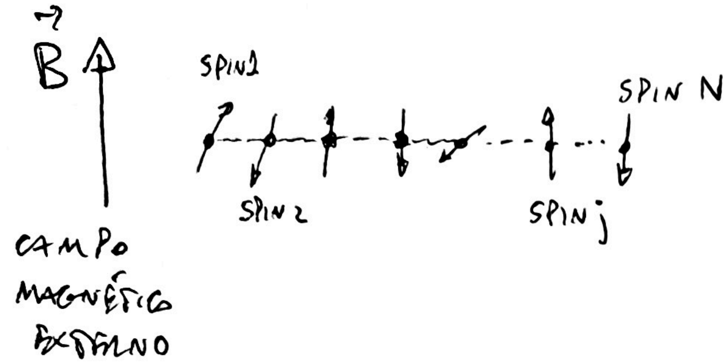
MODELO DE HEISENBERG



→ CADEIA DE N SPINS COM INTERAÇÕES DE PRIMEIROS VIZINHOS.

HAMILTONIANO :

$$\hat{H} = -\frac{1}{2}\sum_{j=1}^{N}\left(J_x \sigma_j^x \otimes \sigma_{j+1}^x + J_y \sigma_j^y \otimes \sigma_{j+1}^y + J_z \sigma_j^z \otimes \sigma_{j+1}^z \right),$$

ONDE O SINAL NEGATIVO INDICA O CASO FERROMAGNÉTICO.

NA PRESENÇA DE UM CAMPO MAGNÉTICO EXTERNO, ADICIONA-SE AO HAMILTONIANO ACIMA O TERMO :

$$\hat{H}_{EXT} = -h\sum_{j=1}^{N}\sigma_j^z ,$$

ONDE DEFINIMOS A DIREÇÃO z COMO PARALELA A $\vec{B}$.

OBS: O SÍMBOLO "$\otimes$" INDICA PRODUTO TENSORIAL.

COMO OS SPINS DAS EXTREMIDADES DA CADEIA DESEMPENHAM UM PAPEL IMPORTANTE E DISTINTO DOS OUTROS (SERÃO SUBMETIDOS A MEDIÇÕES PERIODICAMENTE), VAMOS SEPARÁ-LOS NO HAMILTONIANO:

$$
\begin{aligned}
H = & -\frac{1}{2} J'_x \sigma_1^x \otimes \sigma_2^x - \frac{1}{2} J'_y \sigma_1^y \otimes \sigma_2^y - \frac{1}{2} J'_z \sigma_1^z \otimes \sigma_2^z \\
& - \frac{1}{2} \sum_{j=2}^{N-1} \left(J_x \sigma_j^x \otimes \sigma_{j+1}^x + J_y \sigma_j^y \otimes \sigma_{j+1}^y + J_z \sigma_j^z \otimes \sigma_{j+1}^z \right) \\
& - \frac{1}{2} J''_x \sigma_{N-1}^x \otimes \sigma_N^x - \frac{1}{2} J''_y \sigma_{N-1}^y \otimes \sigma_N^y - \frac{1}{2} J''_z \sigma_{N-1}^z \otimes \sigma_N^z
\end{aligned}
$$

PARA SIMPLIFICAR A ANÁLISE INICIAL, VAMOS CONSIDERAR QUE OS SPINS INTERMEDIÁRIOS OBEDECEM AO MODELO DE ISING ($J_x = J_y = 0$). ALÉM DISSO, VAMOS CONSIDERAR QUE NAS EXTREMIDADES TEMOS $J'_y = J'_z = 0$ E $J''_x = J''_y = 0$

FICAMOS COM

$$\hat{H} = -\frac{1}{2} J'_x \sigma_1^x \otimes \sigma_2^x - \frac{J_z}{2} \sum_{j=2}^{N-1} \sigma_j^z \otimes \sigma_{j+1}^z - \frac{J''_z}{2} \sigma_{N-1}^z \otimes \sigma_N^z$$

COMO COMBINADO, VAMOS CONSIDERAR UMA "CADEIA TESTE" COM APENAS 3 SPINS:

$$\hat{H} = -\frac{J'}{2} \sigma_1^x \otimes \sigma_2^x - \frac{J''}{2} \sigma_2^z \otimes \sigma_3^z , \qquad (*)$$

ONDE DEFINIMOS $J'_x = J'$ E $J''_z = J''$. VAMOS DETERMINAR A MATRIZ QUE REPRESENTA $\hat{H}$ NA BASE DE AUTOVALORES DE σ_z (PARA OS 3 SPINS). NESTA BASE TEMOS

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad E \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

ASSIM

$$\sigma_1^x \otimes \sigma_2^x = \begin{pmatrix} 0 \cdot \sigma_2^x & 1 \cdot \sigma_2^x \\ 1 \cdot \sigma_2^x & 0 \cdot \sigma_2^x \end{pmatrix} = \left(\begin{array}{cc:cc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)$$

(SPINS 1 E 2).

$$\sigma_2^z \otimes \sigma_3^z = \begin{pmatrix} 1.\sigma_3^z & 0.\sigma_3^z \\ 0.\sigma_3^z & -1\,\sigma_3^z \end{pmatrix} = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

(SPINS 2 E 3).

NA VERDADE, EM (*) DEVEMOS ENTENDER

$$\sigma_1^x \otimes \sigma_2^x \quad \text{COMO} \quad \sigma_1^x \otimes \sigma_2^x \otimes \mathbb{1}_3$$

$$\sigma_2^z \otimes \sigma_3^z \quad \text{COM} \quad \mathbb{1}_1 \otimes \sigma_2^z \otimes \sigma_3^z$$

ONDE $\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ É A IDENTIDADE (2×2).

ASSIM, PARA $\sigma_1^x \otimes \sigma_2^x \otimes \mathbb{1}_3$ TEMOS:

$$\left(\begin{array}{cccc|cccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right)$$

PARA A MATRIZ $\mathbb{1}_1 \otimes \sigma_2^z \otimes \sigma_3^z$ TEMOS:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

PORTANTO, A MATRIZ COMPLETA QUE REPRESENTA $\hat{H}$ EM $(*)$ É:

$$-\frac{1}{2}\begin{pmatrix} J'' & 0 & 0 & J' & 0 & 0 & 0 & 0 \\ 0 & -J'' & J' & 0 & 0 & 0 & 0 & 0 \\ 0 & J' & -J'' & 0 & 0 & 0 & 0 & 0 \\ J' & 0 & 0 & J'' & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J'' & 0 & 0 & J' \\ 0 & 0 & 0 & 0 & 0 & -J'' & J' & 0 \\ 0 & 0 & 0 & 0 & 0 & J' & -J'' & 0 \\ 0 & 0 & 0 & 0 & J' & 0 & 0 & J'' \end{pmatrix}$$