1))(2s+4)}{(s^2+4s+8)^2}$

$$\boxed{= \frac{s^2\cos(1) + 4s(\sin(1)+\cos(1)) + 8\sin(1)}{(s^2+4s+8)^2} \cdot e^{-s}}$$

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e) $2t\cos(\omega t+\theta)u(t)$

$\mathcal{L}[t\cos(\omega t+\theta)] = (-1)^n \frac{d}{dt}\left[\frac{s\cos(\theta) - \omega\sin\theta}{s^2+\omega^2}\right]$

$\frac{(s^2+\omega^2)(\cos\theta) - s\cos\theta(2s) + 2s\omega\sin\theta}{(s^2+\omega^2)^2}$

$= s^2\cos\theta + \omega^2\cos\theta - 2s^2\cos\theta + 2s\omega\sin\theta$

$= \omega^2\cos\theta - s^2\cos\theta + 2s\omega\sin\theta$

$\mathcal{L}[2t\cos(\omega t+\theta)u(t)] = (-1)\frac{2(-s^2\cos\theta + \omega^2\cos\theta + 2s\omega\sin\theta)}{(s^2+\omega^2)^2}$

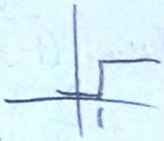
$\frac{d}{dt}\left[\frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}\right] = \frac{\omega^2\cos\theta + 2\omega s\sin\theta - s^2\cos\theta}{(s^2+\omega^2)^2}$

f) $\mathcal{L}\left(\int_0^t \frac{\sin x}{x}dx\right) \Rightarrow \frac{F(s)}{s}$ $\mathcal{L}\left(\frac{\sin t}{t}\right) = \mathcal{L}\,\text{sinc}(t)$

$= \cot^{-1}(s)$

$\frac{\cot^{-1}(s)}{s}$

2) $\frac{s+2}{s^2+4s+20}$ $Y(s) = H(s)U(s)$ $U(s) = \frac{e^{-s}}{s}$ $u(t-1)$ $\frac{1}{s}e^{-1s}$

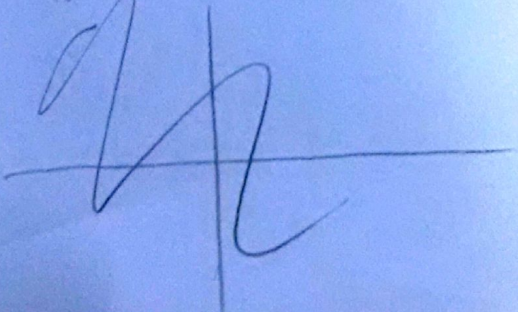


$Y(s) = \frac{e^{-s}}{s}\left(\frac{s+2}{s^2+4s+20}\right)$

$\sin(\omega t) \Rightarrow \left(\frac{\omega}{s^2+\omega^2}\right)$

$Y(s) = \left(\frac{\omega}{s^2+\omega^2}\right)\left(\frac{s+2}{s^2+4s+20}\right)$

$u(t-1)$



$\longrightarrow$

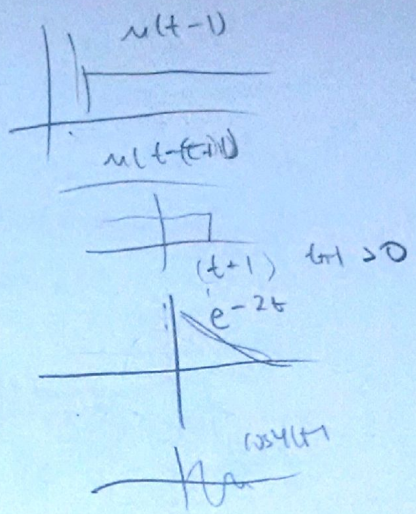
$$\mathcal{L}^{-1}\left[\frac{s+2}{s^2+4s+20}\right] = \frac{1}{2}e^{(-2-4i)t}(1+e^{8it}) = h(t)$$

