Black Hole Formation Through Shockwave Collisions

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Abstract: We considered a rudimentary model for black hole production from two massive particles asymptotically approaching Aichelburg–Sexl shock waves at times before the collision and the Schwarzschild solution after. To connect the solutions, we fit parameters by minimizing the Einstein-Hilbert Action numerically with Stochastic Gradient-Descent. Aside from approximating the final mass of the black hole, we also supported the necessity of gravitational waves, reducing the mass during the collision. We measured the final mass in the black hole as $E_f = 0.82\, E_0$.

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"Before you attribute some phenomena to new physics, you must be sure it cannot be described by old physics."
-Kirill Melnikov

I. Black Hole Creation in Particle Accelerators

As the energy of particle accelerators continues to increase, they will reach a limit where black holes will begin to form due to the high energy density created. [5] In fact, as these energy densities naturally occur in collisions in the outer atmosphere, black holes may already exist around the Earth. [1] Apart from being an interesting natural phenomenon, these micro black holes connect the disciplines of General Relativity and Quantum Field Theory. In their brief existence due to black hole evaporation, the process where strong fields around the event horizon allow for particle creation and the reduction of the central mass, the micro black holes will display a scattering cross section in curved space time. Whether naturally occurring or the result of a high energy experiment, physicists would greatly value the observation of the black hole creation and evaporation.

$$\Delta x_i \geq \frac{\hbar}{\Delta p_i}\left[1 + \left(\alpha\, \mathrm{l_{Pl}} \frac{\Delta p_i}{\hbar}\right)^2\right]$$

(Generalized Uncertainty Principle)

Indeed, as a black hole evaporates it could undergo a quantum process where it jumps between the state of a black hole and known fundamental particles. The generalized uncertainty principle, the extension of the Heisenberg Uncertainty Principle to include effects from graviton, would put a lower limit on the size of black hole, and observing this mass either through creation or evaporation would allow for the measurement of fundamental parameters. [1] In addition the lower bound of energy for black hole creation could give a measurement of the dimensionality of spacetime. As modern theories of quantum gravity include more than three spatial dimensions, with the extra dimensions sufficiently compactified to correspond with our low energy physical theories, black hole creation is a candidate for verifying future theories of physics. [2] Similarly, the scattering cross section, and ratios of particle creation during black hole evaporation give an additional measurement of spatial dimension.

The creation of micro black holes will also enrich our understanding of current theories such as General Relativity. Some experiments with black holes would address some paradoxes that exist in the current theory. The classical path length to the center of a black hole on a geodesic is infinitely long. There does not exist a current verified theory for how a black hole would gain additional mass by particles traveling through the even horizon. Even for the collision of two black holes, a model does not exist to address how the masses will pass through each other's event horizion.

When we do finally reach the energy thresh hold for black hole creation, we will encounter a new era for physics. As with the QED boom of the post war world, the data from black hole experiments will finally give a candidate for a theory that ties the fundamental forces together.

Given the importance of these high energy experiments a strong classically theory of black hole formation needs to exist. In this paper we try to suggest a model to for approximating black hole formation through the collision of particles. The model asymptotically approaches two colliding Aichelburg–Sexl shock waves, a wave representing the space time curvature for a particle traveling the speed of light, as time comes from negative infinity, [3] and exactly forms the Schwarzschild solution for a stationary black hole. From this model we want to calculate the final mass of the black hole and graph the black hole formation.

In this model we assume a space time metric with enough parameters can connect two different solutions. The collision of the particles considered is cylindrically symmetric, as well as symmetric about the x-y plane. The particles do not leave the z-axis and join into one black hole with a mutually shared event horizon that forms in a duck bill fashion as the particles encounter each other.

II. Modifying the Schwarzschild Metric For Black Hole Collisions

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Units: $\hbar = c = G$

Often Physicists call General Relativity one of the beautiful theories of physics. Published in 1915 by Albert Einstein, it would take less than a year for someone to publish the first solution to the equations. Karl Schwarzschild, a lieutenant serving in the first world war, published the solution while serving but tragically died the following year.

This solution would later go on to describe black holes, but also came as a harbinger of future work. Einstein's Field equations that describe General Relativity remain some of the hardest equations to solve in all of physics. They consist of 4th order nonlinear differential equations. They do not respect superposition, and they do not combine with other theories of physics such as Special Relativity and Quantum Field Theory. Testing the theory requires measurements from large bodies in space in rare events where their trajectories differ from Newton's Law of Gravity.

Exact solutions come rarely and come in two forms. Vacuum solutions contain no energy transfer through gravitational waves and energy can only exist as point masses where the coordinates contain non-removeable singularities. These solutions give a zero Ricci or Einstein Tensor. Non-vacuum solutions minimize the Einstein-Hilbert action given by$\int R\sqrt{-g}\, d^4x$, where *R* is the Ricci Scalar and *g* is the determinate of the space-time metric.

A general connection between these terms follows the flow,

$$ds^2 = g_{00}dt^2 + g_{10}\,dx\,dt \ldots + g_{33}\,dz^2$$
$$g^{\mu\nu}g_{\mu\nu} = \delta^{\mu}{}_{\nu}$$

(Christoffel Symbols)

$$\Gamma^{\sigma}_{\mu\delta} = \frac{1}{2}g^{\sigma\nu}\left(\partial_{\delta}g_{\nu\mu} + \partial_{\mu}g_{\nu\delta} - \partial_{\nu}g_{\mu\delta}\right)$$

(Ricci Tensor)

$$\mathrm{R}_{\mu\sigma} = \partial_{\delta}\Gamma^{\delta}_{\mu\sigma} - \partial_{\mu}\Gamma^{\delta}_{\sigma\delta} + \Gamma^{\delta}_{\delta\alpha}\Gamma^{\alpha}_{\mu\sigma} - \Gamma^{\alpha}_{\mu\delta}\Gamma^{\delta}_{\alpha\sigma}$$

(Ricci Scalar)

$$\mathrm{R} = \mathrm{R}_{\mu\nu}g^{\mu\nu}$$

(Einstein Tensor)

$$\kappa \mathrm{T}_{\mu\nu} = \mathrm{R}_{\mu\nu} - \frac{1}{2}\mathrm{R} g_{\mu\nu} [4]$$

In cylindrical coordinates the Schwarzschild solution for the metric is

$$ds^2 = -\left(1 - \frac{2m}{\sqrt{p^2 + z^2}}\right) dt^2$$

$$+ \frac{2m}{\sqrt{p^2+z^2}}\left(1 - \frac{2m}{\sqrt{p^2+z^2}}\right)^{-1}\left(\frac{p\,dp + z\,dz}{\sqrt{p^2+z^2}}\right) + dp^2$$

$$+ p^2 \sin^2\phi \, d\phi^2 + dz^2 \,.$$

Notice this metric contains two different types of discontinuities. At the event horizon there is a removable singularity, which disappears under change of coordinates. At the origin there is a singularity that can not be removed. This represents a point mass. The Ricci Tensor calculated from this metric gives a vacuum solution with no mass, energy, or momentum except for the point at the origin where the coordinates have a hole.

The Aichelburg-Sexl solution comes from the Lorentz Boost in the z-direction of this solution. Although in general you can not mix Special Relativity and General Relativity, this solution gives a delta function traveling at the speed of light for the momentum in the Einstein Tensor

In order to model the two Aichelburg–Sexl waves we locally make the coordinate substitutions,

(1) $\frac{1}{\sqrt{p^2+z^2}} = \frac{1}{\sqrt{p'^2 + z'^2_+}} + \frac{1}{\sqrt{p'^2 + z'^2_-}}$

(2) $z_- = \gamma(z' + vt')$

(3) $z_+ = \gamma(z' - vt')$

(4) $m = \frac{E}{\gamma}$

(5) $-dt^2 + dz^2 = -dt'^2 + dz'^2$.

Notice that these coordinates do not give a direct coordinate change. As the Schwarzschild metric gives a vacuum solution, under coordinate change the Ricci Tensor of the resulting solution will vanish. This will give a non-physical solution as some of the energy should leave the system through gravitational waves. These waves will make a vacuum solution impossible, and this coordinate change only approximates the original metric. Some non-physical solutions to the Einstein Field Equations do exist, but through direct calculation the Ricci Tensor after this coordinate change does not vanish.

Notice in (1), for large *t,* a term on the right side of the equation with a non-removable singularity approaches 0 around the second term with a non-removable singularity. As the metric, under a valid coordinate change should still vanish when calculating the energy momentum tensor these points represent the point masses for a given momentum. As (5) resembles the frame invariance of proper time, if *v* and γ approach their classic values when the non-removable singularities are far away they approach the classic shock wave metric. We will not hold the proper time invariance for all observers between the metrics and simply use this substitution to change to other variables. The choice of variables comes only from their resemblance to asymptotic values and no conservation principles or invariances are implied. If on the other hand *v*=0 and γ= 1 the solution is the Schwarzschild solution.

As with the Aichelburg-solution, we will claim success after the metric meets our expectations. As the Einstein and Ricci Scalars are difficult to calculate, we looked at g_{33}. For the graphs in Figure 1, the asymptotic behavior looks correct. The Aichelburg-Sexl shock wave appears to wider but increasing the classical kinetic energy relative to the mass of the particle with give the wave a stronger localization. As advertised the Schwarzschild solutions look perfect.

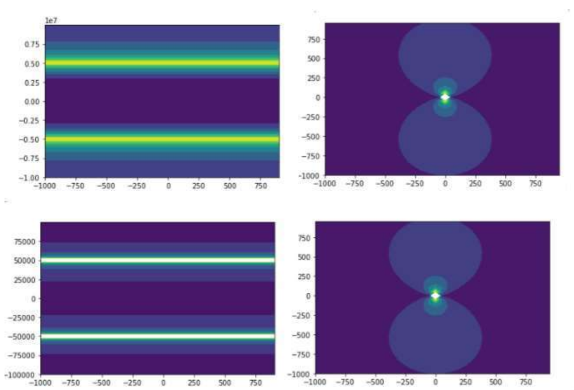

Figure 1: On the top metrics coming from the new coordinates, and on the bottom are the two pictures coming from the asymptotic solutions they are trying to model.

III. The Effect of Parameters on the Collision

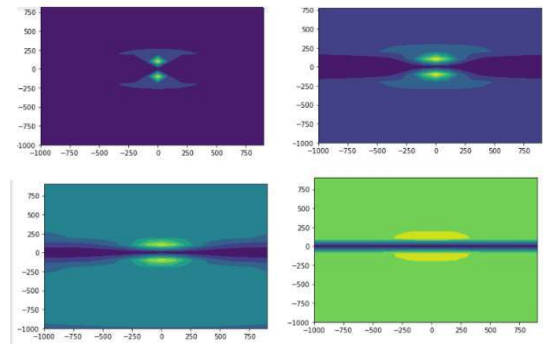

Figure 2: The effect of changing the t coordinate without adjusting the linear z transformation.t is increasing left to right, top to bottom.

In Figure 2 you can see the effect of changing the time coordinate without changing the other parameters. Physically this would correspond to increasing the energy of a particle before the collision. In the diagram the darker spots represent the As you some of these situations might be physically unreasonable. As the time reduces, implying a larger initial energy the shockwave behavior become more noticeable. Particularly in the lower right graph you can see a strong shock wave even though the center of mass of the two particles remains close.

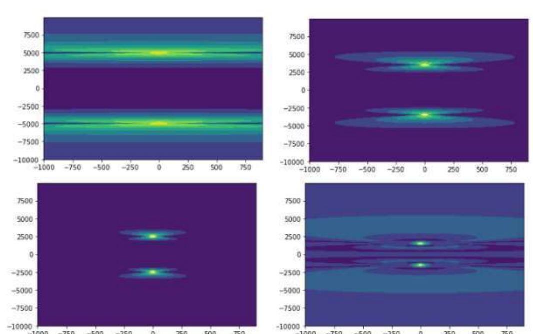

Figure 3: The effect of changing the v parameter in the function and the

asymptotic limit. v decreases going left to right, top to bottom.

In Figure 3 the v parameter is reduced, and you can see the two effects of increasing the v parameter. Increasing v takes the shape of the event horizon and folds it flat. The behavior shows how a black hole can become a shock wave as it folds into itself. The second effect of changing the v parameter is localization. The particles with a greater v parameter appear to less spread out than the particles with a greater v parameter.

In addition to looking at the dependence of a black hole's shape on parameters we also looked at the center of mass energy of the resulting black hole. In a completely classical theory, a black hole is a stationary solution. To find the final mass of the black hole we used a logistic equation to bridge the graph between the parameters with well defined values in the asymptotic solutions.

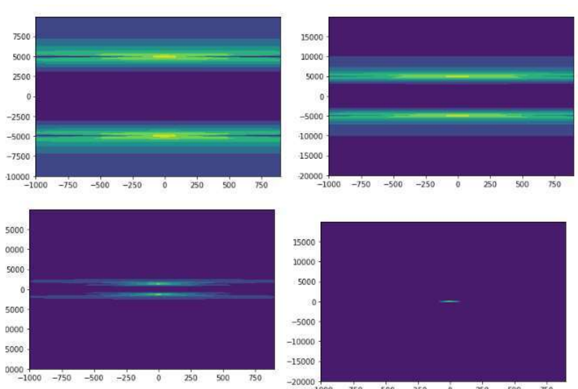

Figure 4: The collision of two Aichelberg-Sexl waves into the Schwarzschild solution

To minimize the action, we used Stochastic Gradient Descent for the Monte Carlo approximation for the Action. This gave a final mass,

$$E_f = 0.81E_0$$

IV. Room for Improvement

The main discrepancy of this analysis comes from the choice of metric. Although the metric close resembled existing metrics asymptotically its approximation has a couple problems.

First its determinant was positive at some points in the space. This meant the Jacobian would give the wrong result for the infinitesimal element of area in the Einstein-Hilbert Action. A better choice would be a metric that based on the asymptotic behavior as well as its Jacobian.

We also reduced the parameters using the same logistic function. Ideally these should be a sperate functions for each parameter. The relationship between the variables should also obey the conservation laws associated with the Ricci Tensor.

The choice of coding was quite limiting. The code ran very slow and it limited the accuracy of the Monte Carlo integral for the action. Maple or Matlab with the Maple

V. Closing Remarks

Even with the setbacks we were able to get the final mass of the of the black hole and minimize the Einstein-Hilbert Action.

References

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