Planetary Orbits Around Binary Stars

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Abstract

This paper looks at the various types of orbits that can occur in binary star systems, examining what conditions are required to produce stable orbits around either the system or a single star and the differences between these orbits and those found in a unary star system. It is found that at large distances from the star system the binary stars behave as a single point of mass and at small distances the planet could only orbit a single star, with the second star making minor perturbations in its orbit.

At medium distances 2 different behaviours were observe depending on whether the planet was orbiting in the same direction as the stars or not. If they were orbiting in the opposite direction to one another, stable orbits were found with minor perturbations. However, when orbiting in the same direction the planet's orbit would always descend into chaotic behaviour eventually causing it to be thrown out into space.

1 Introduction

In our galaxy alone there are over 100 billion stars[1], the vast majority of which are thought to have planets orbiting them[2]. For the ones similar to our solar system i.e. unary star systems, meaning that there is only one star around which the planets orbit. These orbits are simple and can easily be predicted using Newtonian physics. However, many of the stars in our galaxy are part of binary star systems, meaning that in order to calculate the orbits of planets around these systems a three body problem with mutual gravitational interactions must be considered. This proves to be vastly more complicated than the two-body problem, and can present interesting phenomena when analysed numerically.

In order to investigate the planetary orbits in these binary systems, the forces acting on each body are considered and a program is used to calculate the orbits by inputting the initial conditions and iterating over small steps in time, building up a picture of the interaction. Since three body problems are already quite complex, the difference in mass between the two stars is considered to be negligible. The eccentricity of the orbits are assumed to be 0, to further aid in simplifying the problem.

The aim of this paper will be to explore the different types of orbits that can occur in binary star systems and hopefully gain a better understanding of the characteristics of these orbits as well as looking at which types of orbits are most likely to occur.

2 Theory

In order to calculate the orbits of planets in binary systems, we must first consider the stars in the centre of the system. These can be represented in a Cartesian coordinate system as follows:

$$X_1 = R\cos(\Omega t) \qquad Y_1 = R\sin(\Omega t)$$
$$X_2 = -R\cos(\Omega t) \qquad Y_2 = -R\sin(\Omega t)$$

where R is the distance of each star from their common centre of mass, t is the time, Ω is the angular velocity and (X_1, Y_1) and (X_2, Y_2) are the coordinates of the first and second star respectively. These values are related to the mass of the stars by

$$GM = R^3\Omega^2$$

where G is the gravitational constant and M is the mass of each star.

Next we must consider a planet of negligible mass. If we let the coordinates of the planet be *(x, y)* then the distances from the planet to each of the two stars *(r_1, r_2)* are

$$r_1^2 = (x - X_1)^2 + (y - Y_1)^2$$
$$r_2^2 = (x - X_2)^2 + (y - Y_2)^2 \quad .$$

From this we can use Newton's second law of motion and Newton's law of universal gravitation to derive the equations of motion for the planet,

$$\frac{d^2x}{dt^2} = -GM\left(\frac{1}{r_1^2}\frac{x - X_1}{r_1} + \frac{1}{r_2^2}\frac{x - X_2}{r_2}\right)$$

$$\frac{d^2y}{dt^2} = -GM\left(\frac{1}{r_1^2}\frac{y - Y_1}{r_1} + \frac{1}{r_2^2}\frac{y - Y_2}{r_2}\right) \quad .$$

In order to solve these equations, they must first be transformed into first order equations. This can be done fairly easily by introducing velocity and splitting them into four first order equations:

$$\frac{dx}{dt} = v_x$$
$$\frac{dv_x}{dt} = -GM\left(\frac{1}{r_1^2}\frac{x - X_1}{r_1} + \frac{1}{r_2^2}\frac{x - X_2}{r_2}\right)$$
$$\frac{dy}{dt} = v_y$$
$$\frac{dv_y}{dt} = -GM\left(\frac{1}{r_1^2}\frac{y - Y_1}{r_1} + \frac{1}{r_2^2}\frac{y - Y_2}{r_2}\right)$$

where v_x is the velocity in the x-direction and v_y is the velocity in the y-direction.

In order to view the planet's orbit from the reference frame of one of the stars, which is useful in some of the later results of this paper a slight change was made to the coordinate system as follows,

$$a = x + X_1$$

$$b = y + Y_1$$

where *(a, b)* is the coordinates of the planet in one of the star's rotating reference frame.

