Name: ________________

PHYS 335 HW 4

Due 1/28 at 5:00PM

The usual instructions

0. (2 pts) Estimate the time it took to do this problem set. Include any comments about the homework that you'd like to share with Jay.

1. *A supa-fine hyperfine problem...*

(a) Work out the steps to get from $2\hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2$ to $\hat{S}_{1+}\hat{S}_{2-} + \hat{S}_{1-}\hat{S}_{2+} + 2\hat{S}_{1z}\hat{S}_{2z}$ using Eq. 5.11.

(b) Work out the $\langle 3|\,\hat{H}\,|3\rangle$ in the Hamiltonian when represented in the 1,2,3,4 basis as in 5.14. Comment on why the result of part (a) was helpful.

(c) I claim that if a particle starts off in the $|3\rangle$ state, the probability of finding the particle in the $|3\rangle$ state and the $|2\rangle$ state oscillate with a frequency $\nu = 2A/h$ show this. (This frequency happens to correspond to 1420 MHz if you do other physics to work out the parameter A appearing in $\hat{H}$)

2. *You, Rabi, and an ammonia molecule...*

When a spin 1/2 particle is placed in a magnetic field in the z direction, it has energy eigenstates $|\pm z\rangle$. When we added a small, oscillating magnetic field in the x direction, we drove transitions between these original eigenstates. The Hamiltonian for this case is given in 4.38.

(a) By looking at 4.38, you can see that when the x field is zero, $|\pm z\rangle$ will be eigenstates. Say how.

(b) When the x field is turned on, the $|\pm z\rangle$ are no longer the energy eigenstates, and if the system starts in a state $|+z\rangle$ it will transition out of that state and, under suitable conditions, be later found in the $|-z\rangle$ state. This probability is described by the Rabi formula. This behavior is a typical result of off diagonal elements in the Hamiltonian. In fact, when the ammonia molecule is placed in an oscillating electric field, the Hamiltonian can be written in the form 4.61, which is highly analogous to 4.38! Just by mapping symbols, (not by hard-core math) write the analogue of the Rabi formula for an ammonia molecule in an oscillating electric field. Note that in the NMR problem, $\hbar\omega_0$ is the difference between the energy eigenstates when there is no B_x.

(c) Write some words about the transitions your 'Rabi formula' in part (b) describes.

(d) The basis used in 4.61 is the eigenbasis for the system when the oscillating electric field is zero. The Hamiltonian was written in what is perhaps a more natural basis in 4.57. Describe the basis used in 4.57.

(e) The operation you need to transform the hamiltonian in 4.57 to the basis used in 4.61 is as follows

$$\begin{pmatrix} \langle I|1\rangle & \langle I|2\rangle \\ \langle II|1\rangle & \langle II|2\rangle \end{pmatrix} \begin{pmatrix} \langle 1|\hat{H}|1\rangle & \langle 1|\hat{H}|2\rangle \\ \langle 2|\hat{H}|1\rangle & \langle 2|\hat{H}|2\rangle \end{pmatrix} \begin{pmatrix} \langle 1|I\rangle & \langle 1|II\rangle \\ \langle 2|I\rangle & \langle 2|II\rangle \end{pmatrix}$$

Do enough of the matrix multiplication here to provide a convincing arguement that this does the desired transformation. You'll also need the phrase "complete set of states" to reach the desired conclusion.

(f) Carryout the transformation to show that 4.61 is the right Hamiltonian. Note that you'll also need $|\vec{E}| = |\vec{E}_0|\cos\omega t$.

3. *Projecting confidence or projection uncertainty...*

Example 4.3 works out how the time-energy uncertainty arrises when we consider a generic observable A corresponding to an operator $\hat{A}$ that does not commute with the Hamiltonian. (Note that at least my copy of the book as an unfortunate typo, ΔA should be ΔE, in the final line of the solution). I then translated this example in class to the case where A is position to form a more tangible example.

