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# Network Analysis

## Methodological Foundations

Springer

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# Preface

The present book is the outcome of a seminar organized by the editors, sponsored by the *Gesellschaft für Informatik e.V.* (GI) and held in Dagstuhl, 13–16 April 2004.

GI-Dagstuhl-Seminars are organized on current topics in computer science that are not yet well covered in textbooks. Most importantly, this gives young researchers an opportunity to become actively involved in such topics, and to produce a book that can provide an introduction for others as well.

The participants of this seminar were assigned subtopics on which they did half a year of research prior to the meeting. After a week of presentations and discussion at Schloss Dagstuhl, slightly more than another half-year was spent on writing the chapters. These were cross-reviewed internally and blind-reviewed by external experts. Since we anticipate that readers will come from various disciplines, we would like to emphasize that it is customary in our field to order authors alphabetically.

The intended audience consists of everyone interested in formal aspects of network analysis, though a background in computer science on, roughly, the undergraduate level is assumed. No prior knowledge about network analysis is required. Ideally, this book will be used as an introduction to the field, a reference and a basis for graduate-level courses in applied graph theory.

First and foremost, we would like to thank all participants of the seminar and thus the authors of this book. We were blessed with a focused and determined group of people that worked professionally throughout. We are grateful to the GI and Schloss Dagstuhl for granting us the opportunity to organize the seminar, and we are happy to acknowledge that we were actually talked into doing so by Dorothea Wagner who was then chairing the *GI-Beirat der Universitätsprofessor(inn)en*. We received much appreciated chapter reviews from Vladimir Batagelj, Stephen P. Borgatti, Carter Butts, Petros Drineas, Robert Elsässer, Martin G. Everett, Ove Frank, Seokhee Hong, David Hunter, Sven O. Krumke, Ulrich Meyer, Haiko Müller, Philippa Pattison and Dieter Rautenbach. We thank Barny Martin for proof-reading several chapters and Daniel Fleischer, Martin Hoefer and Christian Pich for preparing the index.

December 2004

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