The Structure of White Dwarf Stars

B Taylor
School of Physics and Astronomy
University of Southampton

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Contents

1 Introduction.....1

2 Theory.....2

3 The Computer Program.....3

4 Results and Discussion.....3

5 Conclusion.....6

Abstract

We look at the mass-radius relation for white dwarf stars and see how the composition of the stars affects this relation. We find that as the number of electrons per nucleon in the elements of the star decrease, so does the Chandresekhar limit for the star, with values of 1.4 solar masses for 0.5 electrons per nucleon and 1.2 solar masses for 0.464 electrons per nucleon being obtained. We then look at the composition of specific stars and find that Sirius B has 0.499 electrons per nucleon, 40 Eri B has 0.471, and Stein 2051 has 0.460. Finally, we predict what this could mean for the elements that compose the stars.

1 Introduction

White dwarf stars are one of the possible end results of a star's lifetime, which occurs when the star is low-mid mass. For example, as a mid miss star, our Sun will likely become a white dwarf towards the end of its lifetime[1].

In figure 1, to the right, white dwarfs can be seen on the bottom left of the Hertzsprung-Russell diagram. As can be seen in the diagram they vary in temperature from ~6000K- 30,000K, and in luminosity from 0.0001-1 solar luminositys.

When a star becomes a white dwarf the dominant pressure source, which counteracts gravity and prevents the star collapsing in on itself switches from ideal gas pressure to relativistic electron degeneracy pressure[2]. As a result of this there a limit to the mass of white dwarfs, above which which electron degeneracy pressure can no longer support the star and it will collapse in on itself causing a supernova. This is known as the Chandresekhar limit and is roughly 1.4 solar masses[3].

By relating the gravitational and kinetic energy in a white dwarf It is possible to show that its radius is inversely proportional to the cube of its mass, meaning that as the mass increases, the radius will decrease.

Another of the defining factors of white dwarfs is that nuclear fusion no longer takes place in its core, as all the materials for fusion

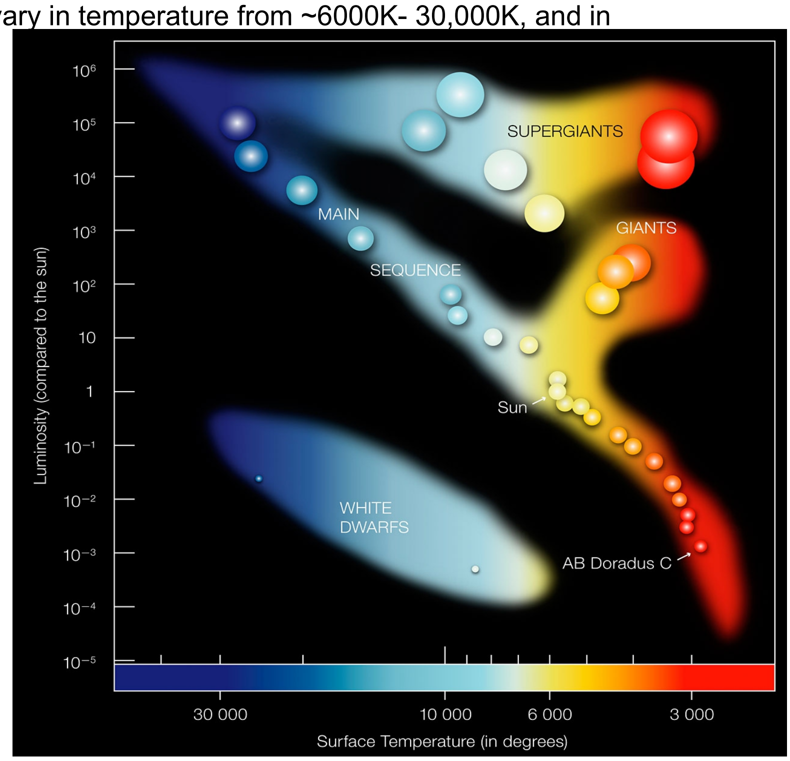


Figure 1 - The Hertzsprung-Russell Diagram.

