Probability Week 3: Historical Problems and Gaming

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Goals

- Understand and use ideas of compound probability
- Use expected value to analyze fairness
- Prove we should believe in God

Origins of Probability

Figure: Astragalas Bone

- Found in ancient Assyrian and Sumerian sites
- Games and Pieces found in Egypt and Babylonian sites (3600 BC)

Game Dates

Event	Date	Region
Dice	2000 BCE	Egypt and Elsewhere
Playing Cards	10th century and 14th century	China then Western Europe
Roulette	1800	France
Poker	1800	Lousiana Territory
Life Insurance	1583	England

Modern Probability

- *The book on Games of Chance* – 1520
- Pascal Fermat correspondence – 1654
- Pascal's wager 1658
- Expectation concept Huygens in 1657 in *Calculating in Games of Chance*
- Jakob Bernoulli *Ars Conjectandi* – 1690

Figure: The Gambling Scholar

Dostoyevsky

"I won 10,400 francs at first, took them home and shut them up in a bag and intended to leave Wiesbaden the next day without going back to the tables; but I got carried away and dropped half my winnings."

A Dice Wager

Dear Blaise,

I've made a lot of money betting with even odds on one six in four rolls. I imagine that rolling at least two sixes in twenty four rolls would be good too. Am I correct?

Sincerely,

Chevalier De mere

Independent Events

Rolls are *independent* because the outcome on each has no bearing on the outcome of any other roll.

$$P(6) \quad \text{at least one} = 1 - P(no \quad six)$$

$$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{625}{1296}$$

$$1 - \frac{625}{1296} = \frac{671}{1296}$$

Problem of Points

Jimmy and Walter's Game: 5 points to win, equal chance each point. Broken up by cops with Jimmy leading 4 games to 3. How should pot be awarded?

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Point 8	Point 9	Overall Winner
J	J	J
J	W	J
W	J	J
W	W	W

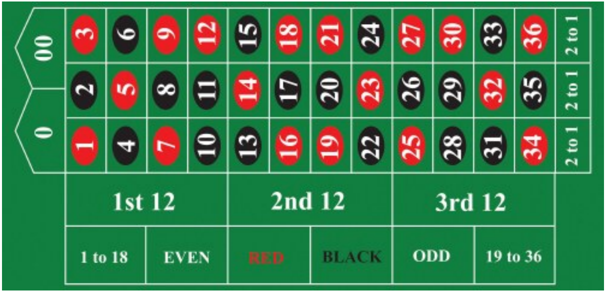
Examples

- The probability of being dealt a pair in two cards is
- The probability of getting a Jack or a Diamond in a single draw is
- If nine cards have been seen by player A, exactly one of which is a Seven, A's probability of the next card being a Seven is
- The probability of getting an Ace and then a second Ace in two consecutive cards is
- The probability of being dealt 5 spades in 5 cards is

Roulette and Odds

$$\text{odds for E} = \frac{p(E)}{1 - p(E)}$$

Odds of spin resulting in 10, 11, or 12?



Two fair coins are flipped, payoffs follow:

- both heads – you win \$2
- both tails – you win \$3
- one of each – you lose \$4

Should you play and, if you play, what do you expect to gain or lose?

You roll a single die; house pays you 12 for rolling a six, 8 for rolling an odd number, nothing otherwise. What should you spend on a roll to make this a fair game?

You are thinking of attending Solange concert on September 30. Non-refundable buy now \$150 before Sept 15, \$200 otherwise. You estimate your probability of attendance is p. For which values should you buy this and which should you pay at concert?

Is Roulette Futile?

$X(\text{red})$

$X(\text{first third})$

$X(\text{single number}$

Pascal's Wager

Let p = the probability that God exists.

	Existence	non-Existence
Belief	x	-z
Nonbelief	$-\infty$	y

St. Petersburg Paradox

You flip a fair coin as many times as you can until you obtain a tail at which time you stop. Payoff to you based on the number of initial "Heads" you get before "tails" stops you. What's expectation?

# of heads	0	1	2	3	...	n	...
Payoff	1	2	4	8	...	2^n	...

Perms and Combs

1. How many different finishes among first 3 places in an 8 horse race?
2. How many different foursomes from 7 golfers?
3. How many different 5 card poker hands are there?
4. How many 9-player batting orders on baseball team with a 25 player roster?
5. How many different spin configurations can the single zero come up 3 times in 10 spins of a roulette wheel?
6. How many seating arrangements are possible at a poker game for 8 where host and hostess must sit at the ends of a narrow long, rectangular 12 seated table?

Poker Probability

Hand Type	Number of Possible Hands	Probability
Straight flush	40	
Four of a kind	624	
Full house	3,744	
Flush	5,108	
Straight	10,200	
Three of a kind	54,912	
Two Pair	123, 552	
One pair	1, 098, 240	
Worse than one pair	1,302, 540	

Definition, connection, example.

$$p(E) = \frac{\text{number of elementary events in E}}{\text{Number of possible equally likely elementary events}}$$

$$p(E) = \frac{\text{number of successful occurrences of E}}{\text{number of trials}}$$