$h(t) * u(t-1)$

2) a)

$$\frac{s+2}{s^2+4s+20} = \frac{s+2}{(s+2)^2+4^2} = \cos 4t\, e^{-2t}$$

$$e^{-2t}\cos 4t * (u(t-1))$$



$$\int_{-\infty}^{\infty} e^{-2\tau}\cos 4(\tau)\cdot u(t-\tau+1)$$

$$\int_0^{t+1} e^{-2\tau}\cos 4\tau$$

$\int uv' = uv - \int u'v$

$u = \cos 4\tau \quad v' = e^{-2\tau}$

$u' = -4\sin 4\tau \quad v = -\frac{1}{2}e^{-2\tau}$

$$\int e^{-2\tau}\cos 4\tau = (\cos 4\tau)\left(-\frac{1}{2}e^{-2\tau}\right) - \int(-4\sin 4\tau)\left(-\frac{1}{2}e^{-2\tau}\right)d\tau$$

$$= -\frac{1}{2}e^{-2\tau}\cos 4\tau - \int 2e^{-2\tau}\sin 4\tau\, d\tau$$

let $u = \sin 4\tau \quad v' = 2e^{-2\tau}$

$u' = 4\cos 4\tau \quad v = -e^{-2\tau}$

$$-\frac{1}{2}e^{-2\tau}\cos 4\tau - \left[-e^{-2\tau}\sin 4\tau + \int 4e^{-2\tau}\cos 4\tau\, d\tau\right]$$

$$\int e^{-2\tau}\cos 4\tau = -\frac{1}{2}e^{-2\tau}\cos 4\tau + e^{-2\tau}\sin 4\tau - 4\int e^{-2\tau}\cos 4\tau\, d\tau$$

$$\frac{1}{5}\,5\int e^{-2\tau}\cos 4\tau = \frac{1}{5}\left[-\frac{1}{2}e^{-2\tau}\cos 4\tau + e^{-2\tau}\sin 4\tau\right]$$

$$-\frac{1}{10}e^{-2\tau}\cos 4\tau + \frac{1}{5}e^{-2\tau}\sin 4\tau \Big|_0^{t+1}$$

$$\boxed{y(t)} = -\frac{1}{10}e^{-2(t+1)}\cos(4t+4) + \frac{1}{5}e^{-2(t+1)}\sin(4t+4) + \frac{1}{10}\cos(0)$$

2) b)

$\dfrac{s+1}{s^3+9s^2+26s+24}$

$\boxed{u(t-1)}$

$\mathcal{L}\ \dfrac{s+1}{(s+2)(s+3)(s+4)}$

$\dfrac{A}{(s+2)} + \dfrac{B}{(s+3)} + \dfrac{C}{(s+4)} = \dfrac{s+1}{(s+2)(s+3)(s+4)}$

$A(s+3)(s+4) + B(s+2)(s+4) + C(s+2)(s+3) = s+1$

$As^2 + A7s + 12A + Bs^2 + B6s + 8B + Cs^2 + 5Cs + 6C = s+1$

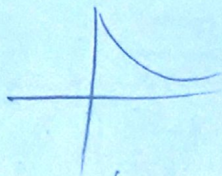
$A+B+C = 0$ $\quad A = -\frac{1}{2} \quad B = 2$

$7A + 6B + 5C = 1$

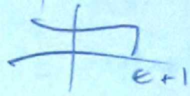
$12A + 8B + 6C = 1$ $\quad C = -\frac{3}{2}$

$\mathcal{L}^{-1}\left(\dfrac{-\frac{1}{2}}{s+2} + \dfrac{2}{s+3} - \dfrac{\frac{3}{2}}{s+4}\right) = \boxed{-\frac{1}{2}e^{-2t} + 2e^{-3t} - \frac{3}{2}e^{-4t}} = h(t)$

$\int_{-\infty}^{\infty} \left(-\frac{1}{2}e^{-2\tau} + 2e^{-3\tau} - \frac{3}{2}e^{-4\tau}\right) u(t-\tau+1)$



$\int_0^{t+1} \left(-\frac{1}{2}e^{-2\tau} + 2e^{-3\tau} - \frac{3}{2}e^{-4\tau}\right) d\tau$

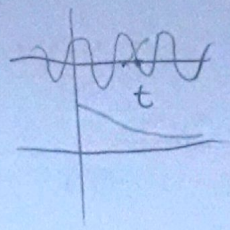


$+\frac{1}{4}e^{-2\tau} - \frac{2}{3}e^{-3\tau} + \frac{3}{8}e^{-4\tau} \Big|_0^{t+1}$