Image from https://www.eso.org/public/images/eso0728c/.

have been used up and instead of Hydrogen or Helium the core is made of a metal, most commonly Iron, but occasionally nucleosynthesis ends early and other metals formed from fusion will occupy the core, such as Carbon.

2 Theory

We start by assuming that the star is spherically symmetrical and not rotating. We then consider an infinitesimally small volume at a distance *r* from the centre of the star, with a height *δh* and area *δA*. The gravitational force acting on this volume will then be:

$$F=\frac{-Gm(r)}{r^2}\rho(r)\partial h\partial A \qquad (1)$$

where *G* is the gravitational constant, *m(r)* is the mass as a function of radius and *ρ(r)* is the density as a function of radius. This force must equal the difference in pressure acting on the top and bottom of the volume, or *δP* times *δA* so that,

$$\frac{dP}{dr}=\frac{-Gm(r)}{r^2}\rho(r) \quad . \qquad (2)$$

which is the equation for hydrostatic equilibrium. We can then split the pressure differential into two products:

$$\frac{dP}{dr}=\frac{d\rho}{dr}\frac{dP}{d\rho} \qquad (3)$$

and the second factor in this equation depends only on the materials of which the star is composed i.e. its equation of state. Combining (2) and (3) we then get:

$$\frac{d\rho}{dr}=-\left(\frac{dP}{d\rho}\right)^{-1}\frac{Gm}{r^2}\rho \quad . \qquad (4)$$

If we then consider a thin spherical shell at a distance *r* from the centre of the star:

$$\partial m=4\pi r^2\rho\partial r \qquad (5)$$

which we can then use to find the mass differential of the star,

$$\frac{dm}{dr}=4\pi r^2\rho \quad . \qquad (6)$$

It will be possible to solve the pair of equations, (4) and (6), numerically assuming we know the equation of state. For a white dwarf a good approximation for the equation of state is the relativistic free Fermi gas:

$$\frac{dP}{d\rho}=Y_e\frac{m_e c^2}{m_p}\gamma\left(\frac{\rho}{\rho_0}\right) \qquad (7)$$

where *Ye* is the number of electrons per nucleon, m_e is the mass of an electron, m_p is the mass of a proton, *c* is the speed of light, *γ* is:

$$\gamma(y)=\frac{y^{2/3}}{3(1+y^{2/3})^{1/2}} \qquad (8)$$

and ρ_0 is:

$$\rho_0 = \frac{m_p m_e^3 c^3}{3\pi^2 \hbar^3 Y_e} \quad . \tag{9}$$

3 The Computer Program

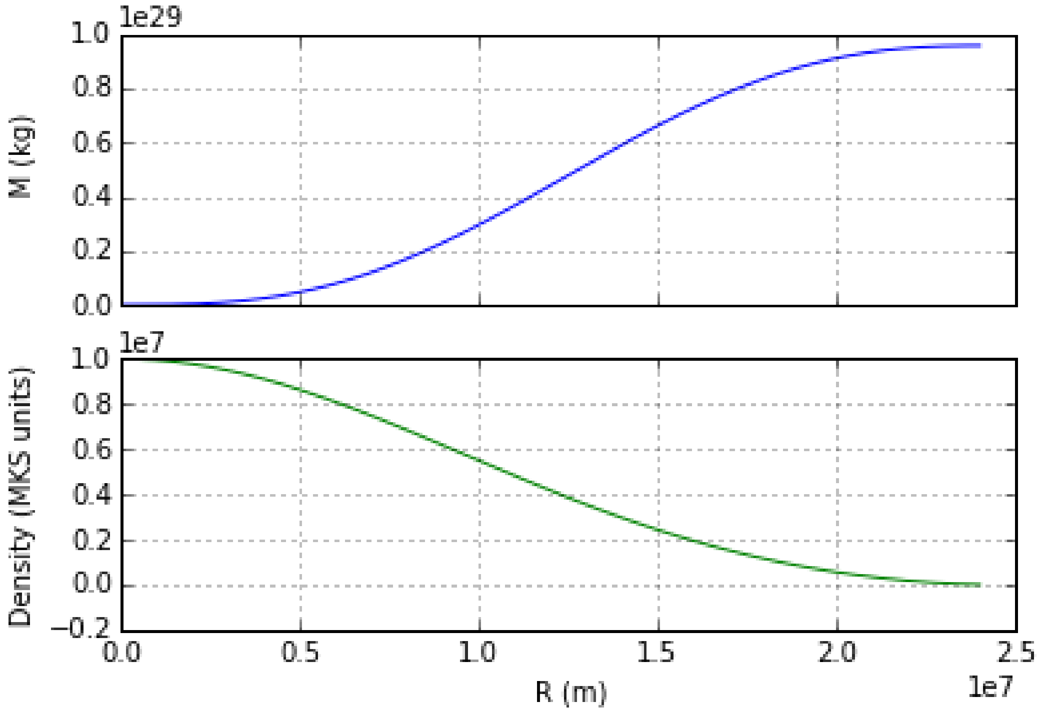
The Runge-Kutta method was used to numerically solve equations (4) and (6), with the equation of state: equation (7). The initial values $r = 0$, $m = 0$ and $\rho = \rho_c$ where ρ_c is the central density of the star. From there the equations were then integrated outwards using the Runge-Kutta method until $\rho = 0$. The radius at which this happens corresponds to the surface of the star. The total mass of the star was then calculated by finding the value of $m(r)$ at the given radius.

In order to find the relationship between the mass and radius of the star the value for the central density was adjusted and a graph of mass against radius was plotted for the different values.

In order to get results which were as accurate as possible, the step size in the Runge-Kutta method was set as small as possible, without taking excessive amounts of computing time.

4 Results and Discussion

Figures 2 and 3 show the results for one of the values of central density integrated out from $r = 0$ and shows the increasing mass and decreasing density as you moving outwards from the centre to the surface of the star.



Figures 2 and 3: The Density and Mass of a white dwarf varying with radius

As you can see from these graphs the density and mass are as would be expected, with density and mass decreasing and increasing respectively at roughly the same rate.

Figure 4 shows the Radius of a white dwarf plotted against Mass for a value of $Ye = 0.5$. The figure shows that as the radius decreases the mass increases, as predicted. It also shows that the radius reaches a minimum, and the mass a maximum at just over 1.4 solar masses, and so it is consistent with the Chandresekhar limit.