Finally, the equations in the program were made dimensionless by setting *R* and Ω to 1 and inputting the values for length and velocity in terms of these dimensionless quantities.

3 Method

To solve the four first order differential equations listed above the Runge-Kutta method (RK4) was used. This algorithm iterates over the functions in steps, starting from an initial value, in order to incrementally calculate the next value of the equation. It does this by approximating the value at the beginning of the current step, twice at the midpoint and at the end of the step (Hence, RK4), then taking an average of these 4 values, weighted towards the midpoint values.

Although this is by no means the best algorithm for solving differential equations, it is reasonably fast, accurate enough for our purposes and is very simple and convenient to use. The main issue with using this algorithm is that the minor errors will build up over multiple iterations, meaning we must be careful to choose a step size that is small enough to produce accurate results. An example of what can happen when these errors build up is shown in figure 1, below.

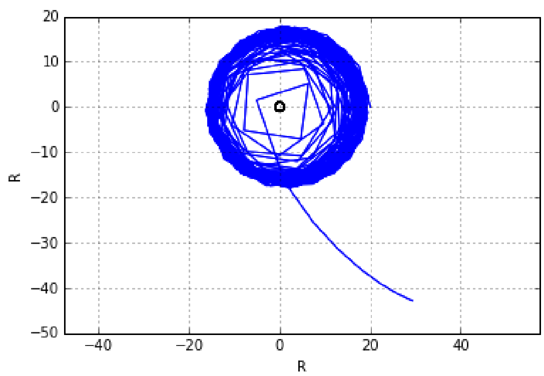

Figure 1 - A planetary orbit with 500 steps

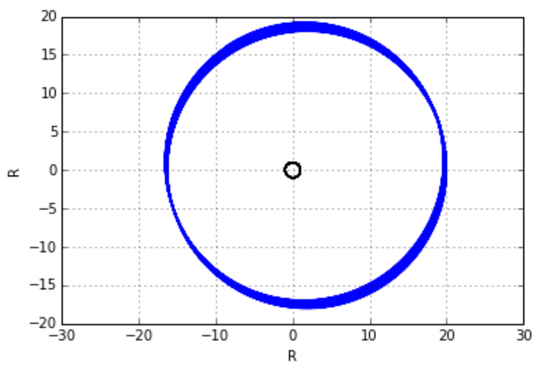

Figure 2 - The same planetary orbit, over the same period of time with 50,000 steps

The axes on these plots are given in units of the dimensionless quantity *R*, which is the radius of the binary stars' orbit. Please note that all future plots of orbits will be given in the same units. The quantity 'v' will similarly be used as an arbitrary measurement of velocity.

For each of the orbital plots shown in this paper black was used to indicate the orbit of the binary stars in the centre of the system, blue was used to represent the orbit of the planet and (with the exception of figures 1 and 2) red was used to represent the distance travelled by the planet in the space of 1 of the stars' orbital period.

When collecting results 2 main variations of the three body system were looked at: when the stars were orbiting in the same direction as the planet, and when the planet and stars were orbiting in opposite directions. In each case the planet was initially placed at a large distance from the centre of the system (20R) with the initial velocity being one that caused roughly a circular orbit. The initial distance from the centre of the system was then decreased until interesting behaviour was observed. The plots for each of the different initial conditions were also looked at from the reference frame of one of the stars in an attempt to better understand the nature of the orbits.

4 Results and Discussion

Firstly, the course of the planet when orbiting in the opposite direction to that of the stars was observed.

