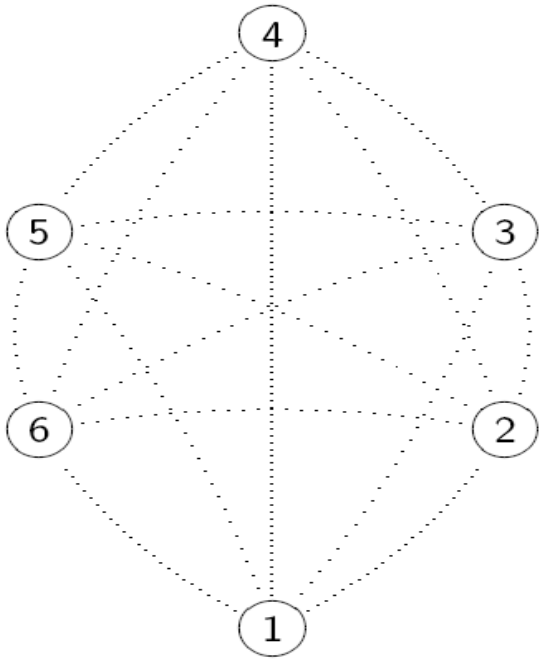
Pesquisa Operacional / Programação Matemática

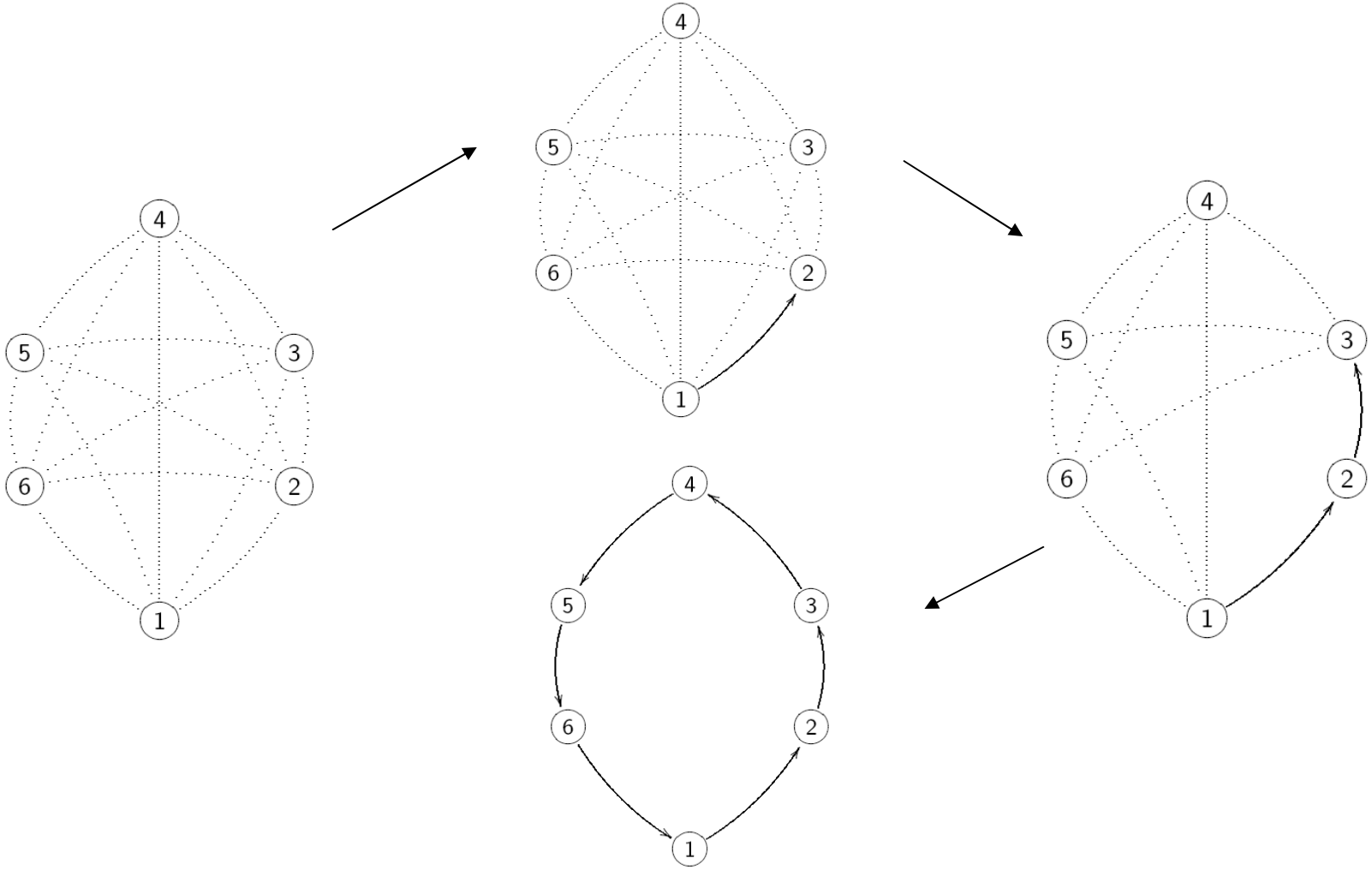
Otimização discreta

Mais problemas

Alysson M. Costa – ICMC/USP