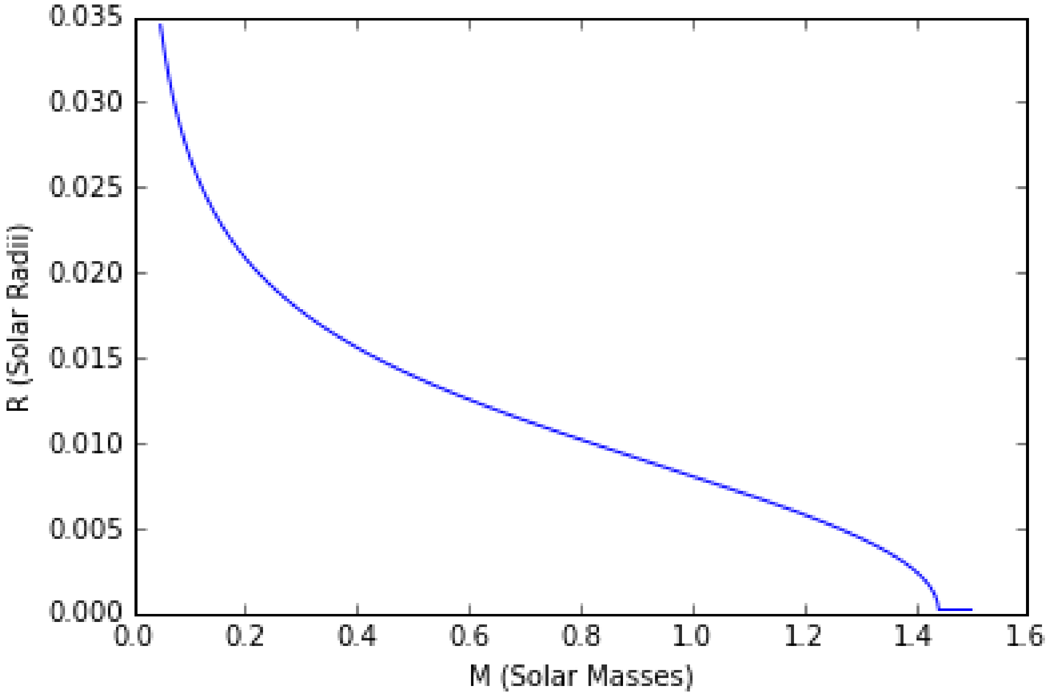


Figure 4: Radius against Mass for Ye = 0.5

Figure 5 shows a plot of Radius against mass for a white dwarf with *Ye = 0.464* which is roughly the value for Iron-56. In this graph we see a very similar shape to figure 4, however the mass limit for this graph is much lower, indicating that the maximum possible mass is just over 1.2 solar masses.