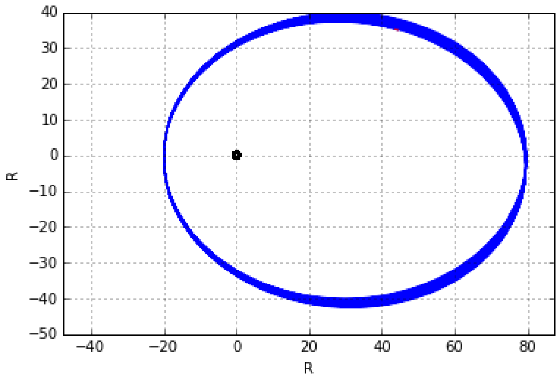

Figure 3 - Distance 20R, velocity 0.4v, opposite direction

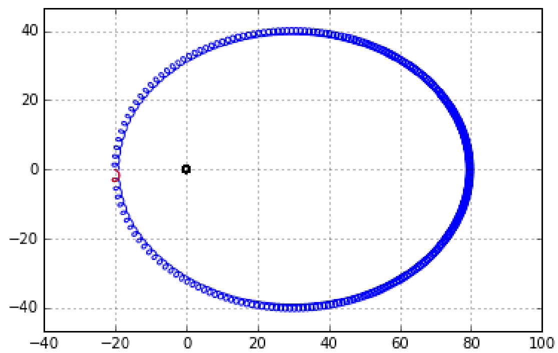

Figure 4 - Distance 20R, velocity 0.4v, opposite direction, star reference frame

As you can see in figure 3, at large distances from the centre of the system the orbit of the planet was almost identical to that of a single star system. The only difference being that due to the orbits of the planet and the stars not being completely in phase with one another the orbit appears to be gradually changing its angle with each rotation. There is a tiny red line on figure 3, on the upper right section of the orbit which represents the distance the planet travelled in 1 of the stars' orbital periods. In this case the planet's orbital period was ~250 times that of the stars'.

Figure 4 is, as one would expect, almost identical except for the perturbations which repeat during each of the stars' oribtal periods and represent the star moving during its orbit. From figure 4 however we can tell that the planet was travelling faster when it was closer to the stars, and slower when further away from the fact that the perturbations are more closely packed together when further away. This is in keeping with Kepler's second law of planetary motion.

Next, the same conditions were looked at when the planet was orbiting in the same direction as the stars.

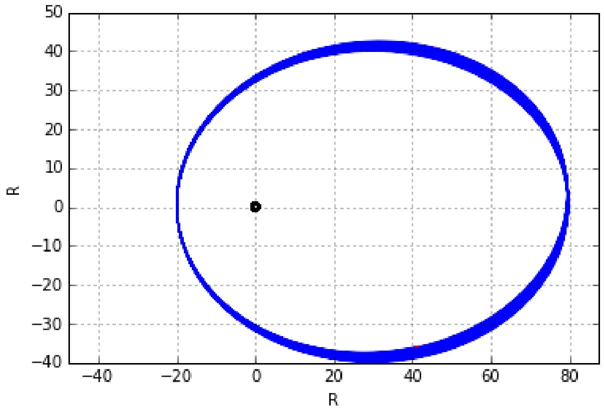

Figure 5 - Distance 20R, velocity 0.4v, same direction

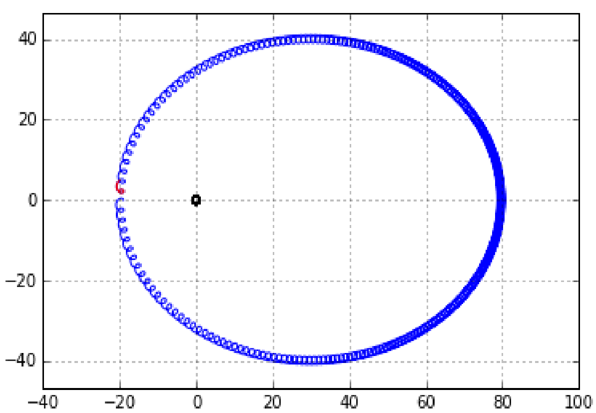

Figure 6 - Distance 20R, velocity 0.4v, same direction, star reference frame

These figures are almost identical to figures 3 and 4. This is what what one would expect, as at these distances from the centre of the system, the orbit behaves very similarly to a unary star system.

As we reduce distance that the planet is orbiting from the stars in the opposite direction, we first start to see major changes at about 2R.

