Robust Adaptive Control of Planar Quadrotor UAV Carrying Unknown Payload

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Background & Motivation

- Many potential uses of rotorcraft UAVs involve the carrying of a payload
- UAVs may have to carry payloads of various sizes, weights, shapes
- In these cases, it is important to have guidance and control strategies which account for the unknown properties of the combined vehicle + payload

Figure : DHL drone delivery prototype
(Credit: MIT Technology Review)

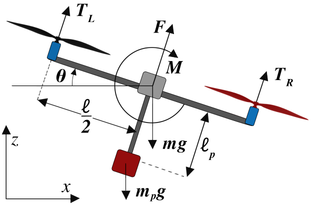
Problem Statement

Planning Given initial state x_0 and goal state x_1, find a dynamically feasible trajectory which

- Avoids obstacles
- Satisfies state and input constraints
- Is optimal, in some sense

Control With a given trajectory from the kinodynamic motion planner, design a tracking controller for the vehicle with

- Underactuated system (nonholonomic constraints)
- Parametric uncertainties $(m_p, \ell_p, c_t, \bar{c}_d)$ which may change during trajectory tracking task
- External disturbances (wind) and unmodeled actuator dynamics



Vehicle Equations of Motion

The state of the tandem-rotor vehicle is

$$X = \begin{bmatrix} x & z & \theta & \dot{x} & \dot{z} & \dot{\theta} \end{bmatrix}^T \qquad (1)$$

Nonlinear vehicle dynamics are

$$\dot{X} = \underbrace{\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{\theta} \\ \frac{-\bar{c}_d |\dot{x}| \dot{x}}{m+m_p} \\ -g - \frac{\bar{c}_d |\dot{z}| \dot{z}}{m+m_p} \\ \frac{m_p \ell_p g \sin(\theta)}{I_{yy}} \end{bmatrix}}_{f(X,t)} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{\sin(\theta)}{m+m_p} & 0 \\ \frac{\cos(\theta)}{m+m_p} & 0 \\ 0 & \frac{1}{I_{yy}} \end{bmatrix}}_{G(X,t)} \underbrace{\begin{bmatrix} F \\ M \end{bmatrix}}_{u(t)} \qquad (2)$$

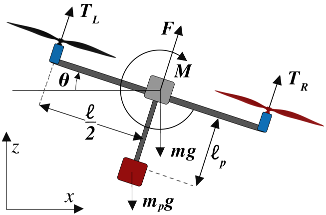


Figure : Diagram of (planar) tandem-rotor vehicle

- $I_{yy} = m\ell^2/8 + m_p\ell_p^2$ is rotational inertia
- Inputs $F = T_L + T_R + F_W$ and $M = (T_L - T_R)\ell/2 + M_W$
- F_W and M_W represent disturbances caused by wind
- Nonlinear damping from air drag, with $\bar{c}_d = \frac{1}{2}\rho A$

Planning: Kinodynamic RRT*

Sampling Based, Kinematically-Dynamically Constrained Motion Planner (kRRT*) [Webb and Van den Berg, 2013]

- Used to find obstacle free and dynamically feasible trajectory between two states
- Linearized vehicle dynamics $\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + \mathbf{c}$
- Cost function $c(\pi) = \int_0^\tau (1 + \mathbf{u}(t)^T R\mathbf{u}(t))dt$
- When optimal trajectory is infeasible (state or input constraints not satisfied or obstacles hit), states are sampled until a chain of states can be "connected" from start to goal which satisfies constraints – this is an asymptotically optimal algorithm
- From this planner, we generate $\pi^*(\mathbf{x}_0, \mathbf{x}_1) = (\mathbf{x}(t), \mathbf{u}(t), \tau^*)$ for use in trajectory tracking controller

Controller Structure

Planar quadrotor control problem:

- Underactuated, non-holonomic vehicle dynamics: nested control structure is used
- **Outer loop** (translation): determines desired force
- **Inner loop** (rotation): determines desired moment, to orient vehicle in direction of desired thrust

