Simple Neurons Network (“Well” Neurons)

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Abstract

Making rhythms with a single layered network and a simple neuron model.

1 The Model

1.1 Terms

Neurons The model is made up of a single layer of neurons. Each neuron takes and input, and returns an output. Each neuron also has an internal state, represented by a scalar. For each of the n neurons, let the output of the neuron be denoted a_i and the internal state be denoted s_i, for $i \in \{1, \ldots, n\}$. Each neuron has associated with it a threshold ϕ_i

Inputs Each neuron receives input, where x_i denotes the input of the i^{th} neuron.

Connections Each input is connected to its corresponding neuron, i.e. input x_1 is connected to the 1^{st} neuron, and input x_2 is connected to the 2^{nd} neuron, etc. The output of each neuron is connected to the input of every neuron, including itself.

Weights Every connection has a corresponding weight associated with it. We will let v_i be the weights of the connections between the inputs and the neurons, i.e. v_1 is the weight of the connection between input x_1 and neuron 1. Let $w_{i,j}$ be the weight of the connection from neuron i to neuron j.

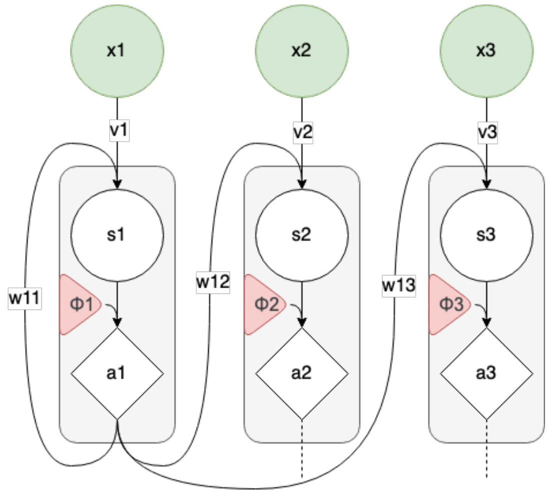

Figure 1: Three neuron system.

1.2 Running the Model

In what follows we use the super indices to indicate the time step of the algorithm, i.e. s_i^t is the state of neuron i on time step t.

Each time set t, we provide an input vector $\vec{x}$ to the network. Each neuron takes as input the corresponding component of this vector, as well as any input that it receives from the outputs of other neurons. It updates its internal state via the following equations:

$$s_i^{t+1} = s_i^t + v_i x_i^t + \sum_j w_{j,i}^t a_j^t, \tag{1}$$

with the output of each neuron, a_i, given by the following equation:

$$a_i^t = \begin{cases} 1 & s_i^{t-1} - \phi_i > 0 \\ 0 & s_i^{t-1} - \phi_i \leq 0 \end{cases} \tag{2}$$

This function is know as the Heaviside step function, applied to $s_i - \phi_i$.

1.3 Diagram

Figure 1 shows the architecture for a three neuron system. Some connections as omitted for simplicity.

1.4 Notes

Damping This is the amount of energy that gets removed from a well when it fires. In previous implementations this has been seen as a separate parameter. However, in this model it is simply the value of the weight w_{ii}, associated with the connection from a neuron i back into itself. If this is set to a negative value, then the neuron will be losing energy every time it fires, recreating the effect of the old damping parameter.

Setting $w_{ii} < 0$ is a natural model for a neuron to lose energy whenever it fires. Whether or not it will be useful to have configurations of a network with $w_{ii} > 0$, such that a neuron is self reinforcing is unknown. Clearly in such a situation there will need to be other mechanisms is place to reduce the value of s_i, e.g. other neurons feeding back negatively into any neurons with $w_{ii} > 0$.

Constant Input Signal This was the way the these neurons have previously been operated. The input vector $\vec{x}$ was always assumed to be the unit vector of dimension n, i.e. all inputs were always 1. The amount of energy flowing into each neuron was then controlled by the parameter v_i. An obvious extension would be to vary the input vector, perhaps feeding in a known rhythm and allowing the network to express the signals as they pass through. This could allow the network to play something “close” to a predetermined rhythm.

1.5 Recommendation

Exploration required.