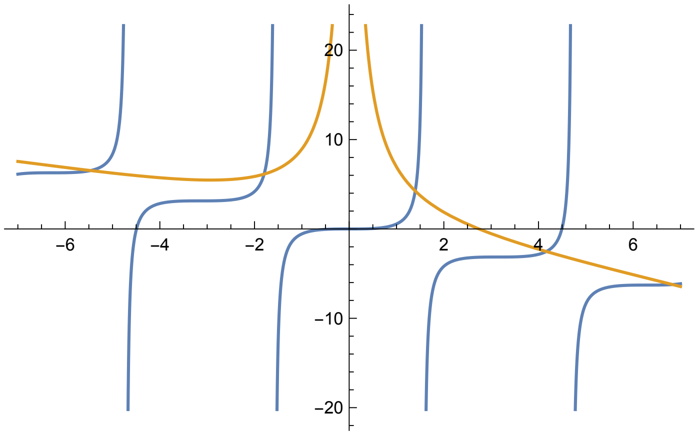
```
Plot[{{s}, {Sqrt[(8 / z) ^2 - 1] - z}}, {z, -7, 7}]
```

Out[∘]=



- We can use Desmos to find the intersections. The intersections correspond to a z-value, which corresponds to an energy eigenvalue. Since there are 5 intersections we know there will be 5 even eigenfunctions (we know they are even since we found them with the condition A=0).

In[∘]:=
```
Integrate[(Sqrt[2] / a) * Cos[3 * Pi * x / (2 * a)] * Cos[Pi * x / a], {x, -a / 2, a / 2}]
```

Out[∘]= $\frac{8}{5\pi}$

In[1]:=
```
DSolve[-psi''[z] + k1 * z * psi[z] == k2 * psi[z], psi[z], z]
```

Out[1]= $\left\{\left\{\text{psi}[z] \to \text{AiryAi}\left[\frac{-k2 + k1\, z}{k1^{2/3}}\right] c_1 + \text{AiryBi}\left[\frac{-k2 + k1\, z}{k1^{2/3}}\right] c_2\right\}\right\}$

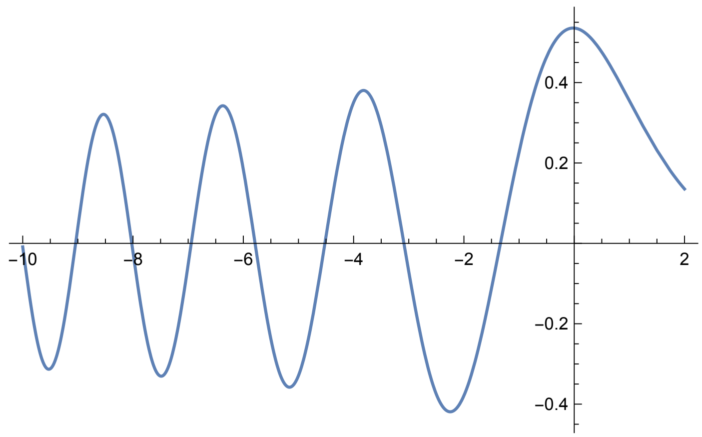
In[54]:=
```
k1 = 1;
k2 = 1;
A = 1;
B = 1;
```

$\text{Plot}\left[\text{AiryAi}\left[\frac{-k2 + k1\, z}{k1^{2/3}}\right] * A, \{z, -10, 2\}\right]$

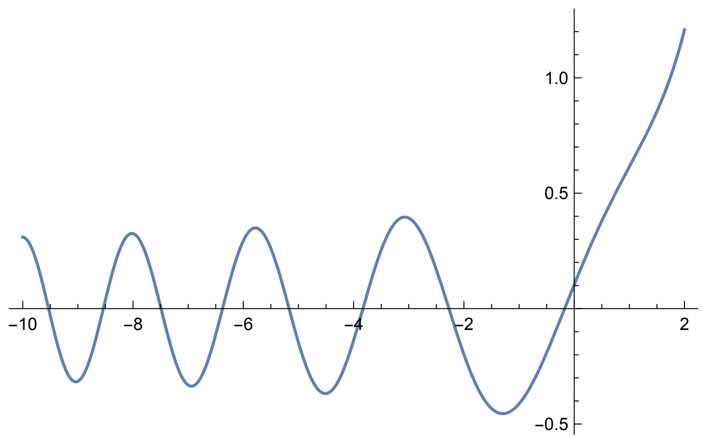
$\text{Plot}\left[\text{AiryBi}\left[\frac{-k2 + k1\, z}{k1^{2/3}}\right] * B, \{z, -10, 2\}\right]$

$\text{Plot}\left[\text{AiryAi}\left[\frac{-k2 + k1\, z}{k1^{2/3}}\right] * A + \text{AiryBi}\left[\frac{-k2 + k1\, z}{k1^{2/3}}\right] * B, \{z, -10, 2\}\right]$

