✅ 자유도 보정된 MSE (Residual Mean Square, MSE_{res})

$$\mathrm{MSE}_{res} = \frac{\mathrm{SSE}}{n-p}$$

- n: 샘플 수
- p: 모델에 포함된 **회귀 계수 수 (절편 포함)**
- 일반적으로 단순 선형회귀에서는 $p = 2$ (절편 + 기울기)

→ 그래서 단순 선형회귀에서는 자유도 $= n - 2$

In [2]:

```python
import numpy as np
import pandas as pd
house=pd.read_csv("https://raw.githubusercontent.com/ADPclass/ADP_book_ver01/main/data/kc_house_data.csv")
house=house[["price","sqft_living"]]
house.corr()
```

Out[2]:

	price	sqft_living
price	1.000000	0.702035
sqft_living	0.702035	1.000000

✅ 3) F-검정 수식 (내부 계산 방식)

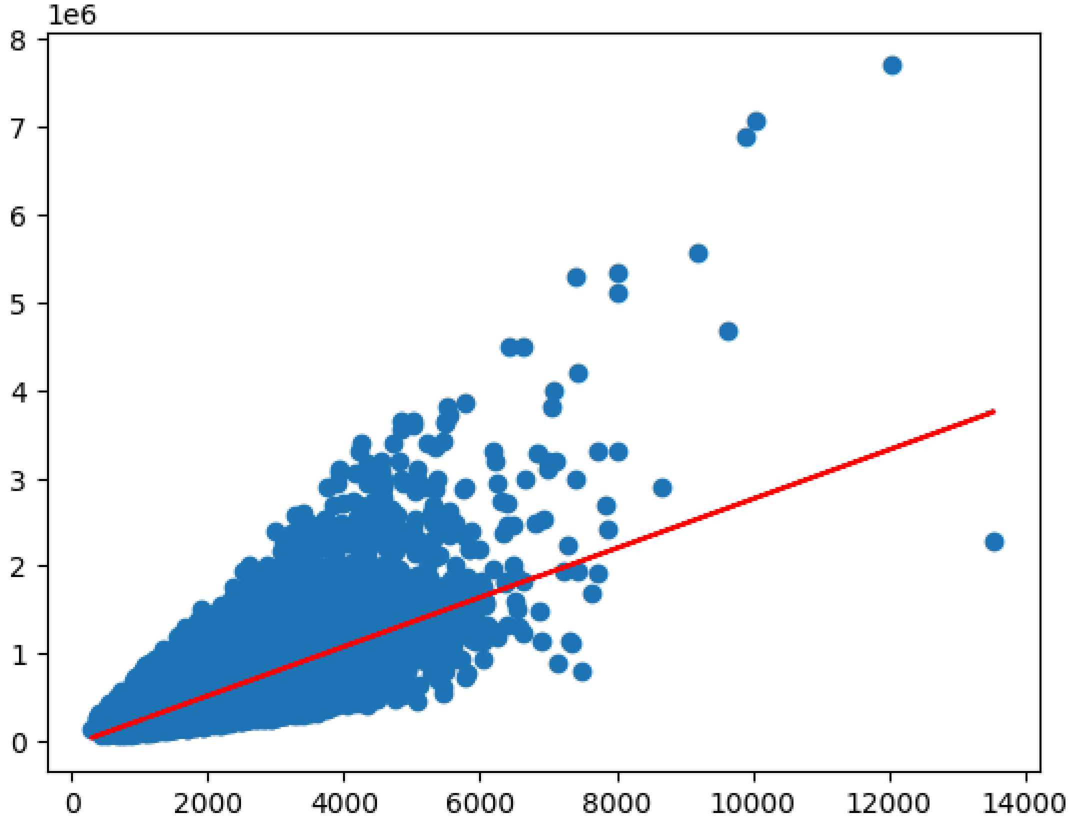
$$F = \frac{\text{회귀 제곱합 SSR}/(p-1)}{\text{잔차 제곱합 SSE}/(n-p)}$$

- p: 회귀계수 수 (절편 포함)
- n: 샘플 수
- SSR: 모델이 설명하는 제곱합 (explained)
- SSE: 잔차 제곱합 (unexplained)

✅ 4) 요약 그림

In [4]:
```python
from statsmodels.formula.api import ols
import matplotlib.pyplot as plt

X=house[['sqft_living']]
y=house['price']
lr=ols("price~sqft_living",data=house).fit()
y_pred=lr.predict()
plt.scatter(X,y)
plt.plot(X,y_pred,color='r')
plt.show()
```



In [5]:
```
lr.summary()
```

Out[5]:

OLS Regression Results

Dep. Variable:	price	**R-squared:**	0.493
Model:	OLS	**Adj. R-squared:**	0.493
Method:	Least Squares	**F-statistic:**	2.100e+04
Date:	Sun, 24 Aug 2025	**Prob (F-statistic):**	0.00
Time:	18:31:14	**Log-Likelihood:**	-3.0027e+05
No. Observations:	21613	**AIC:**	6.005e+05
Df Residuals:	21611	**BIC:**	6.006e+05
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-4.358e+04	4402.690	-9.899	0.000	-5.22e+04	-3.5e+04
sqft_living	280.6236	1.936	144.920	0.000	276.828	284.419

Omnibus:	14832.490	**Durbin-Watson:**	1.983
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	546444.713
Skew:	2.824	**Prob(JB):**	0.00
Kurtosis:	26.977	**Cond. No.**	5.63e+03

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 5.63e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

✅ 2. 선형회귀(OLS)에서 로그우도 $\ln \hat{L}$ 계산

선형회귀는 잔차가 **정규분포를 따른다**는 가정 하에, 아래와 같이 로그우도를 계산할 수 있습니다:

잔차 분포 가정:

$$\varepsilon_i \sim \mathcal{N}(0, \sigma^2) \quad \Rightarrow \quad y_i \sim \mathcal{N}(\hat{y}_i, \sigma^2)$$

그러면 전체 로그우도 함수는:

$$\ln(\hat{L}) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\hat{\sigma}^2) - \frac{1}{2\hat{\sigma}^2} \cdot \mathrm{RSS}$$

여기서 $\hat{\sigma}^2 = \frac{\mathrm{RSS}}{n}$ 을 대입하면:

$$\ln(\hat{L}) = -\frac{n}{2}\left(1 + \ln(2\pi) + \ln\left(\frac{\mathrm{RSS}}{n}\right)\right)$$

✅ 3. AIC로 다시 정리

$$\mathrm{AIC} = 2k - 2\ln(\hat{L}) = 2k + n \cdot \left(1 + \ln(2\pi) + \ln\left(\frac{\mathrm{RSS}}{n}\right)\right)$$

여기서 **상수 항** $n(1 + \ln(2\pi))$는 **모델 비교 시 모든 모델에 공통**이라 보통 생략하거나 무시해도 됩니다.

그래서 **비교 목적의 AIC 간소식**으로 다음을 많이 씁니다:

$$\boxed{\mathrm{AIC} \approx n \ln\left(\frac{\mathrm{RSS}}{n}\right) + 2k}$$

> ✔️ 이게 바로 `statsmodels`나 일반 회귀문헌에서 "직접 계산하는 AIC"에 자주 나오는 식입니다.

In [9]:
```python
# 직접 AIC 계산
n = lr.nobs
rss = np.sum(lr.resid ** 2)
k = lr.df_model + 1  # +1 for intercept

aic_manual = n * np.log(rss / n) + 2 * k

print("Manual AIC:", aic_manual)
print("Model AIC :", lr.aic)
```

```
Manual AIC: 539203.5295452357
Model AIC : 600538.5665815409
```

In [ ]:
```python
from statsmodels.stats.outliers_influence import variance_inflation_factor
vif_df = pd.DataFrame()
vif_df["feature"] = X.columns #다변량에서 ㄱ
vif_df["VIF"] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
```

In [10]:
```python
import pandas as pd
Cars=pd.read_csv("https://raw.githubusercontent.com/ADPclass/ADP_book_ver01/main/data/Cars93.csv")
import numpy as np
import statsmodels.api as sm
import statsmodels.formula.api as smf

Cars.columns=Cars.columns.str.replace(".","")
model=smf.ols(formula="Price~EngineSize+RPM+Weight+Length+MPGcity+MPGhighway",data=Cars)
result=model.fit()
result.summary()
```

Out[10]:

OLS Regression Results

Dep. Variable:	Price	**R-squared:**	0.572
Model:	OLS	**Adj. R-squared:**	0.542
Method:	Least Squares	**F-statistic:**	19.14
Date:	Sun, 24 Aug 2025	**Prob (F-statistic):**	4.88e-14
Time:	18:41:16	**Log-Likelihood:**	-302.94
No. Observations:	93	**AIC:**	619.9
Df Residuals:	86	**BIC:**	637.6
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-32.2157	17.812	-1.809	0.074	-67.625	3.193
EngineSize	4.4732	1.410	3.172	0.002	1.670	7.276
RPM	0.0071	0.001	5.138	0.000	0.004	0.010
Weight	0.0056	0.003	1.634	0.106	-0.001	0.012
Length	-0.0464	0.094	-0.496	0.621	-0.232	0.139
MPGcity	-0.3478	0.448	-0.776	0.440	-1.239	0.544
MPGhighway	0.0582	0.460	0.126	0.900	-0.856	0.973

Omnibus:	62.984	**Durbin-Watson:**	1.446
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	383.289
Skew:	2.074	**Prob(JB):**	5.89e-84
Kurtosis:	12.039	**Cond. No.**	1.61e+05

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 1.61e+05. This might indicate that there are strong multicollinearity or other numerical problems.