Caixeiro viajante

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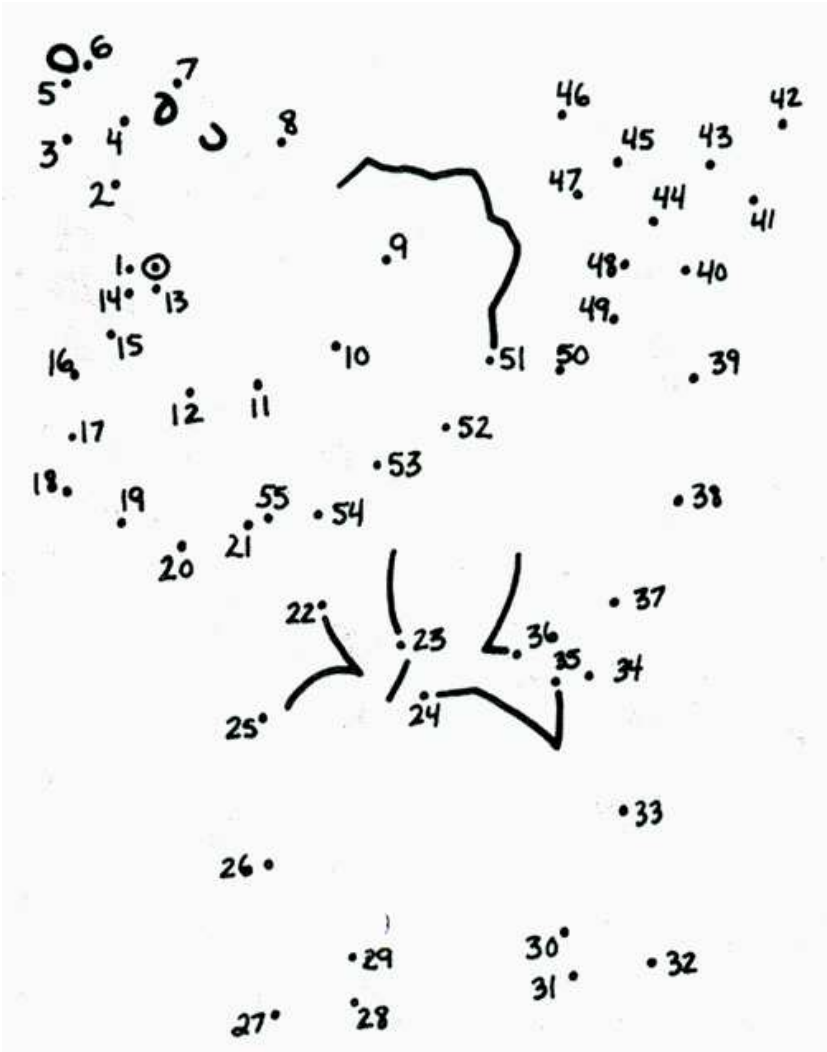


The Travelling Salesman Problem: a computational study. Applegate, Bixby, Chvátal and Cook