$\boxed{y(t) = \frac{1}{4}e^{-2(t+1)} - \frac{2}{3}e^{-3(t+1)} + \frac{3}{8}e^{-4(t+1)} + \frac{1}{4} - \frac{2}{3} + \frac{3}{8}}$

b) Sh! uh— gotta do convolution again I guess

$$\int_{-\infty}^{\infty} e^{-2\tau}\cos 4\tau \cdot \sin(t-w\tau)\, d\tau$$



$$\int_0^t e^{-2\tau}\cos 4\tau\,(-\sin w\tau)$$

Unless w is the same as 4, then you have major problems

$$\int_0^t e^{-2\tau}\cos 4\tau\,(-\sin 4\tau)$$

$\frac{\sin 2x}{2} = \cos \sin$

$$-\int \frac{\sin(2\cdot 4\tau)}{2} e^{-2\tau}\, d\tau = -\frac{1}{2}\left[\int e^{-2t}\sin 8t\, dt\right]$$

let $u = \sin 8\tau$, $du = +8\cos 8\tau$; $dv = e^{-2\tau}$, $v = -\frac{1}{2}e^{-2\tau}$

$$\int e^{-2\tau}\sin 8\tau\, d\tau = \left[-\frac{1}{2}e^{-2\tau}\sin 8\tau - \int -\frac{1}{2}e^{-2\tau}\, 8\cos 8\tau\, d\tau\right]$$

$$= -\frac{1}{2}e^{-2\tau}\sin 8\tau + \int 4e^{-2\tau}\cos 8\tau\, d\tau$$

let $u = \cos 8\tau$, $du = -8\sin 8\tau$; $dv = 4e^{-2\tau}$, $v = -2e^{-2\tau}$

$$= -\frac{1}{2}e^{-2\tau}\sin 8\tau + \left[-2e^{-2\tau}\cos 8\tau - \int -8\sin 8\tau\,(-2e^{-2\tau})\, d\tau\right]$$

$$\int e^{-2\tau}\sin 8\tau\, d\tau = -\frac{1}{2}e^{-2\tau}\sin 8\tau - 2e^{-2\tau}\cos 8\tau - \int 16\sin 8\tau\, e^{-2\tau}\, d\tau$$

$$17\int e^{-2\tau}\sin 8\tau\, d\tau = -\frac{1}{2}e^{-2\tau}\sin 8\tau - 2e^{-2\tau}\cos 8\tau$$

$$\int e^{-2\tau}\sin 8\tau\, d\tau = -\frac{1}{34}e^{-2\tau}\sin 8\tau - \frac{2e^{-2\tau}}{17}\cos 8\tau$$

so

$$\int_0^t e^{-2\tau}\cos 4\tau\,(-\sin 4\tau) = -\frac{1}{2}\left(\int e^{-2\tau}\sin 8\tau\, d\tau\right) = \frac{1}{68}\left(e^{-2\tau}\sin 8t - e^{-2\tau}\cos 8\tau\right)\Big|_0^t$$

$$\boxed{y(t) = \frac{1}{68}\left[e^{-2t}\sin 8t - e^{-2t}\cos 8t\right]}$$

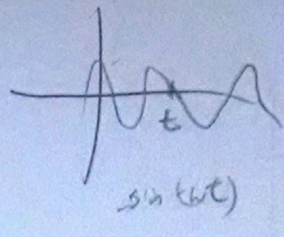
Jesus Christ

2) b) (sin wt) , sin wt

continued

2) b)