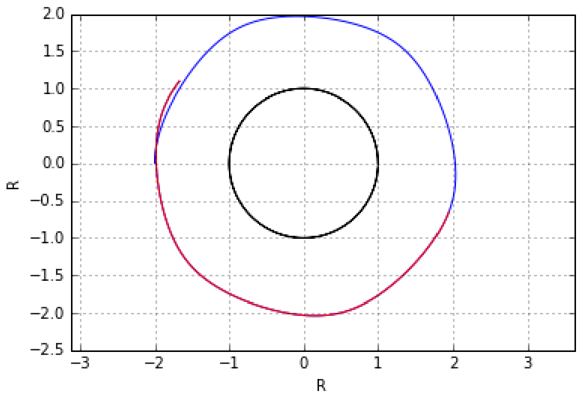

Figure 7 - Distance 2R, velocity 1.19v, opposite direction

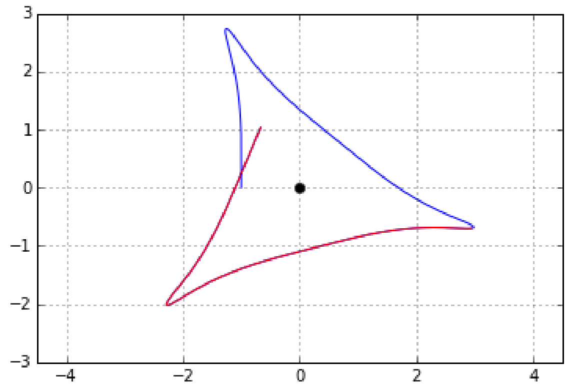

Figure 8 - Distance 2R, velocity 1.19v, opposite direction, star reference frame

As we can see from figure 7, as we have brought the planet in closer to the stars we begin to see some perturbations in its orbit. From the red line we can see that the orbital period of the planet is roughly double that of the stars at this point. Looking at figure 8 we can make a guess as to why this pattern occurs. First, in figure 8, the planet starts directly to the left of the star at (-2R, 0), propelled in an upwards direction, and the star we are taking the reference frame of starts directly to the right of the centre of mass of the system, at (R, 0), propelled in an upwards direction. Bearing this and the orital periods in mind we can deduce that at the top left peak of figure 8 is the first point at which the planet is closest to our star. This happens twice more before the planet completes its orbit. From figure 7 we can see that there are 6 perturbations in the planets orbit, so it is safe to assume that these happen whenever the stars both line up with the planet. This change in the stars' alignment is causing some gravitational perturbation which is in turn affecting the planet's orbit. If we further decrease the radius we can see an even bigger perturbation of the planet's orbit.

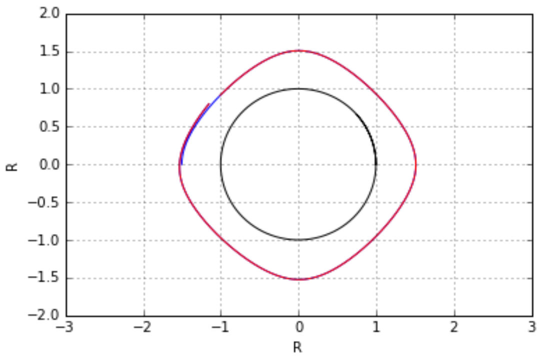

Figure 9 - Distance 1.5R, velocity 1.67v, opposite direction

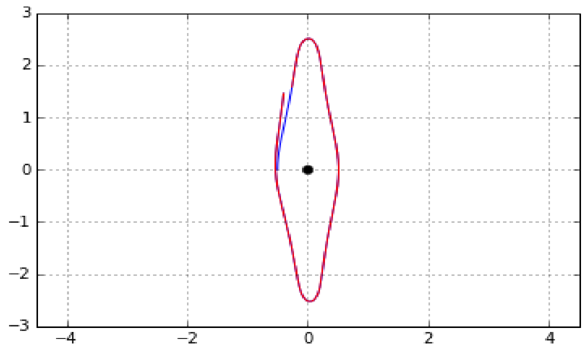

Figure 10 - Distance 1.5R, velocity 1.67b, opposite direction, star reference frame

In these the decrease to the distance from the star has caused the perturbation to the planet's orbit has become even more apparent, now resembling a squared circle. The red line tells us the planet's orbital period is now roughly in line with that of the star's

It should be noted that although in both of the example's given here, the orbital periods of the stars and the planet resonated, similar patterns could be observed at all values of R, with the number of perturbations per orbit varying with each orbit depending on how close to a resonant orbit it was. However, it was difficult to find stable orbits below 1.5R. The ones that were tested had a tendency to collide with the stars' orbit path at some point causing the program to fling the planet into space.

When the radius was decreased further however there was one other interesting phenomana, shown in figure 11, below.