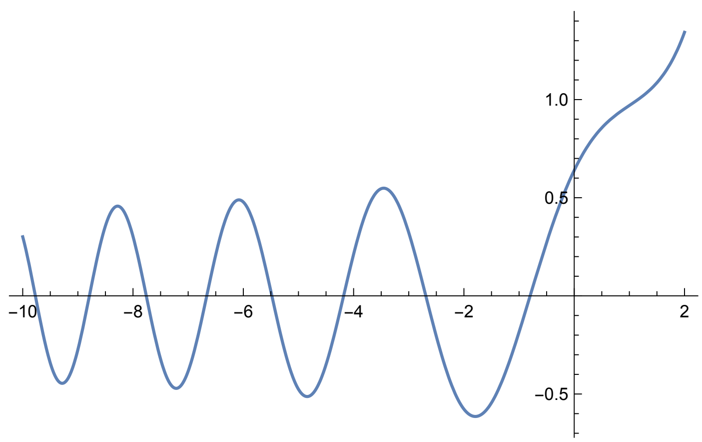
Out[58]=



Out[59]=

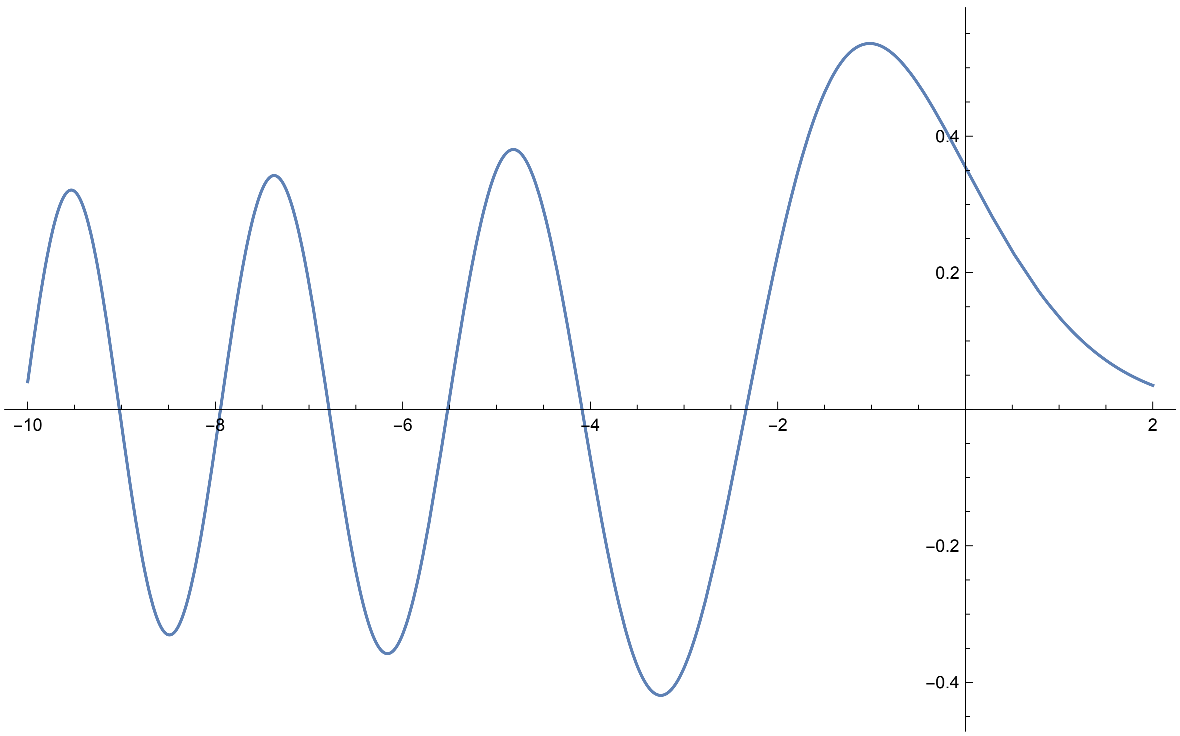


Out[60]=



In[61]:=
```
(* we can approximate the zeros with *)
Plot[AiryAi[z] * A, {z, -10, 2}]
```

Out[61]=



Out[62]= $\frac{1}{3^{2/3}\,\text{Gamma}\left[\frac{2}{3}\right]}$

```
NIntegrate[AiryAi[-2.4 + z]^2, {z, 0, ∞}]
```

Out[64]= 0.491735

In[65]:=
```
Sqrt[1 / 0.4917354066126901]
```

Out[65]= 1.42605

In[66]:=
```
NIntegrate[z * AiryAi[-2.4 + z]^2, {z, 0, ∞}]
```

Out[66]= 0.796859

In[68]:=
```
(1.426^2) * 0.7968593148382319
```

Out[68]= 1.62039

FindRoot : The number of equations does not match the number of variables in

$$\text{FindRoot}\left[\left\{\text{AiryBi}\left[\frac{-\text{k2} + \text{k1}\,z}{\text{k1}^{\text{Times}[\ll 2 \gg]}}\right] A,\ \text{AiryBi}\left[\frac{-\text{k2} + \text{k1}\,z}{\text{k1}^{\text{Times}[\ll 2 \gg]}}\right] B\right\}, \{z, -2\}\right].$$