$ \begin{array}{rcl} & 1001 & = & -7 \\ & +0101 & = & 5 \\ & 1110 & = & -2 \\ & (a) (-7) + (+5) \end{array} $	1100 = -4 +0100 = 4 10000 = 0 (b) (-4) + (+4)
0011 = 3 $+0100 = 4$ $0111 = 7$ $(c) (+3) + (+4)$	1100 = -4 +1111 = -1 11011 = -5 (d) (-4) + (-1)
0101 = 5 + 0100 = 4 1001 = Overflow (e) (+5) + (+4)	1001 = -7 +1010 = -6 10011 = Overflow (f) (-7) + (-6)

Figure 9.3 Addition of Numbers in Twos Complement Representation

$$\begin{array}{c} 0010 = 2 \\ + 1001 \\ \hline 1011 = -5 \\ \end{array} \qquad \begin{array}{c} 0101 = 5 \\ + 1110 = -2 \\ \hline 10011 = 3 \\ \end{array} \\ (a) \ \ M = 2 = 0010 \\ S = 7 = 0111 \\ -S = 1001 \\ \end{array} \qquad \begin{array}{c} (b) \ \ M = 5 = 0101 \\ S = 2 = 0010 \\ -S = 1110 \\ \end{array} \\ \begin{array}{c} 1011 = -5 \\ + 1110 = -2 \\ \hline 11001 = -7 \\ \end{array} \qquad \begin{array}{c} 0101 = 5 \\ + 0010 = 2 \\ \hline 0111 = 7 \\ + 0111 = 7 \\ \hline 1110 = 0 \end{array} \\ (c) \ \ M = -5 = 1011 \\ S = 2 = 0010 \\ -S = 1110 \\ \end{array} \qquad \begin{array}{c} (d) \ \ M = 5 = 0101 \\ S = -2 = 1110 \\ -S = 0010 \\ \end{array} \\ \begin{array}{c} 0111 = 7 \\ + 0111 = 7 \\ \hline 1110 = 0 \end{array} \\ \begin{array}{c} 0111 = 7 \\ + 0111 = 7 \\ \hline 1110 = 0 \end{array} \\ \begin{array}{c} (d) \ \ M = 5 = 0101 \\ S = -2 = 1110 \\ \end{array} \\ \begin{array}{c} 0110 = -6 \\ + 1100 = -4 \\ \hline 10110 = 0 \end{array} \\ \begin{array}{c} 0110 = -6 \\ + 1100 = 0 \end{array} \\ \begin{array}{c} 0110 = -6 \\ + 1100 = 0 \end{array} \\ \begin{array}{c} 0111 = 7 \\ - 0111 = 7 \\ \hline 0111 = 7 \\ - 0111 = 7 \\ \hline 01110 = 0 \end{array} \\ \begin{array}{c} 0111 = 7 \\ \hline 01110 = -6 \\ \hline 01110 = 0 \end{array} \\ \begin{array}{c} 0111 = 7 \\ \hline 01110 = 0 \end{array} \\ \end{array}$$

Figure 9.4 Subtraction of Numbers in Twos Complement Representation (M – S)

```
1001 (-7)
     1001 (9)
    ×0011
            (3)
                                           <u>×0011</u>
                                                   (3)
                                                   (-7) \times 2^0 = (-7)
00001001
            1001 \times 2^{0}
                                       11111001
                                       11110010 \quad (-7) \times 2^{1} = (-14)
00010010 1001 \times 2^{1}
            (27)
                                       11101011
                                                   (-21)
00011011
      (a) Unsigned integers
                                          (b) Twos complement integers
```

Figure 9.11 Comparison of Multiplication of Unsigned and Twos Complement Integers

$ \begin{array}{c} 0111 \\ \times 0011 \\ 11111001 \\ 0000000 \\ 1-1 \\ 000111 \\ 00010101 \\ (21) \end{array} $ (a) (7) × (3) = (21)	$\begin{array}{c} 0111 \\ \times 1101 \\ 11111001 \\ 0000111 \\ 111001 \\ 1-0 \\ 11101011 \\ (-21) \end{array}$ (b) (7) × (-3) = (-21)
$ \begin{array}{c} 1001 \\ \times 0011 \\ 00000111 \\ 1-0 \\ 0000000 \\ 1-1 \\ \underline{111001} \\ 11101011 \\ (c) (-7) \times (3) = (-21) \end{array} $	$ \begin{array}{c} 1001 \\ \times 1101 \\ 00000111 \\ 1-0 \\ 1111001 \\ 000111 \\ 00010101 \\ (21) \end{array} $ $ (d) (-7) \times (-3) = (21) $

Figure 9.14 Examples Using Booth's Algorithm

A	Q	M = 0011	A	Q	M = 1101
0000	0111	Initial value	0000	0111	Initial value
0000 1101 0000	1110 1110	shift subtract restore	0000 1101 0000	1110 1110	shift add restore
0001 1110	1100	shift subtract	0001 1110	1100	shift add
0001	1100	restore	0001	1100	restore
0011 0000	1000	shift subtract	0011 0000	1000	shift add
0000	1001	set $Q_0 = 1$	0000	1001	$set Q_0 = 1$
0001 1110	0010	shift subtract	0001 1110	0010	shift add
0001	0010	restore	0001	0010	restore
	(a) $(7)/(3)$			(b) $(7)/(-3)$	

A	Q	M = 0011	A	Q	M = 1101
1111	1001	Initial value	1111	1001	Initial value
1111	0.01.0	1:0	1111	0010	1 : 6.
1111 0010	0010	shift add	1111 0010	0010	shift subtract
1111	0010	restore	1111	0010	restore
	0010	restore	***	0010	restore
1110	0100	shift	1110	0100	shift
0001		add	0001		subtract
1110	0100	restore	1110	0100	restore
1100	1000	al.: 64	1100	1000	al.:64
1100 1111	1000	shift add	1100 1111	1000	shift subtract
1111	1001	$set Q_0 = 1$	1111	1001	$set Q_0 = 1$
	1001	500 Q ₀ 1		1001	560 Q ₀ 1
1111	0010	shift	1111	0010	shift
0010		add	0010		subtract
1111	0010	restore	1111	0010	restore
	(c) (-7)/(3)			(d) $(-7)/(-3)$	

Figure 9.17 Examples of Twos Complement Division

```
x = 1.000....00 \times 2^{1}
                                                               x = .100000 \times 16^{1}
   -y = 0.111....11 \times 2^{1}
                                                             \underline{-y} = \underline{.0FFFFF} \times 16^1
     z = 0.000....01 \times 2^{1}
                                                               z = .000001 \times 16^{1}
       = 1.000....00 \times 2^{-22}
                                                                  = .100000 \times 16^{-4}
(a) Binary example, without guard bits
                                                      (c) Hexadecimal example, without guard bits
    x = 1.000....00 0000 \times 2^{1}
                                                               x = .100000 00 \times 16^{1}
   -y = 0.111....11 1000 \times 2^{1}
                                                             \underline{-y} = \underline{.0} + \underline{.0} \times 16^{1}
     z = 0.000....00 \ 1000 \times 2^{1}
                                                               z = .000000 \ 10 \times 16^{1}
        = 1.000....00 0000 \times 2^{-23}
                                                                  = .100000 00 \times 16^{-5}
 (b) Binary example, with guard bits
                                                        (d) Hexadecimal example, with guard bits
```

Figure 9.25 The Use of Guard Bits