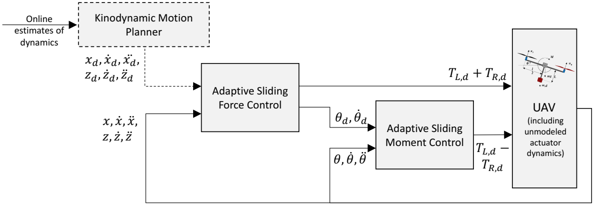


Figure : High-level block diagram of control structure

Outer Loop Controller (1)

Outer loop dynamics (translational dynamics):

$$(m + m_p)\ddot{x} + \bar{c}_d|\dot{x}|\dot{x} = (F_T + F_W)\sin(\theta) \tag{3}$$

$$(m + m_p)(\ddot{z} + g) + \bar{c}_d|\dot{z}|\dot{z} = (F_T + F_W)\cos(\theta) \tag{4}$$

Define sliding variables:

$$s_z = \dot{\tilde{z}} + \lambda\tilde{z} \tag{5}$$

$$s_x = \dot{\tilde{x}} + \lambda\tilde{x} \tag{6}$$

Outer loop control law:

$$\underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{u_{out}} = \underbrace{\begin{bmatrix} \ddot{z}_r + g, & |\dot{z}|\dot{z} \\ \ddot{x}_r, & |\dot{x}|\dot{x} \end{bmatrix}}_{Y_{out}} \underbrace{\begin{bmatrix} \hat{a}_{out,1} \\ \hat{a}_{out,2} \end{bmatrix}}_{\hat{a}_{out}} - \begin{bmatrix} k_z \mathrm{sat}(\frac{s_z}{\Phi_z}) \\ k_x \mathrm{sat}(\frac{s_x}{\Phi_x}) \end{bmatrix} \tag{7}$$

where $\ddot{z}_r = \ddot{z}_d - \lambda\dot{\tilde{z}}$ and $\ddot{x}_r = \ddot{x}_d - \lambda\dot{\tilde{x}}$

Outer Loop Controller (2)

True parameter vector:

$$a_{out} = \begin{bmatrix} \frac{m+m_p}{c_t} \\ \frac{c_d}{c_t} \end{bmatrix} \tag{8}$$

is replaced by the online estimate $\hat{a}_{out}$ with adaptive control law:

$$\dot{\hat{a}}_{out} = -\gamma Y_{out}^T \begin{bmatrix} s_{\Delta z} \\ s_{\Delta x} \end{bmatrix} \tag{9}$$

which makes use of:

$$s_{\Delta z} = s_z - \Phi_z \text{sat}(\frac{s_z}{\Phi_z}) \tag{10}$$

$$s_{\Delta x} = s_x - \Phi_x \text{sat}(\frac{s_x}{\Phi_x}) \tag{11}$$

to create a dead-zone in which adaptation is suspended
The desired control signals are computed using nonlinear transformations:

$$\theta_d = -\tan^{-1}(\frac{u_2}{u_1}) \tag{12}$$

$$F_{T,d} = \sqrt{u_1^2 + u_2^2} \tag{13}$$

Inner Loop Controller

Inner loop dynamics (rotational dynamics of vehicle):

$$I_{yy}\ddot{\theta} - m_p \ell_p g \sin(\theta) = M_T + M_W \quad (14)$$

Define sliding variable:

$$s_\theta = \dot{\tilde{\theta}} + \lambda\tilde{\theta} \quad (15)$$

Inner loop control law:

$$M_{T,d} = u_3 = \underbrace{\begin{bmatrix} \ddot{\theta}_r & \sin(\theta) \end{bmatrix}}_{Y_{in}} \underbrace{\begin{bmatrix} \hat{a}_{in,1} \\ \hat{a}_{in,2} \end{bmatrix}}_{\hat{a}_{in}} - k_\theta \mathrm{sat}\left(\frac{s_\theta}{\Phi_\theta}\right) \quad (16)$$