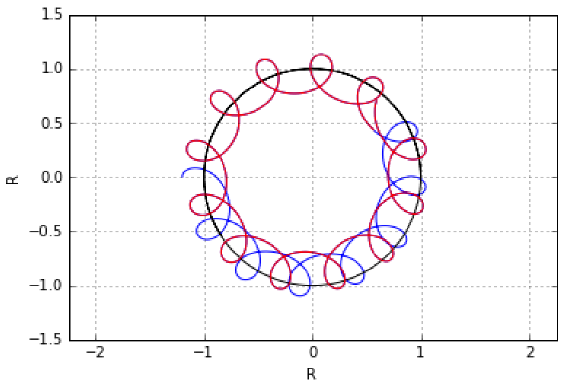

Figure 11 - Distance 1.2R, velocity 1.3v, opposite direction

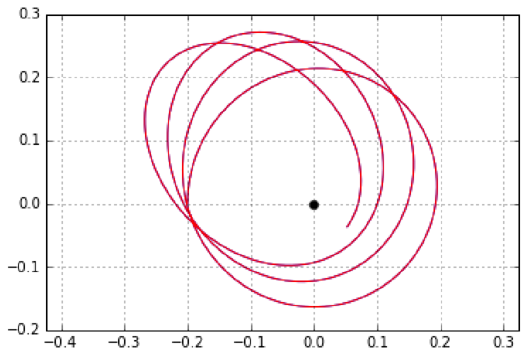

Figure 12 - Distance 1.2R, velocity 1.3v, opposite direction, star reference frame

From figure 11 we can see the planet is orbiting just one of the stars in the system, and in figure 12 we see that it is a reasonably stable orbit, with the second star causing some perturbations.

Now, we will move on to the plots with the stars and planet rotating in the same direction.

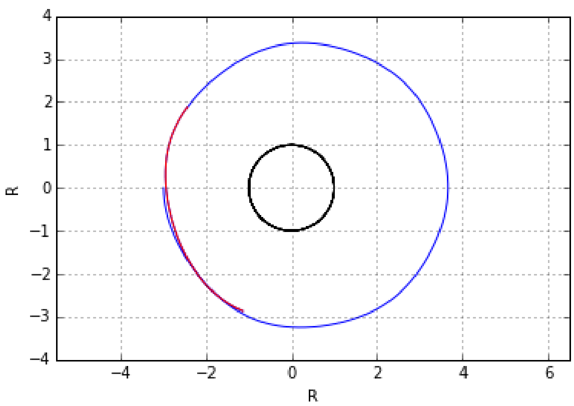

Figure 13 - Distance 3R, velocity -0.85v, same direction

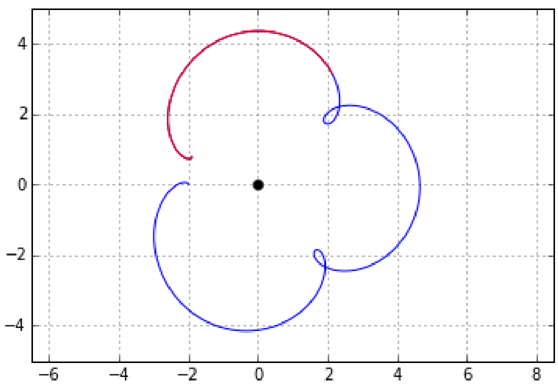

Figure 14 - Distance 3R, velocity -0.85v, same direction, star reference frame

Here we can see a similar perturbation of the planet's orbit to what we saw before. However, this time they have started much sooner. Whereas before we saw similar sized perturbations at 2R, this time they appear to have started at 3R instead.

Below 3R it was impossible to find a stable orbit around both stars. After a certain number of orbits, the planet would always be flung into space as shown in figure 15 and 16, below.