In [11]:
```
Cars[["EngineSize","RPM","Weight","Length","MPGcity","MPGhighway"]].corr()
```

Out[11]:

	EngineSize	RPM	Weight	Length	MPGcity	MPGhighway
EngineSize	1.000000	-0.547898	0.845075	0.780283	-0.710003	-0.626795
RPM	-0.547898	1.000000	-0.427931	-0.441249	0.363045	0.313469
Weight	0.845075	-0.427931	1.000000	0.806274	-0.843139	-0.810658
Length	0.780283	-0.441249	0.806274	1.000000	-0.666239	-0.542897
MPGcity	-0.710003	0.363045	-0.843139	-0.666239	1.000000	0.943936
MPGhighway	-0.626795	0.313469	-0.810658	-0.542897	0.943936	1.000000

In [18]:
```
from patsy import dmatrices
from statsmodels.stats.outliers_influence import variance_inflation_factor
y,X=dmatrices("Price~EngineSize+RPM+Weight+Length+MPGcity+MPGhighway",data=Cars,return_type="dataframe")
vif_list=[]
for i in range(1,len(X.columns)):
    vif_list.append([variance_inflation_factor(X.values,i),X.columns[i]])
pd.DataFrame(vif_list,columns=['vif','variable'])
```

Out[18]:

	vif	variable
0	4.605118	EngineSize
1	1.446859	RPM
2	8.685973	Weight
3	4.013002	Length
4	13.668288	MPGcity
5	12.943133	MPGhighway

In [20]:
```
X.columns
```

Out[20]:
```
Index(['Intercept', 'EngineSize', 'RPM', 'Weight', 'Length', 'MPGcity',
       'MPGhighway'],
      dtype='object')
```

In [22]:

```
model = smf.ols(formula="Price~EngineSize+RPM+Weight+Length+MPGhighway", data=Cars)
result=model.fit()
result.summary()
```

Out[22]:

OLS Regression Results

Dep. Variable:	Price	**R-squared:**	0.569
Model:	OLS	**Adj. R-squared:**	0.544
Method:	Least Squares	**F-statistic:**	22.95
Date:	Sun, 24 Aug 2025	**Prob (F-statistic):**	1.28e-14
Time:	19:12:01	**Log-Likelihood:**	-303.27
No. Observations:	93	**AIC:**	618.5
Df Residuals:	87	**BIC:**	633.7
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-35.8122	17.158	-2.087	0.040	-69.916	-1.709
EngineSize	4.6591	1.386	3.361	0.001	1.904	7.415
RPM	0.0071	0.001	5.173	0.000	0.004	0.010
Weight	0.0053	0.003	1.567	0.121	-0.001	0.012
Length	-0.0194	0.087	-0.224	0.823	-0.191	0.153
MPGhighway	-0.2500	0.231	-1.082	0.282	-0.709	0.209

Omnibus:	61.903	**Durbin-Watson:**	1.397
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	363.806
Skew:	2.044	**Prob(JB):**	1.00e-79
Kurtosis:	11.785	**Cond. No.**	1.56e+05

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.56e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