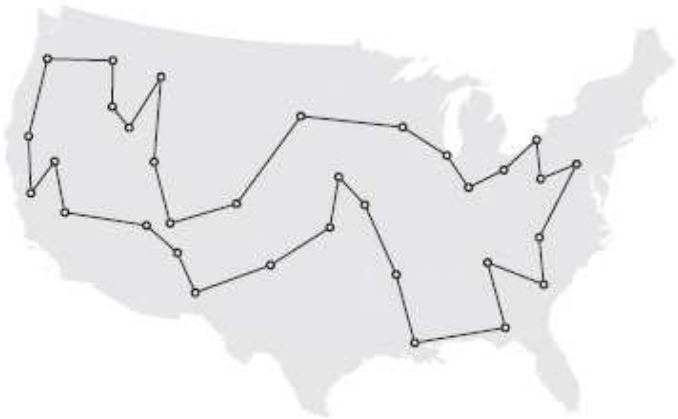
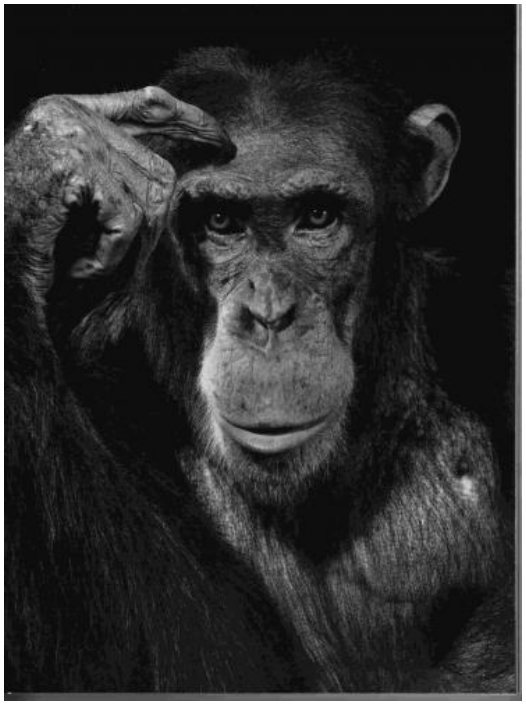


Figure 1.10 Optimal 33-city tour.

The Travelling Salesman Problem: a computational study. Applegate, Bixby, Chvátal and Cook



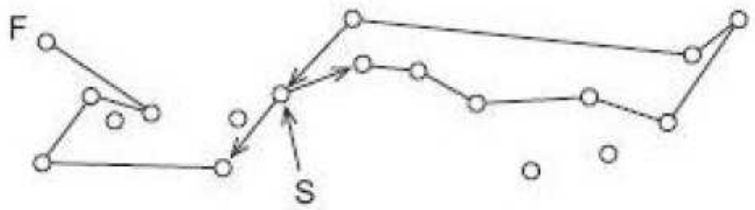
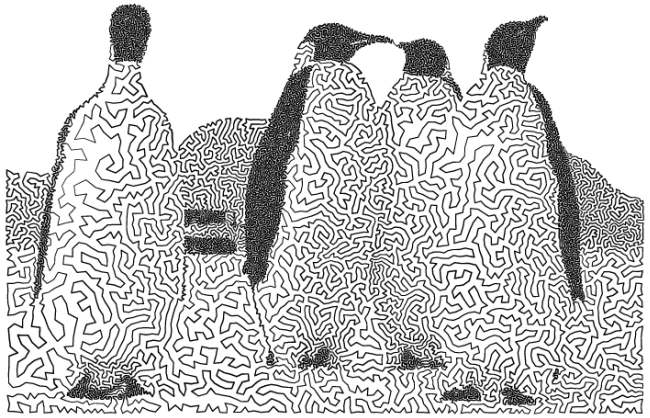


