D26: A digraph D is ***strongly connected*** (or ***strong***) if D contains both a directed u-v path and a directed v-u path for every pair u, v of distinct vertices of D.

Radius and Diameter in Strong Digraphs

The definitions of eccentricity, radius, and diameter in a digraph are analogous to those in an undirected graph (see Definitions 3 and 4).

DEFINITION

D27: The ***eccentricity*** $e(v)$ of v in a strong digraph D is the greatest directed distance from v to a vertex of D. The minimum eccentricity among the vertices of D is its ***radius***, $rad(D)$, and the maximum eccentricity is its ***diameter***, $diam(D)$.

EXAMPLES

E13: There are three directed u-v paths in the digraph D of Figure 9.1.13. A shortest directed u-v path has length 2 and so $\vec{d}(u, v) = 2$. On the other hand, there is no directed v-u path in D. In fact, there is no directed x-u path in D for any vertex x $(\neq u)$ of D since the indegree of u is 0. Therefore, D is not a strong digraph.

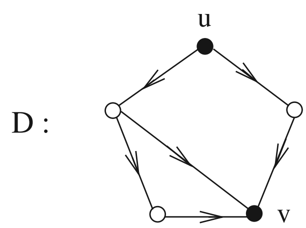

Figure 9.1.13 **A digraph that is not strong.**

E14: The vertices of the strong digraph D of Figure 9.1.14 are labeled by their eccentricities. Observe that $rad(D) = 2$ and $diam(D) = 5$. So, in general, it is not true that $diam(D) \leq 2rad(D)$.

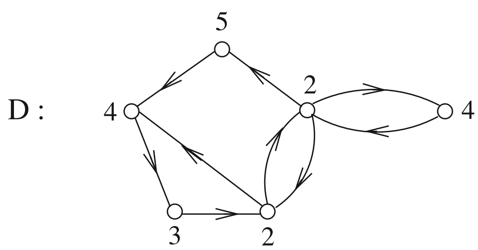

Figure 9.1.14 **The eccentricities of the vertices of a strong digraph.**

FACT

F37: [ChJoTi92] For every two positive integers a and b with $a \leq b$, there exists a strong digraph D with $rad(D) = a$ and $diam(D) = b$.