Name: ________________

PHYS 335 HW 5

Due 2/18 at 5:00PM

The usual instructions

0. (2 pts) Estimate the time it took to do this problem set. Include any comments about the homework that you'd like to share with Jay.

1. *What the Bell?*

(a) Argue that

$$N_1 + N_2 \leq (N_2 + N_4) + (N_1 + N_5) \tag{1}$$

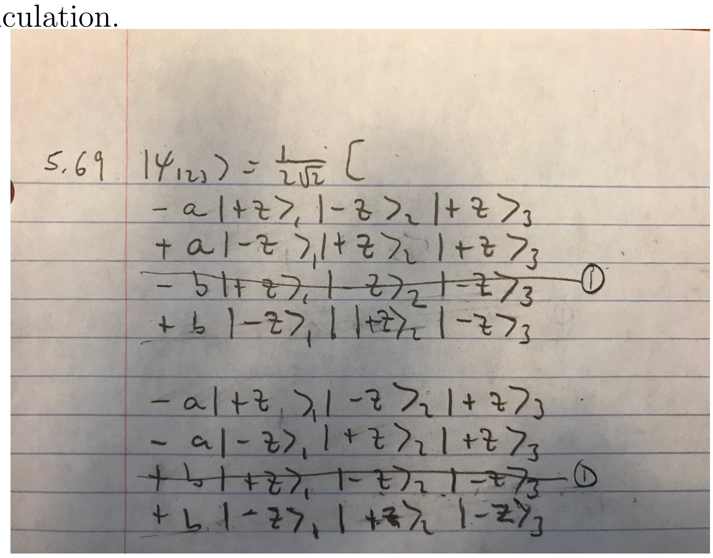
is an inequality that is predicted by the hidden variable model of the EPR experiment developed in the neighborhood of Eq. 5.41.

(b) Show that this prediction is at odds with QM when the angles between the SG apparatuses are as follows: $\theta_{ac} = 16^o$, $\theta_{ab} = 270^o$, and $\theta_{cb} = 254^o$, where θ_{ac} is the angle from the $+a$ direction to the $+c$ direction, and analogously for the others.

2. *You, Alice, and Bob.*

(a) In our teleportation setup, we begin with particles 2 and 3 entangled and particle 1, the one we want to teleport, not entangled as described by Eq. 5.65 or equivalently 5.66. To make the trick work, Alice is going to make a measurement on particles 1 and 2. Prior to discussing teleportation, we'd used 2 different basis for the 4-d space that describes systems of 2 spin 1/2 particles. What are these 2 basis? Describe them and list the states for each case.

(b) Alice could make measurements to force the state she has into one of the 4 possible eigenstates in each of the basis above. But that wouldn't help her know what the state of particle 3 is when it arrives at Bob (at least it won't garantee that she can). She instead chooses a new basis for this 4-d space, the Bell basis, and forces the system into one of these 4 states by making an appropriate measurement on particles 1 and 2. To make this easier, Alice (aka Townsend) writes the whole 3-particle state from 5.66 in terms of the Bell basis vectors to generate Eq. 5.69. Show that Eq. 5.69 is equivalent to Eq. 5.66. Jay did this by multiplying out Eq. 5.69, then showing that terms can be grouped and canceled to get back to Eq. 5.66. It helps to identify pairs that either add or cancel. See the first few lines of Jay's work below to help you think about how to organize the calculation.



5.69 $|\psi_{123}\rangle = \frac{1}{2\sqrt{2}}[$

$- a|+z\rangle_1|-z\rangle_2|+z\rangle_3$

$+ a|-z\rangle_1|+z\rangle_2|+z\rangle_3$

$\cancel{- b|+z\rangle_1|-z\rangle_2|-z\rangle_3}$ 1

$+ b|-z\rangle_1|+z\rangle_2|-z\rangle_3$

$- a|+z\rangle_1|-z\rangle_2|+z\rangle_3$

$- a|-z\rangle_1|+z\rangle_2|+z\rangle_3$

$\cancel{+ b|+z\rangle_1|-z\rangle_2|-z\rangle_3}$ 1

$+ b|-z\rangle_1|+z\rangle_2|-z\rangle_3$

(c) If Alice finds her particles in the $\left|\Psi_{12}^{(+)}\right\rangle$ bell state, what should she tell Bob to do with the particle he gets in order to complete the teleportation process? To help figure it out, first see what happens if Bob rotates his state by π around the x axis. This can be done most straightforwardly by first writing the rotation operator in the form

$$\hat{R}(\theta i) = \cos\frac{\theta}{2} - \frac{2i}{\hbar}\hat{S}_x \sin\frac{\theta}{2}, \tag{2}$$

then applying it to the relevant state in matrix form in the z basis. Once you see the answer to this, see what Bob should do next to finish the teleportation process. (If you want to see where Eq. (2) comes from, see Townsend problem 3.5.)

3. *A weighty problem...some might say it's....*
Townsend 5.18

4. *The position operator in the "momentum basis"*
Townsend 6.2

These proofs require that you insert complete sets of states like $\int dx\,|x\rangle\langle x|$ in strategic places, and that you notice $xe^{-ipx/\hbar} = i\hbar\frac{\partial}{\partial p}e^{-ipx/\hbar}$. In addition to the questions Townsend asks, use the second result to write the expectation value of x in terms of momentum space wave functions $\psi(p)$.

5. *The more you know....about p...*
Townsend 6.5, but with the following twists:

In part (b) write your answer in terms of sin, rather than complex exponentials. When you get to part (c), don't sketch the amplitudes, sketch the probabilities instead. If you want to use Mathematica to plot instead of sketching, it's ok, but you'll either need to pick some representative values for variables or use dimensionless variables. Label the first zeros of $|\langle x|\psi\rangle|^2$ on your sketch with their x values. When it comes to guessing the uncertainties, use the distance to the first zero of $|\langle x|\psi\rangle|^2$ as your guess for Δx. Write some words about why this is a reasonable thing to do. Then write some words about how your guesses align with Heisenberg. Then write some words about what happens to your x plot as you shrink P, and why this behavior is expected.

6. Nothing to submit here, but when you have an idea for your project, email Jay. The information about the project is either on Moodle, or about to be, depending on when you read this. You don't need to get this done by the due date for this homework, but for many reasons, the sooner the better.

Academic Honesty Statement

____I did not receive help on this problem set. I did all the work on my own.
____I received on this problem set from:
____Jay on problems ____________
____Student Assistants on problems ____________
____Other Students (please name ________________) on problems ____________
____Other Instructor (please name ________________) on problems ____________
____A resource other than our text (please name ________________)
on problems ____________

By way of acknowledgements, several of these problems were adapted from Townsend.