where $\ddot{\theta}_r = \ddot{\theta}_d - \lambda\dot{\tilde{\theta}}$

The true parameter vector:

$$a_{in} = \begin{bmatrix} \frac{I_{yy}}{c_t} \\ \frac{-m_p \ell_p}{c_t} \end{bmatrix} \quad (17)$$

is estimated by $\hat{a}_{in}$ with adaptive control law:

$$\dot{\hat{a}}_{in} = -\Gamma Y_{in}^T s_{\Delta\theta} \quad (18)$$

where

$$s_{\Delta\theta} = s_\theta - \Phi_\theta \mathrm{sat}\left(\frac{s_\theta}{\Phi_\theta}\right) \quad (19)$$

Commands sent to vehicle motors are:

$$T_{L,d} = \frac{F_{T,d}}{2} + \frac{M_{T,d}}{\ell} \quad (20)$$

$$T_{R,d} = \frac{F_{T,d}}{2} - \frac{M_{T,d}}{\ell} \quad (21)$$

Stability Outline: Inner Loop

Define Lyapunov function candidate

$$V = \frac{1}{2} s_{\Delta\theta} I_{yy} s_{\Delta\theta} + \frac{1}{2} \tilde{a}_{in}^T \Gamma^{-1} \tilde{a}_{in} \tag{22}$$

and differentiate, to get

$$\begin{aligned} \dot{V} &= I_{yy} s_{\Delta\theta} \dot{s} + \dot{\hat{a}}_{in} \Gamma^{-1} \tilde{a}_{in} \\ &= s_{\Delta\theta}(I_{yy}\ddot{\theta} - I_{yy}\ddot{\theta}_r) + \dot{\hat{a}}_{in} \Gamma^{-1} \tilde{a}_{in} \\ &= s_{\Delta\theta}(u - Y_{in} a_{in} + M_W) + \dot{\hat{a}}_{in} \Gamma^{-1} \tilde{a}_{in} \\ &= -s_{\Delta\theta} k_\theta \mathrm{sat}\left(\frac{s_\theta}{\Phi_\theta}\right) + s_{\Delta\theta}(Y_{in}\tilde{a}_{in} + M_W) + \dot{\hat{a}}_{in} \Gamma^{-1} \tilde{a}_{in} \end{aligned}$$

Substituting in the adaptation law,

$$\dot{V} = -k_\theta |s_{\Delta\theta}| + s_{\Delta\theta} M_W \tag{23}$$

- For a negative semi-definite $\dot{V}$, it is necessary to choose $k_\theta = |M_{W,max}| + \eta$, $\eta > 0$ so that $\dot{V} \leq -\eta |s_{\Delta\theta}|$
- Then by Barbalat's Lemma, $\dot{V} \to 0$ as $t \to \infty$, implying $s_{\Delta\theta} \to 0$ and $\theta \to \theta_d$

Stability Outline: Outer Loop

For approximated holonomic system (control inputs in x and z directions), Lyapunov function candidate

$$V = \frac{1}{2}\mathbf{s}_\Delta^T(m + m_p)\mathbf{s}_\Delta + \frac{1}{2}\tilde{\mathbf{a}}_{out}^T\Gamma^{-1}\tilde{\mathbf{a}}_{out} \tag{24}$$

leads to

$$\dot{V} = -\mathbf{s}_\Delta^T\mathbf{k}\,\mathrm{sat}\left(\frac{\mathbf{s}}{\Phi}\right) + \mathbf{s}_\Delta^T\mathbf{F}_W \tag{25}$$

where $\mathbf{s}_\Delta = \begin{bmatrix} s_{\Delta x}, & s_{\Delta z}\end{bmatrix}^T$, $\mathbf{k} = \mathrm{diag}\begin{bmatrix} k_x, & k_z\end{bmatrix}$, and
$\mathbf{F}_W = \begin{bmatrix} F_W\sin(\theta), & F_W\cos(\theta)\end{bmatrix}^T$

