Class 3

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Reading Questions

- most were linearly independent of each other
- some I'll address in the course of the class
- some I don't know the answer to
- some have answers off of our critical path
- if your question is adjacent to our discussion, raise hand
- **if you want an answer and you didn't get one, see me!**

key points

- you can't completely nail down which way an angular momentum points in QM
- more fun with a 2d state space

language and symbols

- state vector, Hilbert space vector, state $\rightarrow |\psi\rangle$ or $\langle\psi|$
- eigenstate, eigenvector, state of definite (some observable) $\rightarrow |+z\rangle$
- probability amplitude, amplitude $\rightarrow \langle +x|+z\rangle$ or $a, \phi, \ldots$
- probability $\rightarrow |\langle +x|+z\rangle|^2$
- expectation value $\langle S_z^2 \rangle$

Jeoporstate with Alex Dirac

a, b are complex numbers and the states are electron spin states
True of False

1. $\langle +z|\psi\rangle = \langle\psi|+z\rangle$
2. $|\langle +z|\psi\rangle|^2 = |\langle\psi|+z\rangle|^2$
3. $\langle +z|\psi\rangle\langle\psi|+z\rangle = \langle\psi|+z\rangle\langle +z|\psi\rangle$
4. $\langle +z|\, a\, |\psi\rangle = a\,\langle +z|\psi\rangle$
5. $|\psi\rangle = a\,|+z\rangle + b\,|-z\rangle$
6. the expectation value is the most likely value to be observed in a given measurement

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spin

spin is (intrinsic) angular momentum

PHYSICAL REVIEW D **78**, 092006 (2008)

Preferred-frame and *CP*-violation tests with polarized electrons

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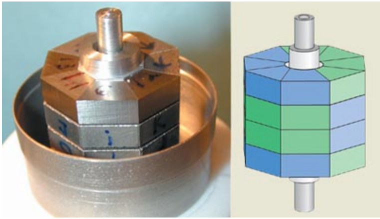
D. Gyro-compass effect


Physics World

... so it's angular momentum, but nothing is spinning ...

B-fields (along side the point)

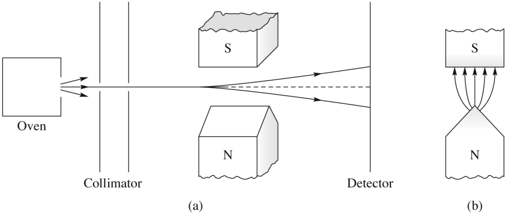


Figure 1.1
Townsend A MODERN APPROACH TO QUANTUM MECHANICS 2nd edition

$$\begin{aligned}
\vec{F} &= -\vec{\nabla} U && (1)\\
&= -\vec{\nabla}(-\vec{\mu} \cdot \vec{B}) && (2)\\
&= \vec{\nabla}(\mu_z B_z) && (3)\\
F_z &= \mu_z \frac{\partial B_z}{\partial z} && (4)
\end{aligned}$$

B gets smaller as z bets bigger, that's a negative derivative so negative μ_z goes up but the negative charge means spin up goes up

an SG is a sorter, of sorts...

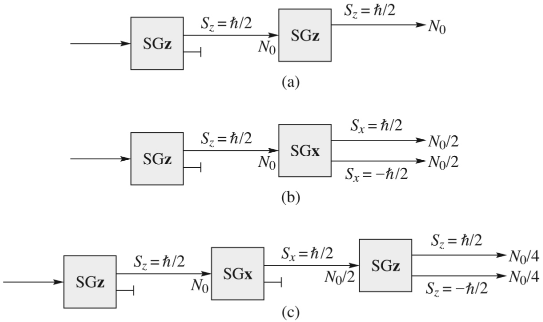


Figure 1.3
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experiment 3

modifed SG ... an interferometer like the 2-slit!

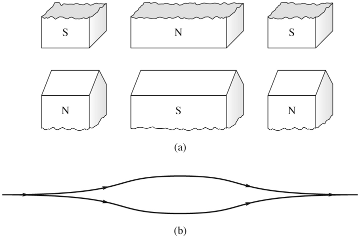


Figure 1.4
Townsend A MODERN APPROACH TO QUANTUM MECHANICS 2nd edition

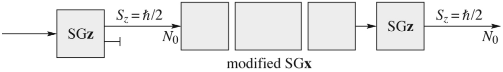


Figure 1.6
Townsend A MODERN APPROACH TO QUANTUM MECHANICS 2nd edition

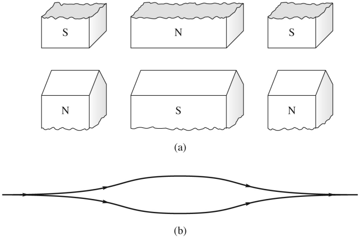
experiment 4

1. Use Eq. 1.29 and it's partner, aka $|\pm x\rangle = \frac{1}{\sqrt{2}}\,|+z\rangle \pm \frac{1}{\sqrt{2}}\,|-z\rangle$ and the "laws of quantum mechanics" to show that we get 100% $S_z = \hbar/2$

laws of quantum mechanics

1. Add the amplitudes for multiple indistinguishable ways an event can occur
2. Add the probabilities if the ways are distinguishable
3. Multiply the amplitudes for successive events

modifed SG ... an interferometer like the 2-slit!



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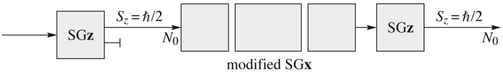

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Townsend A MODERN APPROACH TO QUANTUM MECHANICS 2nd edition

experiment 4

2. Now suppose that we insert some apparatus to check whether our electron takes the upper or the lower path. What is the probability of spin up now?

a 2 slit problem

Consider the classic 2 slit experiment with electrons. The amplitude to arrive at the screen at a particular location having traveled from the source to the screen (step A), gone through one of the holes (step B), then traveled from the hole to the screen (step C) can be thought of as the product of the amplitudes for each step. Let's focus first on the electrons that travel through hole 1, and let's call the amplitudes for each of the 3 steps in the process a_{A1}, a_{B1}, and a_{C1} respectively. Now we could consider the analogous amplitudes for the path through hole number 2: a_{A2}, a_{B2}, and a_{C2}. On Monday morning you measure the probability of finding an electron at the central maximum. Write an expression for the probability in terms of these amplitudes.

a 2 slit problem

On Monday night, under cover of darkness, Royal Paine-Días sneaks into your lab and inserts a divice that alters the amplitude to make it through hole 2 such that $a_{B2} \rightarrow 0.2e^{i\pi/2}a_{B2}$. Assuming the paths were originally symmetric such that $a_{A1} = a_{A2}$, $a_{B1} = a_{B2}$, and $a_{C1} = a_{C2}$, by what factor is the probability to find an electron at the central max reduced when you do the experiment on Tuesday morning relative to what you found before?