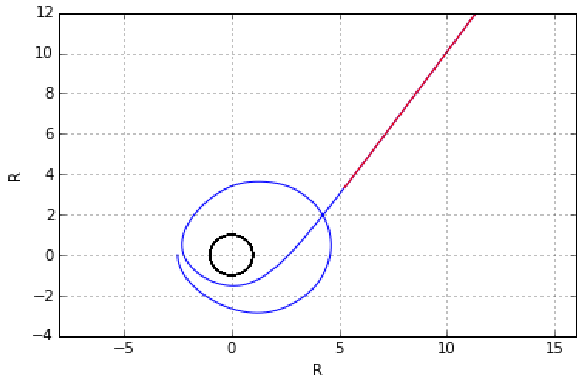

Figure 15 - Distance 2.5R, velocity -0.9v, same direction

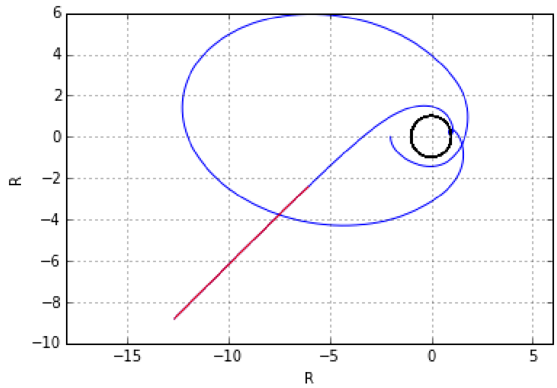

Figure 16 - Distance 2R, velocity -0.97v, same direction

In figure 15 we can see the planet make 1 full orbit, then fly off into space, and in figure 16 we see the planet make one large orbit (despite a relatively low starting velocity), then orbit one of the stars once before also flying off into space. My hypothesis for why this happens here, and not when the planet is rotating in the opposite direction to the stars is that the planet is more likely to spend more time close to one of the stars (since they will rotate together) when close to the binary orbit. When this happens the planet gets affected much more by the gravity of the nearer star and gets pulled in by it. This means the planet can't find a stable orbit and its behaviour becomes chaotic, causing the planet to eventually get thrown into space.

Finally, it was also possible to find a stable orbit around a single star with the same initial conditions as show in figures 17 and 18, below.

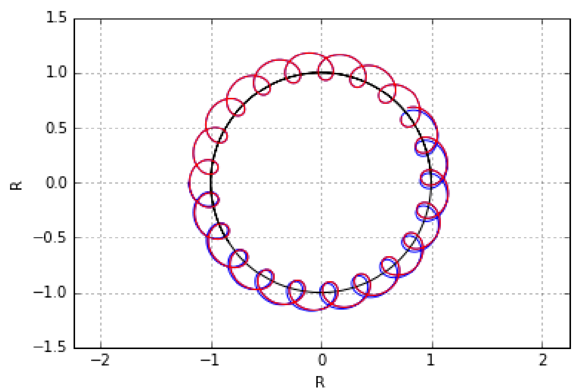

Figure 17 - Distance 1.2R, velocity -2.5v, same direction

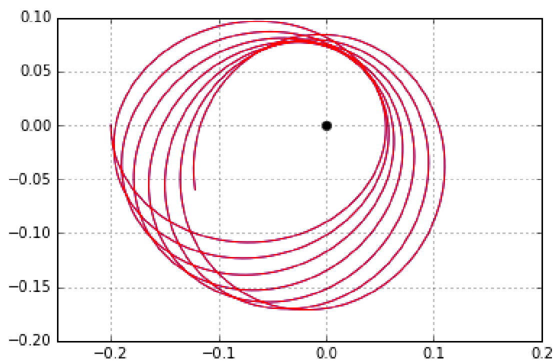

Figure 18 - Distance 1.2R, velocity -2.5v, same direction, star reference frame

Here we see a very similar orbit to figures 11 and 12, however this time a much higher initial velocity was needed to make it work. This is likely due to the planet needing to 'catch up' to the star as they are travelling in the same direction.

5 Conclusions

From these findings we can conclude that while the planet is a significant distance (>3R) away from the centre of mass of the system, there is very little difference between a binary star system and a unary star system, with the third body causing only negligible perturbations in the orbit of the planet. However as the distance between the star system and the planet becomes smaller the differences become more and more apparent.

When the stars were rotating in the opposite direction to the planet it became easy to find a stable orbit with small perturbations dependant on the orbital period of the planet compared to that of the star. However when the directions of orbit were the same the orbits had chaotic behaviour with little predictability and a likelihood of being thrown into space eventually.

Finally, it was also possible for the planet to orbit a single one of the stars, with the second star causing only minor perturbations in this orbit.

References

[1] Howell, Elizabeth. "How Many Stars Are In The Milky Way?". *Space.com*. N.p., 2017. Web. 19 May 2017.

[2] Wall, Mike. "Nearly Every Star Hosts At Least One Alien Planet". *Space.com*. N.p., 2017. Web. 19 May 2017.