- Choosing k_x and k_z such that $|\mathbf{k}| = |\mathbf{F}_W| + \eta, \eta > 0$, gives $\dot{V} \leq -\eta|s_\Delta|$
- By Barbalat's Lemma, $\dot{V} \to 0$ as $t \to \infty$, implying $\mathbf{s}_\Delta \to 0$ and $(x, \quad z) \to (x_d, \quad z_d)$

Stability Outline: Outer Loop

- For this stability argument, the rotational dynamics of the vehicle must be neglected
- This implies that the bandwidth of the outer loop will need to be low compared to that of the inner loop ($\lambda_{ol} < \lambda_{il}$)
- When the vehicle's actuators themselves have dynamics, the same applies to the inner loop and thus we want $\lambda_{ol} < \lambda_{il} < \lambda_{act}$
- This is the design principle used in the following numerical example, where the controller is demonstrated in simulation with unmodeled actuator dynamics and external disturbances (wind)

Numerical Simulation

Trajectory for controller tuning (with $T_1 = 20s$ and $T_2 = 5s$):

$$\underbrace{\begin{bmatrix} x_d \\ \dot{x}_d \\ \ddot{x}_d \\ z_d \\ \dot{z}_d \\ \ddot{z}_d \end{bmatrix}}_{\mathbf{x}_d} = \begin{bmatrix} 10\cos(\frac{2\pi t}{T_1}) + 0.2\cos(\frac{2\pi t}{T_2}) \\ -20\frac{\pi}{T_1}\sin(\frac{2\pi t}{T_1}) - 0.4\frac{\pi}{T_2}\sin(\frac{2\pi t}{T_2}) \\ -40(\frac{\pi}{T_1})^2\cos(\frac{2\pi t}{T_1}) - 0.8(\frac{\pi}{T_2})^2\cos(\frac{2\pi t}{T_2}) \\ 10\sin(\frac{2\pi t}{T_1}) + 0.2\cos(\frac{2\pi t}{T_2}) \\ 20\frac{\pi}{T_1}\cos(\frac{2\pi t}{T_1}) - 0.4\frac{\pi}{T_2}\sin(\frac{2\pi t}{T_2}) \\ -40(\frac{\pi}{T_1})^2\sin(\frac{2\pi t}{T_1}) - 0.8(\frac{\pi}{T_2})^2\cos(\frac{2\pi t}{T_2}) \end{bmatrix} \tag{26}$$

	$t < 150$ s	$t \geq 150$ s
ℓ (m)	0.5	-
ℓ_p (m)	0.4	0.3
m (kg)	3.0	-
m_p (kg)	2.0	1.0
I_{yy} (kg·m^2)	0.414	0.184
c_t (s)	1.0	0.92
$\bar{c}_d$ (s)	1.57	-
τ (s)	0.05	-
T_{max} (N)	80	-

Table : Vehicle parameters

Parameter	$(\cdot)_x$	$(\cdot)_z$	$(\cdot)_\theta$
Φ	0.4	0.4	0.1
λ	2.0	2.0	6.0
k	6.0	6.0	0.05
γ	0.2	0.2	2.0