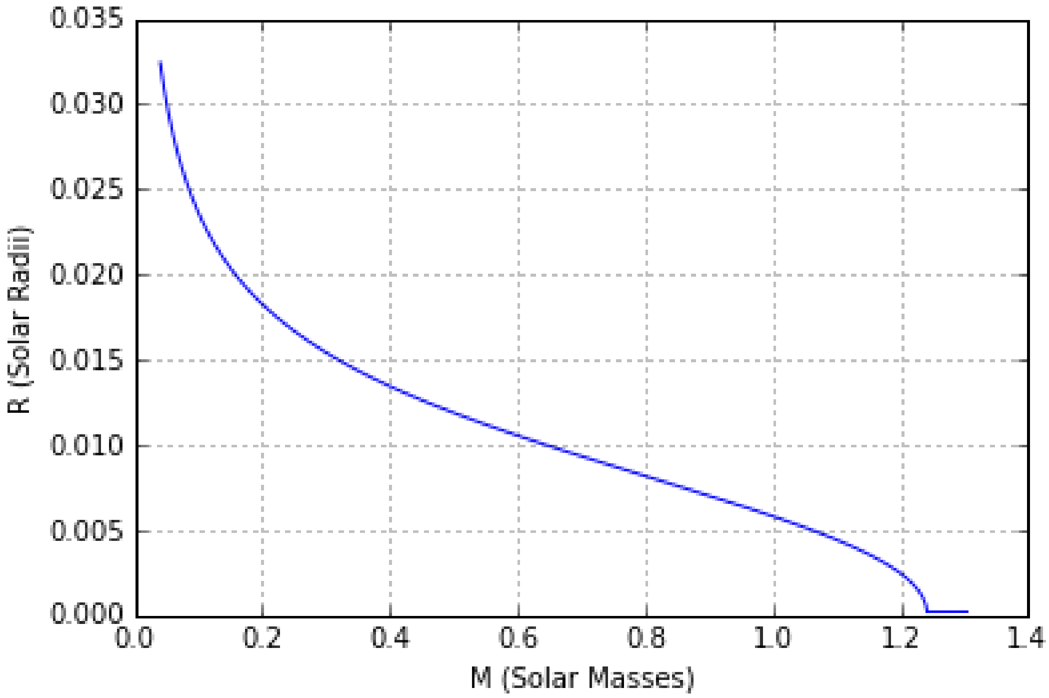


Figure 5: Radius against Mass for Ye = 0.464

From these two graphs a reasonable conclusion can be drawn that although the maximum mass for any white dwarf is 1.4 solar masses, this only applies to white dwarfs which have ended their nucleosynthesis early and have cores of lower mass elements, such as Carbon, Oxygen or Silicon. Stars with cores made up of heavier elements, in fact, have a lower limit placed on their mass. In the case of Iron this limit appears to be around 1.2 solar masses.

Table 1 shows the masses and radii of 3 different white dwarfs that we have values for. Using the graphs above we can look at the masses on the table and find the corresponding radius on the graphs to see if these observations are consistent with our results.

	Mass	Radius
Sirius B	1.053 ± 0.028	0.0074 ± 0.0006
40 Eri B	0.48 ± 0.02	0.0124 ± 0.0005
Stein 2051	0.50 ± 0.05	0.0115 ± 0.0012

Table 1: The Mass and Radius of different white dwarfs

	Y_e
Sirius B	0.499±0.019
40 Eri B	0.471±0.015
Stein 2051	0.460±0.038

Table 2: The electrons per nucleon of different white dwarfs

By adjusting the number of electrons per nucleon in our program we can find the value of Y_e which allows the observational values of mass and radius to match up. The results of doing this are shown in table 2. Using this information we can then infer which element might compose the majority of each stars core. However, since there is a large amount of error in these measurements due to the uncertainty of the observational data, these predictions should be taken lightly.

Sirius B appears to have a core with roughly double the amount of nucleons per electron implying nucleosynthesis ended early in its lifetime and it has a core made of a lighter element such as Carbon-12, Oxygen-16 or Silicon-28.

40 Eri B looks like it has a core slightly lighter than Iron, and therefore could be composed of a mixture of Iron-56 and another element with more electrons per nucleon, for example Nickel-60 or Silicon-28. Alternatively, the margin for error has the electron per nucleon value for Iron-56 within it, so the core could be made of pure Iron-56 after all.

Stein 2051 appears to have a core made of something with less electrons per nucleon than Iron-56. However this is extremely unlikely, because it is impossible to create elements heavier than Iron through nuclear fusion without huge amounts of energy, such as the energy levels found in supernovae and although there are stable isotopes of Iron with electron per nucleon values similar to this they are very rare. The much more likely explanation is that since the mass and radius measurements we have for Stein 2051 we have have such relatively large error margins, this causes the prediction made here to have a large error margin as well, which makes it impossible to accurately predict the materials in the core. Although, since the value is fairly close to that of Iron, and Iron is the most common element in the core of white dwarfs, it would be the best guess.

5 Conclusion

We can conclude from these results that it is possible to calculate the mass-radius relation for white dwarf stars using the equation of state for the relativistic free Fermi gas for different values of electrons per nucleon. Using these relations we can then calculate the mass limit for different values of Y_e and show that as this number gets smaller, so does the mass limit.

Using the observational data gathered from different white dwarf stars we then found it was possible to predict the electron per nucleon values for these stars and thus determine their composition.

However due to the uncertainty in the observational data, the results obtained all had a relatively large margin for error and should be taken only as a guideline, not as absolute fact.

References

[1] O'Neill, Ian. "The Sun As A White Dwarf Star - Universe Today". *Universe Today*. N.p., 2015. Web. 28 Mar. 2017.

[2] S.L.Shapiro and S.A.Teukolsky, Black Holes, White Dwarfs, and Neutron Stars, (J Wiley and Sons, Inc., New York 1983) Pages 57-60.

[3] S.L.Shapiro and S.A.Teukolsky, Black Holes, White Dwarfs, and Neutron Stars, (J Wiley and Sons, Inc., New York 1983) Pages 64-65.