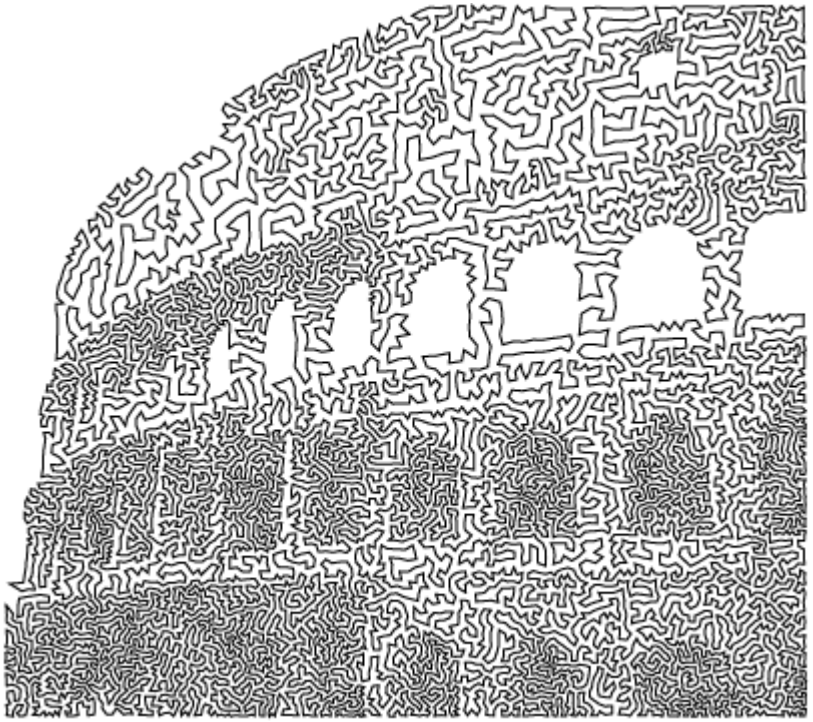
Figure 1.40 Chimpanzee tour (Bido).

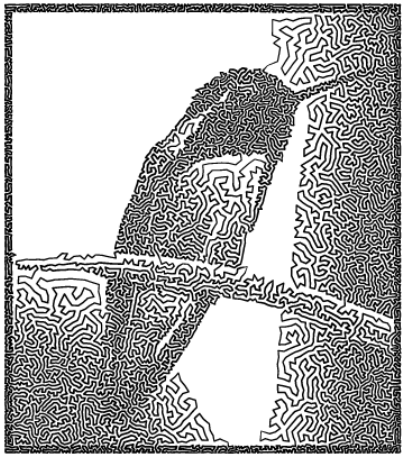
The Travelling Salesman Problem: a computational study. Applegate, Bixby, Chvátal and Cook

Figure 1.41 Pigeon solving a TSP. Images courtesy of Brett Gibson.

The Travelling Salesman Problem: a computational study. Applegate, Bixby, Chvátal and Cook







http://www.cgl.uwaterloo.ca/~csk/projects/tsp

Caixeiro viajante (estética!)

Figure 1.23 *Traveling Salesman*, by Julian Lethbridge. Image courtesy of Julian Lethbridge and United Limited Art Editions.

The Travelling Salesman Problem: a computational study. Applegate, Bixby, Chvátal and Cook

Polivanova, N. I.
Functional and structural aspects of the visual components of intuition in problem solving
Voprosy Psikhologii, **1974**, *4*, 41-51

- problemas representados geometricamente × problemas onde as distâncias são dadas para cada par de cidades.

Vickers, D.; Butavicius, M.; Lee, M. & Medvedev, A.
Human performance on visually presented Traveling Salesman problems
Psychological Research, **2001**, *65*, 34-45

Otimização x Estética

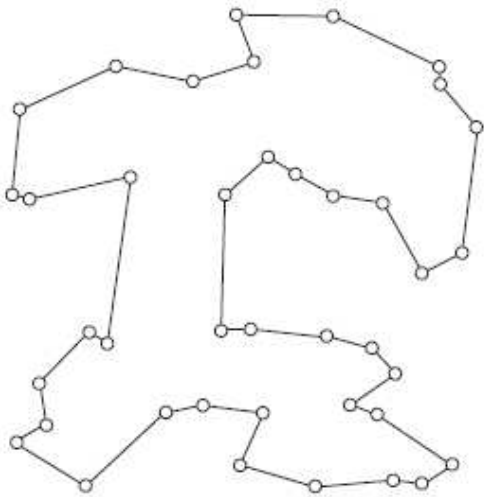
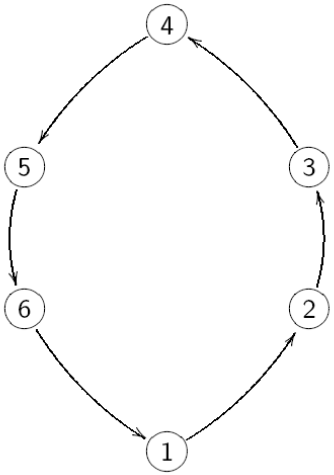


Figure 1.32 Tour found by member of the Gestalt group. Vickers et al. [539].

Formulação matemática



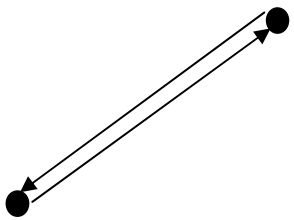
$$\text{Min} \sum_{(i,j)\in A} c_{ij}x_{ij}$$

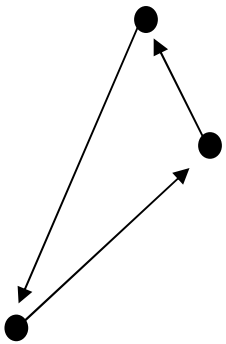
$$\sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, \ldots, n$$

$$\sum_{j=1}^{n} x_{ji} = 1 \quad i = 1, \ldots, n$$

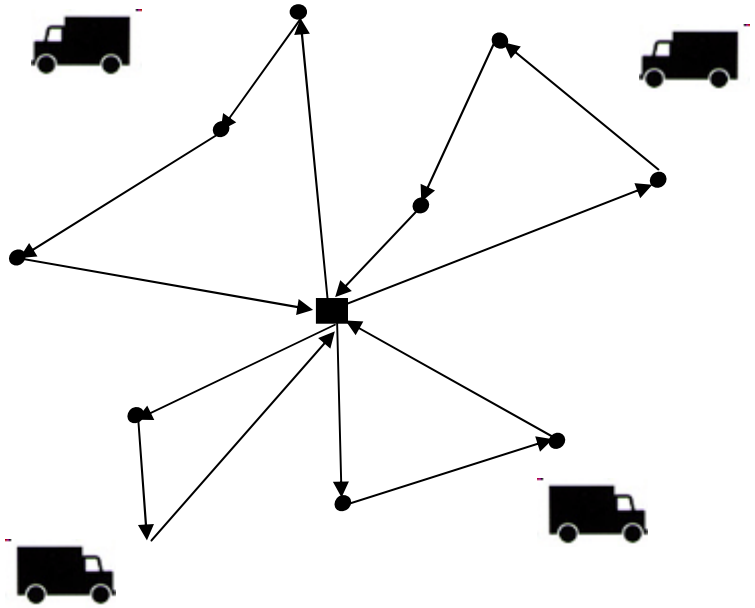
$$\sum_{i,j\in S} x_{ij} \leq |S| - 1, \quad S \subseteq N - \{1\}, |S| \geq 2$$

$$x_{ij} \in \{0, 1\}, \quad (i, j) \in A$$

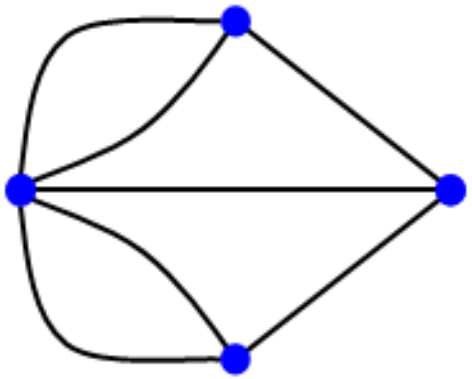


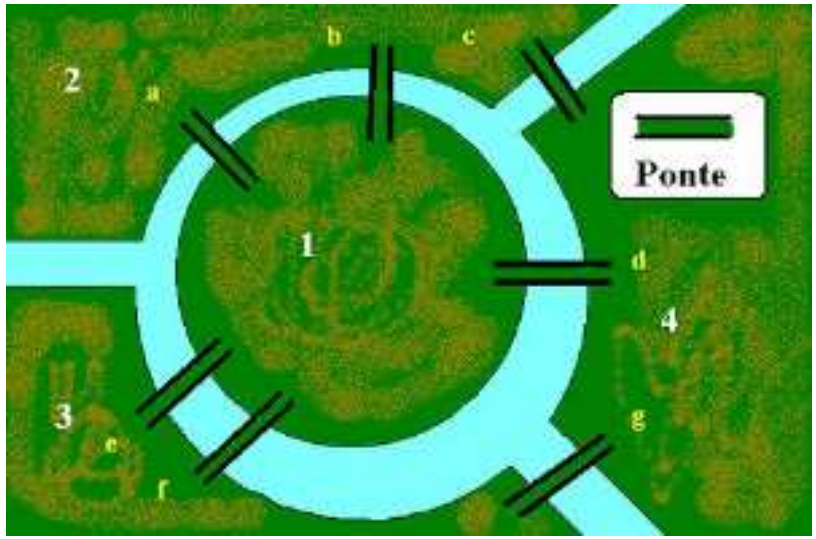


Roteamento de veículos

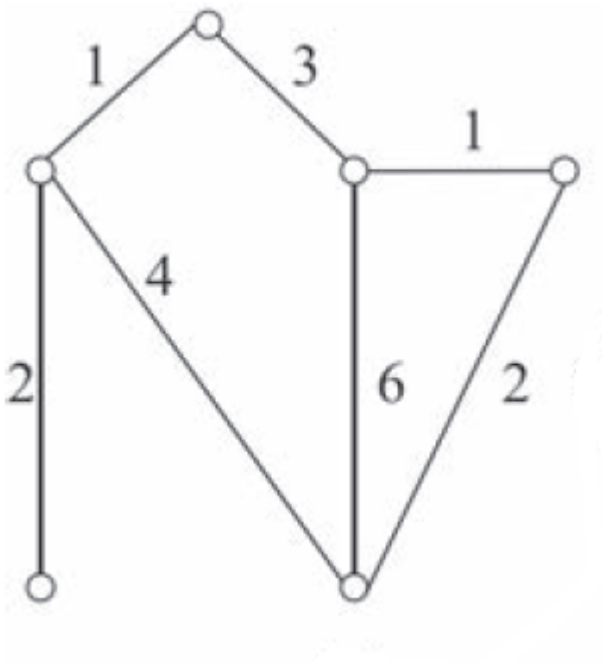


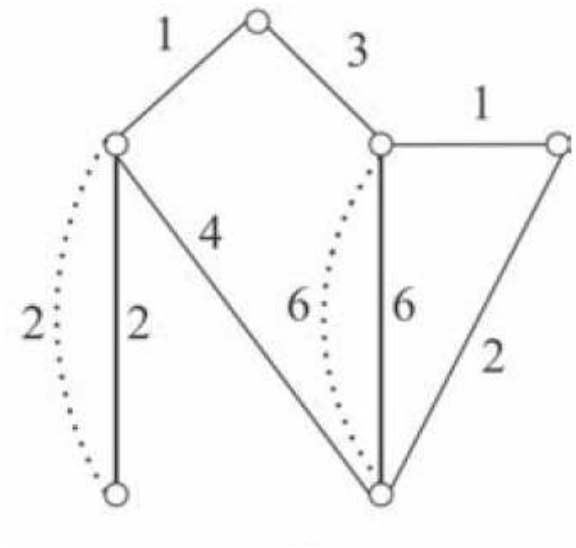
Problema do carteiro chinês (origem)

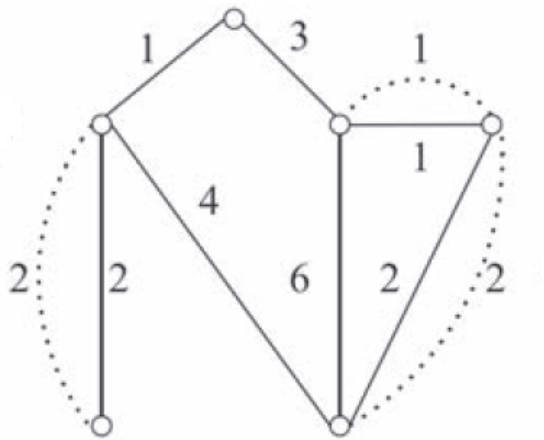




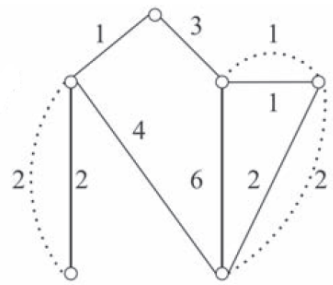
Problema do carteiro chinês







Problema do carteiro chinês



x_{ij} = número de vezes que o arco (i, j) é atravessado

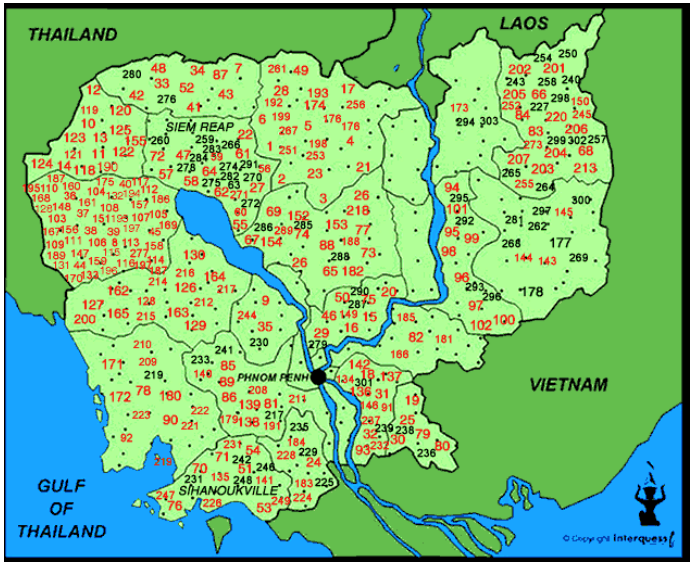
$$\min \sum_{(i,j)\in E} c_{ij} x_{ij}$$

$$\sum_{\{j:(i,j)\in E\}} x_{ij} - \sum_{\{j:(j,i)\in E\}} x_{ji} = 0, \quad i \in N$$

$$x_{ij} \geq 1, \quad \forall (i, j) \in E$$

$$\mathbf{x} \in Z_{+}^{|E|}$$

Localização de facilidades



http://www.camnet.com.kh/cambodiaschools/schools_map/schools_map.htm

P-medianas

- c_{ij}: distância do *consumidor* j à facilidade i.
- p: número de facilidades a instalar.

$$y_i = \begin{cases} 1, & \text{se a facilidade é aberta no local } i \\ 0, & \text{caso contrário} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{se o cliente } j \text{ é designado à facilidade localizada em } i \\ 0 & \text{caso contrário} \end{cases}$$

$$y_i = \begin{cases} 1, & \text{se a facilidade é aberta no local } i \\ 0, & \text{caso contrário} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{se o cliente } j \text{ é designado à facilidade localizada em } i \\ 0 & \text{caso contrário} \end{cases}$$

$$\min \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$$

$$\sum_{i \in I} x_{ij} = 1, \quad \forall j \in J$$

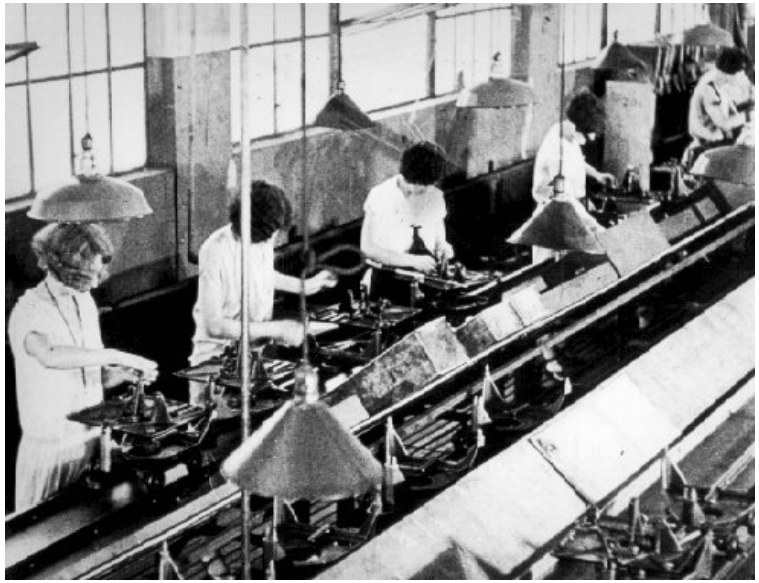
$$x_{ij} \leq y_i, \quad \forall i \in I, \quad \forall j \in J$$

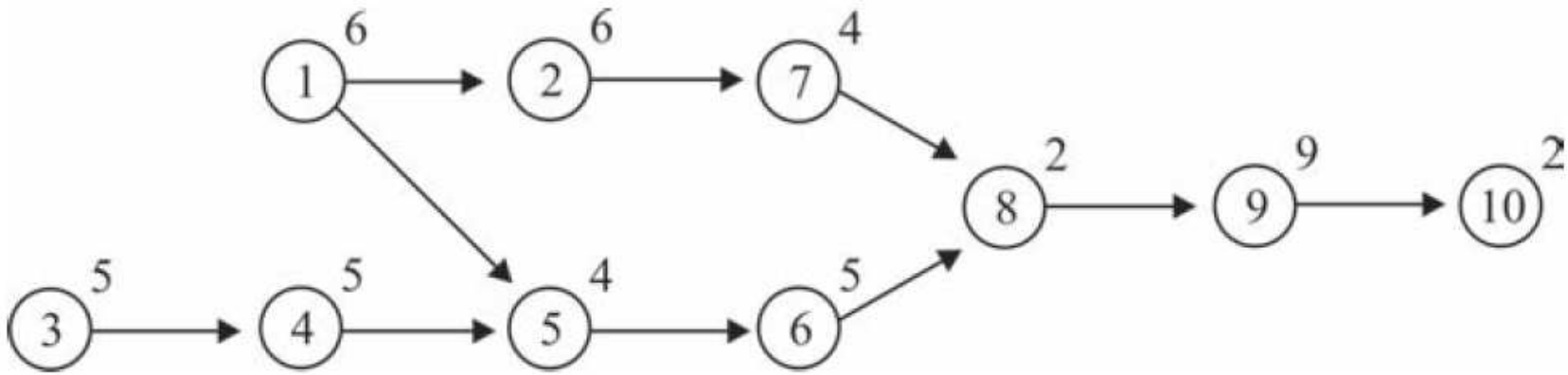
$$\sum_{i \in I} y_i = p$$

$$\mathbf{x} \in B^{|I||J|}, \mathbf{y} \in B^{|I|}$$

Balanceamento de linhas de montagem

- Precedências





Balanceamento de linhas de montagem

$$\text{Min } C$$

subject to

$$\begin{aligned}
\sum_{s \in S} x_{si} &= 1, && \forall i \in N, \\
\sum_{s \in S} s \cdot x_{si} &\leq \sum_{s \in S} s \cdot x_{sj}, && \forall j \in N, i \in D_j, \\
\sum_{i \in N} t_i \cdot x_{si} &\leq C, && \forall s \in S, \\
x_{si} &\in \{0, 1\} && \forall s \in S, \forall i \in N.
\end{aligned}$$

Sir John Everett Millais - The Blind Girl (1856)

Costa, Moreira, Santos, Miralles (2009)

$$\text{Min } C$$

subject to

$$\sum_{w \in W} \sum_{s \in S} x_{swi} = 1, \qquad \forall i \in N,$$

$$\sum_{s \in S} y_{sw} = 1 \qquad \forall w \in W,$$

$$\sum_{w \in W} y_{sw} = 1 \qquad \forall s \in S,$$

$$\sum_{w \in W} \sum_{s \in S} s \cdot x_{swi} \leq \sum_{w \in W} \sum_{s \in S} s \cdot x_{swj} \qquad \forall j \in N, i \in D_j,$$

$$\sum_{w \in W} \sum_{i \in N} p_{wi} \cdot x_{swi} \leq C \qquad \forall s \in S,$$

$$\sum_{i \in N} x_{swi} \leq M y_{sw} \qquad \forall w \in W, \forall s \in S,$$

$$y_{sw} \in \{0, 1\} \qquad \forall s \in S, \forall w \in W,$$

$$x_{swi} \in \{0, 1\} \qquad \forall s \in S, \forall w \in W, \forall i \in N.$$

Família	Grupo	Média SALBP	Média ALWABP	Perda de eficiência
Heskia	1	76,20	78,13	2,53%
Heskia	2	88,10	90,45	2,67%
Heskia	3	132,10	135,35	2,46%
Heskia	4	133,40	136,40	2,25%
Heskia	5	26,20	26,47	1,04%
Heskia	6	32,20	32,41	0,67%
Heskia	7	65,20	65,29	0,13%
Heskia	8	55,00	55,23	0,42%
Média		**76,05**	**77,47**	**1,52%**

Família	Grupo	Média SALBP	Média ALWABP	Perda de eficiência
Tonge	1	48,20	48,60	0,83%
Tonge	2	50,60	50,90	0,59%
Tonge	3	71,10	71,70	0,84%
Tonge	4	77,60	78,40	1,03%
Tonge	5	18,30	18,30	0,00%
Tonge	6	27,10	27,10	0,00%
Tonge	7	45,50	45,50	0,00%
Tonge	8	36,70	36,95	0,68%
Média		**46,89**	**47,18**	**0,50%**