Note that you can take derivatives with respect to something like cos θ as follows

```
? D
```

D[f, x] gives the partial derivative $\partial f / \partial x$.

D[f, {x, n}] gives the multiple derivative $\partial^n f / \partial x^n$.

D[f, x, y, …] differentiates f successively with respect to x, y, ….

D[f, {{x_1, x_2, …}}] for a scalar f gives the vector derivative $(\partial f / \partial x_1, \partial f / \partial x_2, \ldots)$.

D[f, {$array$}] gives a tensor derivative. >>

```
D[Cos[θ]^2, Cos[θ]]
```

2 Cos[θ]

Note that higher derivatives can be taken like follows. Here I've done the second derivative with respect to Cos θ.

```
D[Cos[θ]^2, {Cos[θ], 2}]
```

2

The spherical Harminics are in Mathematica. This would be another way to check your answer after you work it out from the Legendra functions yourself.

```
? SphericalHarmonicY
```

SphericalHarmonicY[l, m, θ, ϕ] gives the spherical harmonic $Y_l^m(\theta, \phi)$. >>

You can then plot your probability density in theta and phi using the following tools to reproduce things like what's seen in Figure 9.11 of Townsend.

```
? SphericalPlot3D
```

SphericalPlot3D[r, θ, ϕ] generates a 3D plot with a spherical radius r as a function of spherical coordinates θ and ϕ.

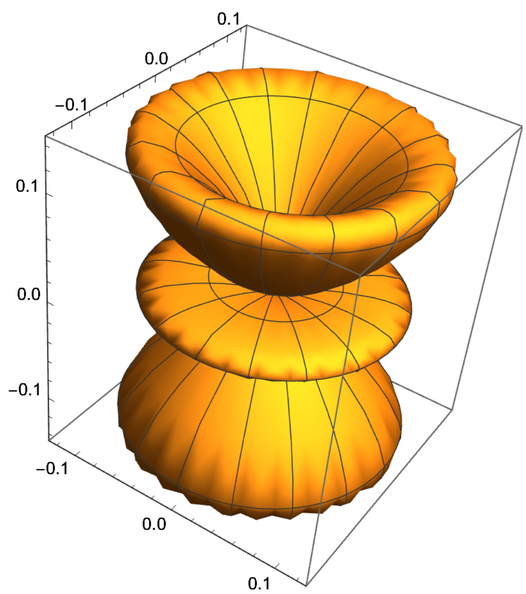
SphericalPlot3D[r, {θ, θ_{min}, θ_{max}}, {ϕ, ϕ_{min}, ϕ_{max}}] generates a 3D spherical plot over the specified ranges of spherical coordinates.

SphericalPlot3D[{r_1, r_2, …}, {θ, θ_{min}, θ_{max}}, {ϕ, ϕ_{min}, ϕ_{max}}] generates a 3D spherical plot with multiple surfaces. >>

```
? Conjugate
```

Conjugate[z] or z* gives the complex conjugate of the complex number z. >>

```
SphericalPlot3D[SphericalHarmonicY[4, 2, θ, ϕ] *
  Conjugate[SphericalHarmonicY[4, 2, θ, ϕ]], {θ, 0, Pi}, {ϕ, 0, 2 * Pi}]
```



We can work on Hw08 stuff here:

Problem #1

```
In[35]:= Clear[theta, n, c, r, rho, Z, R, a]

In[1]:= Integrate[(Sin[theta]^3) * (Cos[theta]^2), {theta, 0, Pi}]
```

Out[1]= $\frac{4}{15}$

```
In[27]:= n = 3;

In[28]:= R := (c / r) * (rho^3) * Exp[-rho / 2]

In[29]:= rho := 2 * Z * r / (n * a)
```

In[30]:=
```
Integrate[R * R * r * r, {r, 0, ∞}]
```

Out[30]= $\boxed{\frac{1080\, a\, c^2}{Z} \text{ if } \mathrm{Re}\left[\frac{Z}{a}\right] > 0}$

In[31]:=
```
Solve[1080 a c^2 / Z == 1, c]
```

Out[31]= $\left\{\left\{c \to -\frac{\sqrt{Z}}{6\sqrt{30}\sqrt{a}}\right\}, \left\{c \to \frac{\sqrt{Z}}{6\sqrt{30}\sqrt{a}}\right\}\right\}$

In[44]:=
```
psi := constant * Sin[theta] * Cos[theta] * Exp[i * phi]
(* we can call everything that is not theta or phi a
  constant since it will be treated as so with the L-operator *)
```

In[52]:=
```
L2op := -(hbar^2) * ((1 / Sin[theta]) * D[(Sin[theta] * D[psi, theta]), theta] +
    (1 / (Sin[theta]^2)) * D[psi, {phi, 2}])
```

In[53]:=
```
Simplify[L2op / psi]
```

Out[53]= $6\, \mathrm{hbar}^2$

Here we expect 6hbar^2 and that is exactly what we get.

In[54]:=
```
Lop := (hbar / i) * D[psi, phi]
```

In[56]:=
```
Simplify[Lop / psi]
```

Out[56]= hbar

Here we expect hbar and that is exactly what we get.

Problem #2

In[62]:=
```
Psi := 2 * (1 / a)^(3/2) * Exp[-r / a]
```

In[63]:=
```
Integrate[Psi^2 * r^2, {r, 2 a, ∞}]
```

Out[63]= $\boxed{\frac{13}{e^4} \text{ if } \mathrm{Re}[a] > 0}$

In[64]:=
```
me = 9.109 * 10^-31
e = 1.602 * 10^-19
hbar = 1.055 * 10^-34
```

Out[64]= 9.109×10^{-31}

Out[65]= 1.602×10^{-19}

Out[66]= 1.055×10^{-34}

In[67]:=
```
μ = me / 2
```

Out[67]= 4.5545×10^{-31}

In[68]:=
```
ao = hbar^2 / (μ * e^2)
```

Out[68]= 0.952224

Problem #5

In[71]:=
```
Series[Sqrt[1 + x^2], {x, 0, 5}]
```

Out[71]= $1+\frac{x^2}{2}-\frac{x^4}{8}+O[x]^6$