Consider a particle in a magnetic field that points in the x direction such that the Hamiltonian can be written $\hat{H} = \omega_0 \hat{S}_x$.

(a) Show using matrix mechanics in the S_z basis that the projection operator $\hat{P}_{+z}$ does not commute with this Hamiltonian.

(b) Follow example 4.3 to show that this feature implies a time energy uncertainty. Interpret the Δt in this context

(c) Consider the state $|+x\rangle$. What would ΔE be here? What would be the uncertainty in the probability to be in the $|+z\rangle$ state? What would be the rate of change of the expectation value $\langle P_{+z}\rangle$? Do all of these results make sense in the context of the time energy uncertainty? Explain.

4. *~~Flash~~ err... Clebsch-Gordon*

Consider a multiparticle system involving a spin 3/2 particle, call it particle 1, (this could be, for example, a nucleus) and a spin 1/2 particle, say an electron, call it particle 2. Let's begin by naming our states of the system using the following notation $|s_1, s_2, m_1, m_2\rangle$, where here we'll always have $s_1 = 3/2$ and $s_2 = 1/2$.

(a) What are the allowed values of m_1 and m_2 for this system?

(b) We could also express the states of the system using the total angular momentum and the z component of the total angular momentum in the form $|s_t, m_t\rangle$. (Note that these states of well defined total angular momentum will typically be superpositions of the states considered in part (a), but both form a basis for the states of the 2-particle system.) List the possible states of the system in this notation. You can get this answer efficiently by using the Clebsch-Gordon table or by "just thinking about it". No need for long calculations, but say what you did.

(c) Consider the state $|s_t, m_t\rangle = |1, -1\rangle$, which should be one element of your list in part (b). Write this state in terms of $|s_1, s_2, m_1, m_2\rangle$ states. Use a figure or some words to show how you get it from the Clebsch-Gordon table.

(d) Your state in part (c) should be an eigenstate of the total spin z operator defined by equation 5.26b. By definition, when it operates on the left side of your equality in part (c), $|1, -1\rangle$, the eigenvalue will be $-\hbar$, since the $m_t = -1$ tells us that this state is an eigenstate of the total spin z operator with eigenvalue $-\hbar$. Show that your equality in (c) has the right behavior when you operate with the total spin operator on the other side that you expressed in terms of the $|s_1, s_2, m_1, m_2\rangle$ states. It better if the result in (c) is right!

(e) [Extra Credit] Repeat the exploration you did in (c) with the total spin operator defined in 5.26a. You might find the relation in 5.10 helpful. Jay found it challenging to organize the calculation and to avoid arithmetic errors. Maybe there is a clever way to set it up in Mathematic or in a spreadsheet, or... That's why it's extra credit, but if you have time, it is satisfying.

5. *EPR__ ... insert your last initial in the blank*

Townsend 5.4.

6. *Free points*

Based on your first approximately 5 weeks of quantum, give Jay some feedback about how

things are going. Start by writing a paragraph of at least 5 sentences addressing the question, "What is going well for you in QM and what is not." Then answer the following questions
Have you used the PSF sessions? If so, how often and have you found them helpful? If not, why not?
Have you used Jay's office hours? If so, how often and have you found them helpful? If not, why not?
How is the pace of the course? Would you like to go faster and cover more? Would you like to go slower and have more time to digest, or go into more depth?
Are you working with a group of your peers on the material? If not, why not?
To what extent are you doing all of the readings actively?
Don't put your name on this page of your work. After the grader checks that you've provided a complete-looking submission, they will record your score, rip off the page, and give it to me annonymously.

Academic Honesty Statement

____I did not receive help on this problem set. I did all the work on my own.
____I received on this problem set from:
  ____Jay on problems ______________
  ____Student Assistants on problems ______________
  ____Other Students (please name __________________) on problems ______________
  ____Other Instructor (please name __________________) on problems ______________
  ____A resource other than our text (please name __________________)
    on problems ______________

By way of acknowledgements, several of these problems were adapted from Townsend, and number 6 was adapted from Bill Titus.