1. Ask Jay if needed

2. See attached CG
   - we need section * $3/2 \times 1$
   - Info we neet for $|3/2\ {-1/2}\rangle$ is at **

$$|\tfrac{3}{2}\ {-\tfrac{1}{2}}\rangle = \sqrt{\tfrac{8}{15}}\,|\tfrac{3}{2}\ 1\ \tfrac{1}{2}\ {-1}\rangle - \sqrt{\tfrac{1}{15}}\,|\tfrac{3}{2}\ 1\ {-\tfrac{1}{2}}\ 0\rangle - \sqrt{\tfrac{2}{5}}\,|\tfrac{3}{2}\ 1\ {-\tfrac{3}{2}}\ 1\rangle$$

3. $$(\hat{S}_{1z} + \hat{S}_{2z})\left(\sqrt{\tfrac{8}{15}}\,|\tfrac{3}{2}\ 1\ \tfrac{1}{2}\ {-1}\rangle - \sqrt{\tfrac{1}{15}}\,|\tfrac{3}{2}\ 1\ {-\tfrac{1}{2}}\ 0\rangle - \sqrt{\tfrac{2}{5}}\,|\tfrac{3}{2}\ 1\ {-\tfrac{3}{2}}\ 1\rangle\right)$$

$$= \left(\tfrac{\hbar}{2} - \hbar\right)\sqrt{\tfrac{8}{15}}\,|\tfrac{3}{2}\ 1\ \tfrac{1}{2}\ {-1}\rangle - \left(-\tfrac{\hbar}{2} + 0\right)\sqrt{\tfrac{1}{15}}\,|\tfrac{3}{2}\ 1\ {-\tfrac{1}{2}}\ 0\rangle - \left(-\tfrac{3\hbar}{2} + 1\right)\sqrt{\tfrac{2}{5}}\,|\tfrac{3}{2}\ 1\ {-\tfrac{3}{2}}\ 1\rangle$$

$$= \left(-\tfrac{\hbar}{2}\right)\left(\sqrt{\tfrac{8}{15}}\,|\tfrac{3}{2}\ 1\ \tfrac{1}{2}\ {-1}\rangle - \sqrt{\tfrac{1}{15}}\,|\tfrac{3}{2}\ 1\ {-\tfrac{1}{2}}\ 0\rangle - \sqrt{\tfrac{2}{5}}\,|\tfrac{3}{2}\ 1\ {-\tfrac{3}{2}}\ 1\rangle\right)$$

$$= \left(-\tfrac{\hbar}{2}\right)|\tfrac{3}{2}\ {-\tfrac{1}{2}}\rangle \quad ✓$$

4. I expect $\tfrac{3}{2}\left(\tfrac{3}{2}+1\right)\hbar^2\,|\tfrac{3}{2}\ {-\tfrac{1}{2}}\rangle$

It would go nasty but in principle doable. There are 5 operators acting on a superposition of 3 states for 15 terms to add up.

Particle Data Book

42. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND *d* FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

	J	J	...
	M	M	...
m_1 m_2			
m_1 m_2	Coefficients		
⋮ ⋮			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2×1/2

J	1
M	+1
+1/2 +1/2	1

J	1	0
M	0	0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

J	1
M	−1
−1/2 −1/2	1

1×1/2

J	3/2
M	+3/2
+1 +1/2	1

J	3/2	1/2
M	+1/2	+1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

J	3/2	1/2
M	−1/2	−1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

J	3/2
M	−3/2
−1 −1/2	1

2×1/2

J	5/2
M	+5/2
+2 +1/2	1

J	5/2	3/2
M	+3/2	+3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

J	5/2	3/2
M	+1/2	+1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

J	5/2	3/2
M	−1/2	−1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

J	5/2	3/2
M	−3/2	−3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

J	5/2
M	−5/2
−2 −1/2	1

3/2×1/2

J	2
M	+2
+3/2 +1/2	1

J	2	1
M	+1	+1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

J	2	1
M	0	0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

J	2	1
M	−1	−1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

J	2
M	−2
−3/2 −1/2	1

1×1

J	2
M	+2
+1 +1	1

J	2	1
M	+1	+1
+1 0	1/2	1/2
0 +1	1/2	−1/2

J	2	1	0
M	0	0	0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

J	2	1
M	−1	−1
0 −1	1/2	1/2
−1 0	1/2	−1/2

J	2
M	−2
−1 −1	1

2×1

J	3
M	+3
+2 +1	1

J	3	2
M	+2	+2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

J	3	2	1
M	+1	+1	+1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10

J	3	2	1
M	0	0	0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

J	3	2	1
M	−1	−1	−1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

J	3	2
M	−2	−2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

J	3
M	−3
−2 −1	1

3/2×1

J	5/2
M	+5/2
+3/2 +1	1

J	5/2	3/2
M	+3/2	+3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

J	5/2	3/2	1/2
M	+1/2	+1/2	+1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