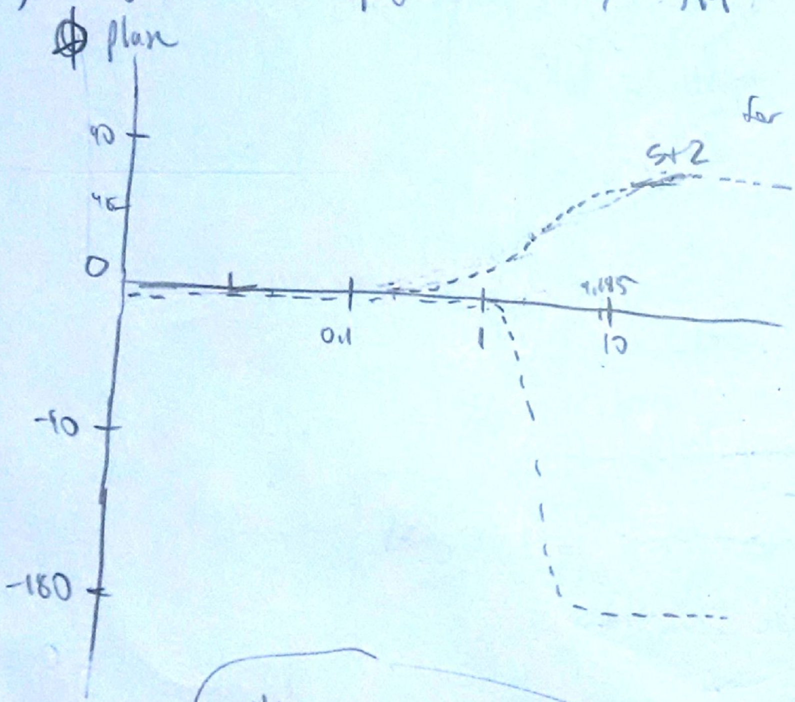
$$\int_0^t -\frac{1}{2}e^{-2\tau} + 2e^{-3\tau} - \frac{3}{2}e^{-4\tau} \cdot \sin(t-w\tau)$$



$$\int_0^t -\frac{1}{2}e^{-2\tau} + 2e^{-3\tau} - \frac{3}{2}e^{-4\tau} (\sin w\tau)$$

$$y(t) = \frac{-e^{-2t}(2\sin wt + w\cos wt)}{2(w^2+2^2)} + \frac{2e^{-3t}(3\sin wt + w\cos wt)}{w^2+3^2} + \frac{3e^{-4t}}{2}\frac{(4\sin wt + w\cos wt)}{(w^2+4^2)}$$

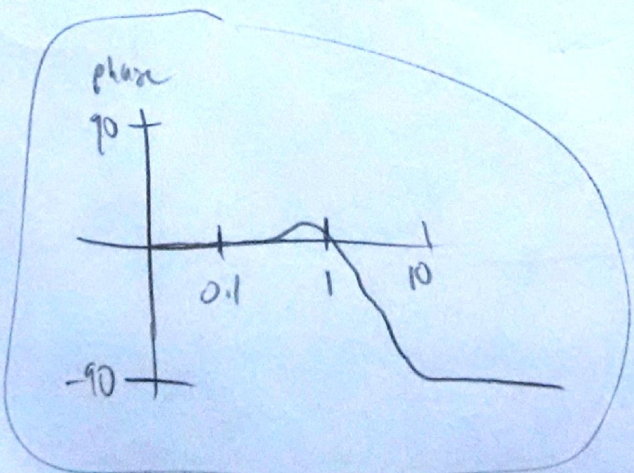
3) mag plot is on page 1 back, log paper



phase

for $w = \frac{w_0}{10^{\zeta}} = \frac{4.472}{10^{0.4472}} = 1.596$

$w = w_0 5^{\zeta} = 9.185$



looks like this?