In [36]:

```
import time
import itertools
import warnings
warnings.filterwarnings('ignore')
def processSubset(X,y,feature_set):
    model= sm.OLS(y,X[list(feature_set)])
    regr=model.fit()
    AIC=regr.aic
    return {'model':regr,"AIC":AIC}

def forward(X,y, predictors):
    remaining_predictors=[p for p in X.columns.difference(['Intercept']) if p not in predictors]
    results=[]
    for p in remaining_predictors:
        results.append(processSubset(X,y,predictors+[p]+['Intercept']))
    models=pd.DataFrame(results)
    best_model=models.loc[models['AIC'].argmin()]
    print("Processed",models.shape[0],"model on",len(predictors)+1, "predictors in")
    print("Selected Predictors",best_model['model'].model.exog_names,"AIC",best_model[0].aic)
    return best_model

def backward(X,y,predictors):
    tic= time.time()
    results=[]
    for combo in itertools.combinations(predictors,len(predictors)-1):
        results.append(processSubset(X=X,y=y,feature_set=list(combo)+['Intercept']))
    models=pd.DataFrame(results)
    best_model=models.loc[models['AIC'].argmin()]
    print("Processed",models.shape[0],"model on",len(predictors)+1, "predictors in")
    print("Selected Predictors",best_model['model'].model.exog_names,"AIC",best_model[0].aic)
    return best_model

def Stepwise_model(X,y):
    Stepmodels=pd.DataFrame(columns=['AIC','model'])
    tic=time.time()
    predictors=[]
    Smodel_before = processSubset(X,y,predictors+['Intercept'])['AIC']
    for i in range(1,len(X.columns.difference(['Intercept']))+1):
        Forward_result=forward(X,y,predictors)
        print('forward')
        Stepmodels.loc[i]=Forward_result
        predictors=Stepmodels.loc[i]["model"].model.exog_names
        predictors=[k for k in predictors if k!="Intercept"]
        Backward_result=backward(X,y,predictors)
        if Backward_result['AIC']<Forward_result['AIC']:
            Stepmodels.loc[i]=Backward_result
```

```
            predictors=Stepmodels.loc[i]["model"].model.exog_names
            predictors=[k for k in predictors if k!="Intercept"]
            print('backward')
        if Stepmodels.loc[i]['AIC']>Smodel_before:
            break
        else:
            Smodel_before=Stepmodels.loc[i]["AIC"]
    toc=time.time()
    print("Total elapsed time",(toc-tic),"seconds")
    return Stepmodels['model'][len(Stepmodels['model'])]
```

In [37]:

```
Stepwise_best_model=Stepwise_model(X,y)
```

```
Processed 6 model on 1 predictors in
Selected Predictors ['Weight', 'Intercept'] AIC 638.2790669305498
forward
Processed 1 model on 2 predictors in
Selected Predictors ['Intercept'] AIC 686.7529864151061
Processed 5 model on 2 predictors in
Selected Predictors ['Weight', 'RPM', 'Intercept'] AIC 624.5241529342444
forward
Processed 2 model on 3 predictors in
Selected Predictors ['Weight', 'Intercept'] AIC 638.2790669305498
Processed 4 model on 3 predictors in
Selected Predictors ['Weight', 'RPM', 'EngineSize', 'Intercept'] AIC 616.0976497740975
forward
Processed 3 model on 4 predictors in
Selected Predictors ['Weight', 'RPM', 'Intercept'] AIC 624.5241529342444
Processed 3 model on 4 predictors in
Selected Predictors ['Weight', 'RPM', 'EngineSize', 'MPGcity', 'Intercept'] AIC 616.1664162275082
forward
Processed 4 model on 5 predictors in
Selected Predictors ['Weight', 'RPM', 'EngineSize', 'Intercept'] AIC 616.0976497740975
backward
Processed 3 model on 4 predictors in
Selected Predictors ['Weight', 'RPM', 'EngineSize', 'MPGcity', 'Intercept'] AIC 616.1664162275082
forward
Processed 4 model on 5 predictors in
Selected Predictors ['Weight', 'RPM', 'EngineSize', 'Intercept'] AIC 616.0976497740975
backward
Processed 3 model on 4 predictors in
Selected Predictors ['Weight', 'RPM', 'EngineSize', 'MPGcity', 'Intercept'] AIC 616.1664162275082
forward
Processed 4 model on 5 predictors in
Selected Predictors ['Weight', 'RPM', 'EngineSize', 'Intercept'] AIC 616.0976497740975
backward
Total elapsed time 0.030241012573242188 seconds
```

In [38]:

```
Stepwise_best_model.summary()
```

Out[38]:

OLS Regression Results

Dep. Variable:	Price	**R-squared:**	0.561
Model:	OLS	**Adj. R-squared:**	0.547
Method:	Least Squares	**F-statistic:**	37.98
Date:	Sun, 24 Aug 2025	**Prob (F-statistic):**	6.75e-16
Time:	19:51:17	**Log-Likelihood:**	-304.05
No. Observations:	93	**AIC:**	616.1
Df Residuals:	89	**BIC:**	626.2
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Weight	0.0073	0.002	3.372	0.001	0.003	0.012
RPM	0.0071	0.001	5.208	0.000	0.004	0.010
EngineSize	4.3054	1.325	3.249	0.002	1.673	6.938
Intercept	-51.7933	9.106	-5.688	0.000	-69.887	-33.699

Omnibus:	62.441	**Durbin-Watson:**	1.406
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	361.880
Skew:	2.076	**Prob(JB):**	2.62e-79
Kurtosis:	11.726	**Cond. No.**	8.27e+04

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 8.27e+04. This might indicate that there are
strong multicollinearity or other numerical problems.

In [ ]: