Name: ________________

# PHYS 335 HW 8

Due 3/11 at 5:00PM

The usual instructions

**0. (2 pts)** Estimate the time it took to do this problem set. Include any comments about the homework that you'd like to share with Jay.

**1.** *Catch a wave(function)*

You did hard labor in classes 24 and 25 to find the eigenvalues and eigenfunctions for the two body central force problem, but now you can use those results to find any eigenstate for a large number of problems. Let's begin by demonstrating that we can find any eigenfunction we want.

(a) Use the $\Phi$ solution, the general form of the associated Legendre functions, the series solution for the radial wave function with the associated recursion relation, and appropriate normalization conditions (these can all be found in the work sheets from classes 24 and 25) to construct the position space wave function $\langle \vec{r}|3, 2, 1\rangle$, where I'm labeling the state with the values of $n, l, m$. (We previously used $E$, the energy eigenvalue to label the first entry in this state, but now that we've solved the radial wave function, it makes more sense to label with the value of $n$ to which $E$ is proportional.) Some of what you need to find is in Townsend, so you could check your answers with what's there. You may want to set up some of these calculations in Mathematica. I've uploaded a Mathematica file that shows off some tricks. If you follow the notes, you'll find that the sign of the normalization constant for the angular wave function is not determined. You'll notice that the standard spherical harmonics pick this sign according to $(-1)^m$.

(b) Discuss some qualitative features of this wave function using the plots provided in the worksheets/Townsend and/or ones you make. In other words, comment on where the electron is likely to be found.

(c) Check explicitly by differentiation that your eigenfunction has the right eigenvalues of $\hat{\vec{L}}^2$ and $\hat{L}_z$. You can use Mathematica if you want.

**2.** *Fruit of your labor*

We've been thinking of this problem as the hydrogen atom, but it's actually the solution to lots of problems. For each of the following find the ground state energy and the Bohr radius as a fraction of what it is for hydrogen. You may need to look up some masses (What's another name for the helium nucleus?) and you can use things like $m_e + m_p \approx m_p$ to generate approximate answers.

(a) Positronium (a bound state of an electron and a positron)

(b) A helium ion (a helium nucleus with one electron)

**3.** *A break from calculating*

Consider a bound state of spinless particles having charges $\pm e$ in an external magnetic field. I claim that the hamiltonian is

$$\hat{H} = \frac{\hat{\vec{P}}^2}{2\mu} - \frac{e^2}{|\vec{r}|} + \omega_0 \hat{L}_z. \tag{1}$$

This is like the Zeeman effect without intrinsic spin effects. This can be realized in pionic hydrogen (a hydrogen-like atom with a pion instead of an electron).

(a) Explain why this Hamiltonian makes sense by addressing the basic structure of each term.

(b) I claim that this hamiltonian has the same eigenfunctions as hydrogen. Explain why.

(c) I claim that this hamiltonian has different energy eigenvalues. Explain how they are different.

**4.** *Hey electron, you're out of bounds!*

Calculate the probability that an electron in the ground state of hydrogen is outside of the classically allowed region. Clearly you'll first need to consider the question of what range of $r$ values are allowed in classical mechanics.

**5.** *A perturbation to our usual pattern...*

Perturbation theory is super important because as usual in physics, there are a lot of interesting problems that don't have exact solutions. I wanna give you a chance to apply this to something super cool here in problem 5, but I need a little more time to identify the spiffiest problem, so something will appear here between now and Tuesday evening.

**It's here!! See attached.**

**6.** *Review*

Nothing to submit on paper here, but submit reviews of the projects to which you were assigned here: `https://forms.gle/GYcQVEqjjdKQDYSY7`.

<u>Academic Honesty Statement</u>

___I did not receive help on this problem set. I did all the work on my own.
___I received on this problem set from:
  ___Jay on problems __________
  ___Student Assistants on problems __________
  ___Other Students (please name ______________) on problems __________
  ___Other Instructor (please name ______________) on problems __________
  ___A resource other than our text (please name ______________)
    on problems __________

By way of acknowledgements, several of these problems were adapted from Townsend.

# The relativistic correction

Throughout the class, our Hamiltonians have involved the nonrelativistic kinetic energy. One trick you can pull with perturbation thry is to work out the leading correction in $v/c$ to your energies.

Consider the relativistic kinetic energy from intro special relativity

$$K = \underbrace{\sqrt{p^2c^2 - (mc^2)^2}}_{\text{total relativistic energy}} - mc^2$$

Rest energy

$$= mc^2\left(\sqrt{1 + \frac{p^2}{m^2c^2}} - 1\right)$$

Q1 Show that if we "talor" this to suit our needs it can be written:

$$= \frac{p^2}{2m} - \frac{p^4}{\boxed{\phantom{xxxx}}} + \ldots$$

Blank to fill in

Q2 If we include the 2nd term above in our analysis of the harmonic oscillator, Identify $H_0$ and $H_1$

Q3 Use time independent perturbation theory along with the position space wave functions for the energy eigenstates of the SHO to find the leading relativistic correction to the ground state energy of the SHO.