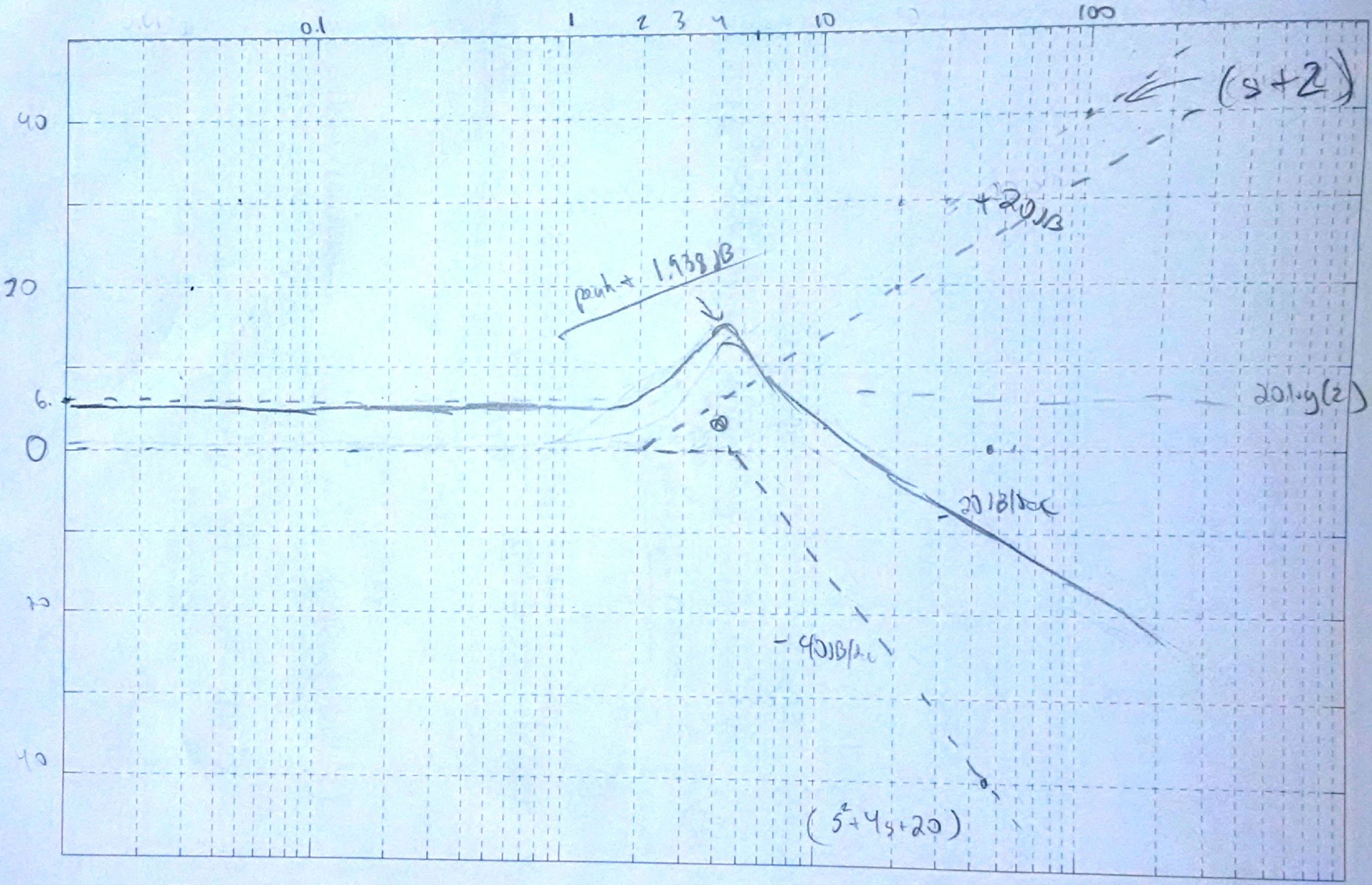
Semi-log Paper for Bode Plots

For $\frac{s+2}{s^2+4s+20}$ $2\,\frac{\frac{s}{2}+1}{s^2+4s+20}$

a zero at -2 poles at $-2 \pm 4i$

$s^2 + 2\zeta\omega_0 s + \omega_0^2$

$\omega_0 = \sqrt{20} = 4.472$ $2\zeta\omega_0 = 4$ $\zeta = \frac{4}{2\sqrt{20}} = 0.4472$



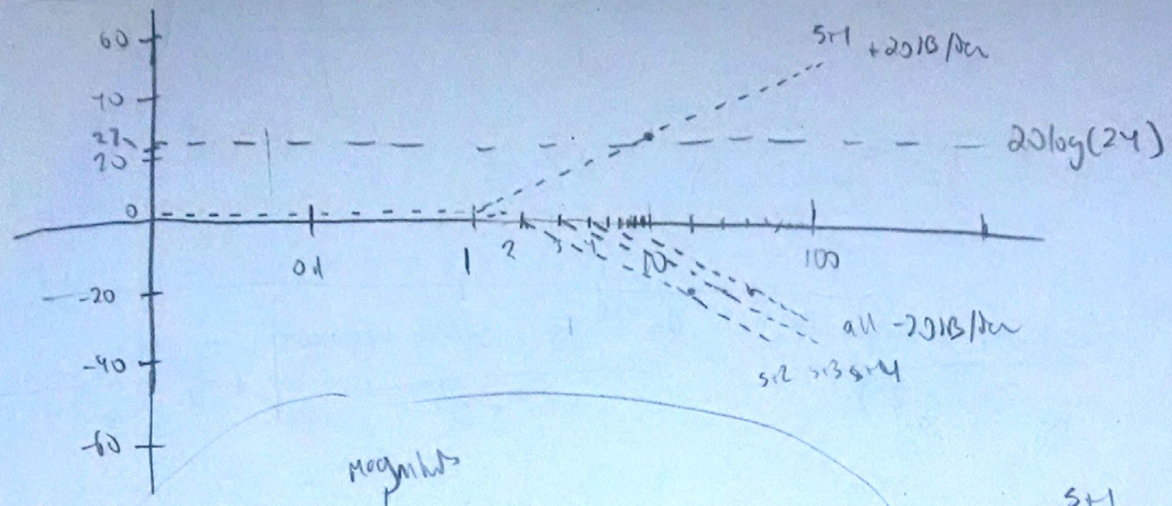
peak height $= -20\log_{10}\left(2\zeta\sqrt{1-\zeta_0^2}\right) = -20\log(0.0961) = 1.938\ dB$