Table : Controller parameters

Numerical Simulation

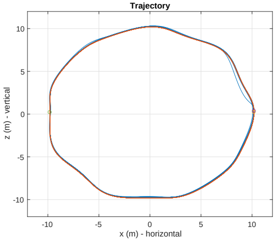


Figure : Tandem-rotor vehicle trajectory - 15 “laps” with adaptation

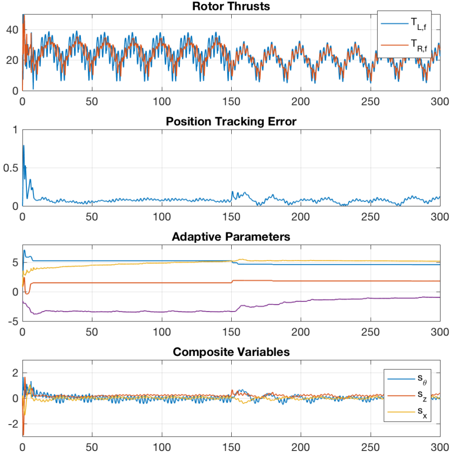
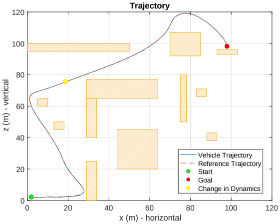


Figure : Simulation results

Numerical Simulation



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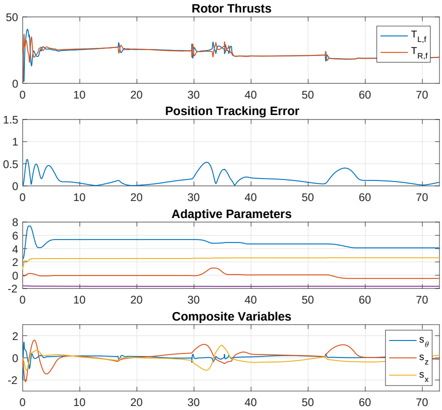

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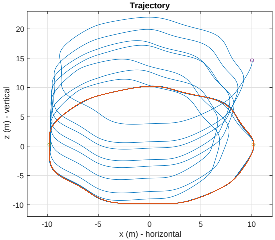
Numerical Simulation - No Adaptation



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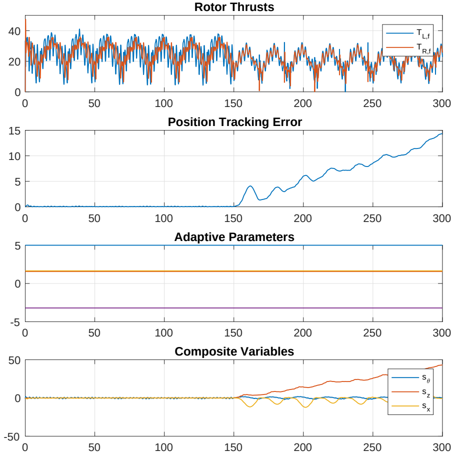
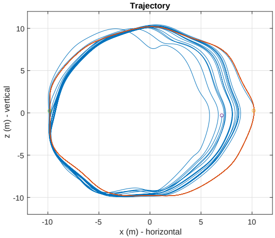


Figure : Simulation results

Numerical Simulation - Linearized About Hover



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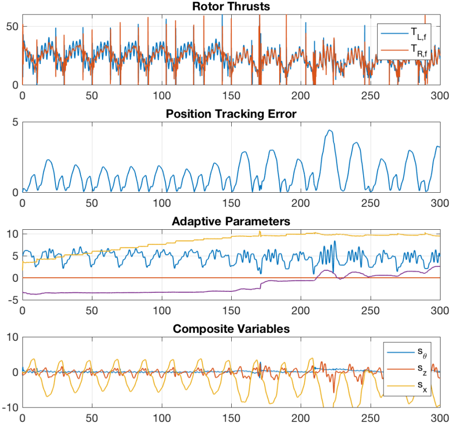

Figure : Simulation results

Summary

Conclusions:

- Trajectory-tracking controller designed for planar quadrotor UAV with unknown parameters $m_p, \ell_p, \bar{c}_d, c_t$
- Ability to adapt online to changes in parameters demonstrated through simulations
- Adaptive controller shows robustness to unmodeled actuator dynamics and external disturbances

Possible extensions:

- Full dynamics of quadrotor UAV moving in 3D space
- Use vehicle parameters identified online to update kinodynamic motion plan
- Estimate slowly time-varying wind online

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Thanks for listening! Questions?

References

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