J	5/2	3/2	1/2
M	−1/2	−1/2	−1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

J	5/2	3/2
M	−3/2	−3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

J	5/2
M	−5/2
−3/2 −1	1

3/2×3/2

J	3
M	+3
+3/2 +3/2	1

J	3	2
M	+2	+2
+3/2 +1/2	1/2	1/2
+1/2 +3/2	1/2	−1/2

J	3	2	1
M	+1	+1	+1
+3/2 −1/2	1/5	1/2	3/10
+1/2 +1/2	3/5	0	−2/5
−1/2 +3/2	1/5	−1/2	3/10

J	3	2	1	0
M	0	0	0	0
+3/2 −3/2	1/20	1/4	9/20	1/4
+1/2 −1/2	9/20	1/4	−1/20	−1/4
−1/2 +1/2	9/20	−1/4	−1/20	1/4
−3/2 +3/2	1/20	−1/4	9/20	−1/4

J	3	2	1
M	−1	−1	−1
+1/2 −3/2	1/5	1/2	3/10
−1/2 −1/2	3/5	0	−2/5
−3/2 +1/2	1/5	−1/2	3/10

J	3	2
M	−2	−2
−1/2 −3/2	1/2	1/2
−3/2 −1/2	1/2	−1/2

J	3
M	−3
−3/2 −3/2	1

2×3/2

J	7/2
M	+7/2
+2 +3/2	1

J	7/2	5/2
M	+5/2	+5/2
+2 +1/2	3/7	4/7
+1 +3/2	4/7	−3/7

J	7/2	5/2	3/2
M	+3/2	+3/2	+3/2
+2 −1/2	1/7	16/35	2/5
+1 +1/2	4/7	1/35	−2/5
0 +3/2	2/7	−18/35	1/5

J	7/2	5/2	3/2	1/2
M	+1/2	+1/2	+1/2	+1/2
+2 −3/2	1/35	6/35	2/5	2/5
+1 −1/2	12/35	5/14	0	−3/10
0 +1/2	18/35	−3/35	−1/5	1/5
−1 +3/2	4/35	−27/70	2/5	−1/10

J	7/2	5/2	3/2	1/2
M	−1/2	−1/2	−1/2	−1/2
+1 −3/2	4/35	27/70	2/5	1/10
0 −1/2	18/35	3/35	−1/5	−1/5
−1 +1/2	12/35	−5/14	0	3/10
−2 +3/2	1/35	−6/35	2/5	−2/5

J	7/2	5/2	3/2
M	−3/2	−3/2	−3/2
0 −3/2	2/7	18/35	1/5
−1 −1/2	4/7	−1/35	−2/5
−2 +1/2	1/7	−16/35	2/5

J	7/2	5/2
M	−5/2	−5/2
−1 −3/2	4/7	3/7
−2 −1/2	3/7	−4/7

J	7/2
M	−7/2
−2 −3/2	1

2×2

J	4
M	+4
+2 +2	1

J	4	3
M	+3	+3
+2 +1	1/2	1/2
+1 +2	1/2	−1/2

J	4	3	2
M	+2	+2	+2
+2 0	3/14	1/2	2/7
+1 +1	4/7	0	−3/7
0 +2	3/14	−1/2	2/7

J	4	3	2	1
M	+1	+1	+1	+1
+2 −1	1/14	3/10	3/7	1/5
+1 0	3/7	1/5	−1/14	−3/10
0 +1	3/7	−1/5	−1/14	3/10
−1 +2	1/14	−3/10	3/7	−1/5

J	4	3	2	1	0
M	0	0	0	0	0
+2 −2	1/70	1/10	2/7	2/5	1/5
+1 −1	8/35	2/5	1/14	−1/10	−1/5
0 0	18/35	0	−2/7	0	1/5
−1 +1	8/35	−2/5	1/14	1/10	−1/5
−2 +2	1/70	−1/10	2/7	−2/5	1/5

J	4	3	2	1
M	−1	−1	−1	−1
+1 −2	1/14	3/10	3/7	1/5
0 −1	3/7	1/5	−1/14	−3/10
−1 0	3/7	−1/5	−1/14	3/10
−2 +1	1/14	−3/10	3/7	−1/5

J	4	3	2
M	−2	−2	−2
0 −2	3/14	1/2	2/7
−1 −1	4/7	0	−3/7
−2 0	3/14	−1/2	2/7

J	4	3
M	−3	−3
−1 −2	1/2	1/2
−2 −1	1/2	−1/2

J	4
M	−4
−2 −2	1

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 42.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).