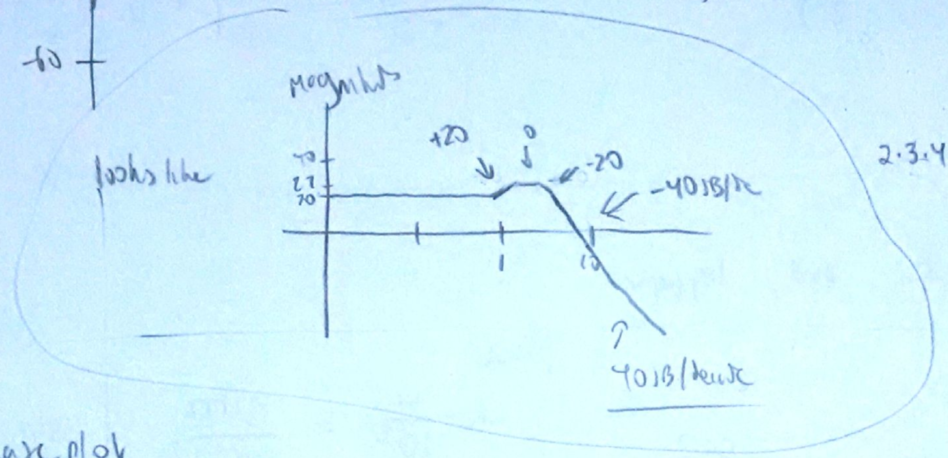
mag

$$\frac{s+1}{(s+2)(s+3)(s+4)}$$

zeros − 1

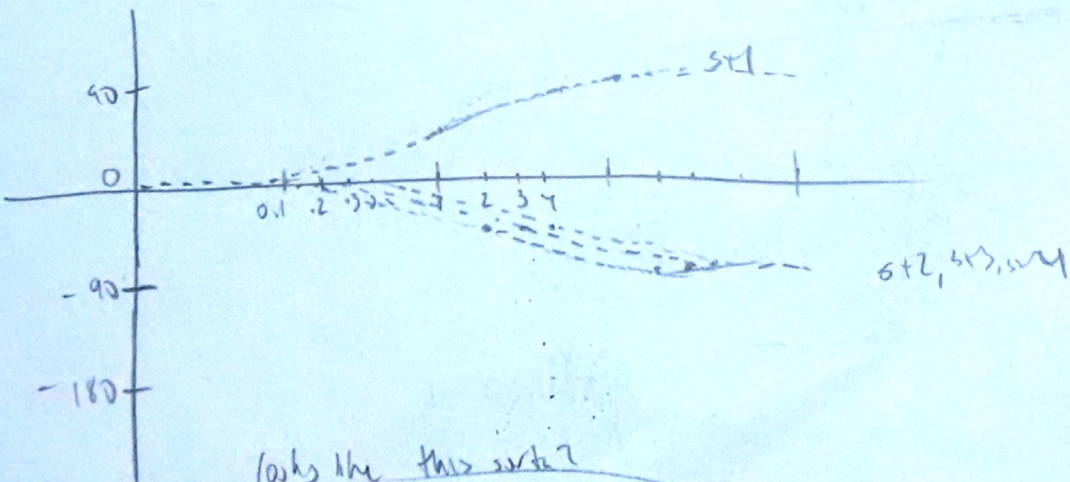
poles (2)(3)(4)





$$\frac{s+1}{2\cdot3\cdot4\left(\frac{s}{2}+1\right)\left(\frac{s}{3}+1\right)\left(\frac{s}{4}+1\right)}$$

phase plot



looks like this sorta?