Name: ________________

PHYS 335 HW 1

Due 1/14 at 5:00PM

Before starting this homework, you should make sure you have read the homework section of the syllabus and the homework rubric from the homework folder on Moodle. To keep the accounting simple, all problems are worth 10 pts unless otherwise noted. In some cases a lot may be required for your 10 points, in other cases a little. Be sure to fill out the academic honesty statement at the end.

0. (2 pts) Estimate the time it took to do this problem set. Include any comments about the homework that you'd like to share with Jay.

1.
Consider the classic 2 slit experiment with electrons. The only modification to the usual setup is that one of the slits is much smaller than the other such that the probability of getting through slit 1 is 100 times smaller than the probability of getting through slit 2. What is the approximate ratio of the probability for an electron to arrive at the central max to the probability to arrive at the first minimum? There are a variety of ways to address this question associated with the approximations you choose to make in modeling the system and the way you organize the steps of the calculation. The steps to follow guide you through one possibility.

(a) First imagine that we insert some apparatus that tracks which hole the electrons go through. Make a very rough qualitative sketch of the probabilities to arrive at various locations on the screen associated with electrons having gone through hole 1 and electrons having gone through hole 2. Then draw another sketch of (in a very rough qualitative way), what you expect the pattern to look like if you do not track what hole the electrons go through and let them interfere. The idea is to produce sketches analogous to Feynman figure 1-3. You could come back and tweak this up after you calculate below, but get a rough sense of at least some features to expect at the start.

(b) The amplitude to arrive at the screen at a particular location having traveled from the source to the screen (step A), gone through one of the holes (step B), then traveled from the hole to the screen (step C) can be thought of as the product of the amplitudes for each step. Let's focus first on the electrons that travel through hole 1, and let's call the amplitudes for each of the 3 steps in the process a_{A1}, a_{B1}, and a_{C1} respectively. We can similarly name the amplitudes associated with travel through hole 2 with the replacement $1 \rightarrow 2$. Let's start by making a few observations about these amplitudes. I claim:
(i) $a_{A1} = a_{A2}$ at least to a good approximation
(ii) $10a_{B1} = a_{B2}$ if we assume that the hole size does not effect the phase, but only magnitude of the amplitude
(iii) the amplitudes in step C are a function of position on the screen. I claim $a_{C1}(x = 0) = a_{C2}(x = 0)$, where $x = 0$ is the position of the central maximum.
(iv) I further claim, $a_{C1}(x = x_{\text{min}}) \approx a_{C2}(x = x_{\text{min}})e^{i\pi}$ if we're dealing with one of the first minima, and that $|a_{C1}(x = x_{\text{min}})| \approx |a_{C1}(x = 0)|$
Write some words explaining each of these observations that would convince a skeptical classmate that they are a reasonable way to model the problem. Be sure to address what we're neglecting with the"≈" in step (iii).

(c) Write an expression for the ratio we seek in terms of the amplitudes introduced above. Use the observations from part (b) to turn it into a number.

You should conclude from the above that the presence of interference makes it easier to detect the small hole next to the big one over the case where interference is not present. This is a generic technique in precision measurement.

2.

General Lee Oriented of the military's quantum information division has an expression for the state of a spin 1/2 particle that is spin up along the $\hat{n}$ direction in space, where

$$\hat{n} = \sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k}. \tag{1}$$

Here θ, ϕ are the usual (physics style) polar and azimuthal angles in spherical coordinates. So the particle is spin up along an arbitrary direction in space specified by the angles θ and ϕ. General Oriented knows that the particle's state is described by the Hilbert space vector

$$|{+n}\rangle = \cos\tfrac{\theta}{2}\,|{+z}\rangle + e^{i\phi}\sin\tfrac{\theta}{2}\,|{-z}\rangle \tag{2}$$

(we'll see how to show this later in the course).

(a) Show that General Oriented's state reduces to the correct answer for $|{+y}\rangle$ with the correct choice of θ and ϕ.

(b) Suppose that General Oriented makes a measurement of S_z on their state. As a function of θ and/or ϕ, what is the probability of getting $\hbar/2$ and what is the probability of getting $-\hbar/2$? What would be a nifty way to check your answer? Do your check!

(c) What would be the uncertainty in General Oriented's measurement of S_z? Under what conditions is the uncertainty zero? Does this make sense?

3.

(a) Consider a particle in the $|{+n}\rangle$ state from the prior problem. What is the amplitude to find the particle with $S_y = \hbar/2$? What is the probability? (These answers may be functions of θ and/or ϕ.)

(b) Check your answer to part (a) by evaluating your answer for a convenient choice of θ and ϕ.

(c) What are the amplitude and probability to find a particle that Hu-Wi prepares in the $|{+y}\rangle$ state in the state $|{+n}\rangle$? Either be tricky about getting this from your answer to part (a) or say how you could have been tricky. (Again, these answers may be functions of θ and/or ϕ.)

(d) The state

$$|{-n}\rangle = \sin\tfrac{\theta}{2}\,|{+z}\rangle - e^{i\phi}\cos\tfrac{\theta}{2}\,|{-z}\rangle \tag{3}$$

along with the state $|{+n}\rangle$ form an orthonormal basis for describing a spin 1/2 particle. Show the orthogonality of these two potential basis vectors and show that $|{-n}\rangle$ is properly normalized.

4.

In the Stern-Gerlach experiment shown below, the first and last SG devices are standard SGz setups. The middle SG is set up such that it passes particles with $\phi = 0$ and some angle θ with respect to the z axis in the $x - z$ plane.

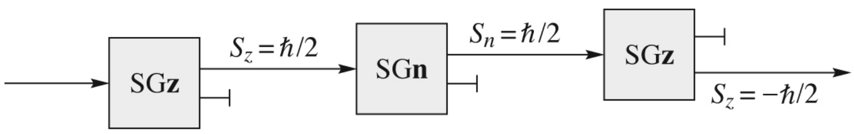

Figure 1.12
Townsend A MODERN APPROACH TO QUANTUM MECHANICS 2nd edition

(a) What fraction of the particles that are transmitted by the first SGz make it out the end of the experiment (that is, are transmitted out the end of the last SGz)?

(b) What is the ideal orientation (ideal θ) for the middle SG such that the transmission through the final SG is maximized? What fraction of particles that are transmitted by the first SG are transmitted by the final SG for this orientation?

(c) What fraction of particles entering the first SG emerge from the final SG if the middle SG is removed?

(d) Fay Zschift inserts a device after the second SG that performs the following change to the state of the particles that are transmitted from the SGn device: $|+n\rangle \rightarrow e^{i\delta}|+n\rangle$ for some real number δ. What effect does Fay's addition have on the probability of a particle that makes through the first SGz to make it through the last SGz? Include some math and words to justify your answer as usual.

5.
Iona Beam (Jim Beam's sister) has a beam of identically prepared particles for which a measurement of S_z yields $\hbar/2$ with 36% probability and a measurement of S_x yields $\hbar/2$ with 50% probability. Determine the state of Iona's particles as completely as possible from this information. Iona suggests that you begin with an arbitrary state, like $|\psi\rangle = a_1|+z\rangle + a_2|-z\rangle$ and start using the given information about the various measurement results to constrain the arbitrary complex numbers a_1 and a_2. Don't forget that they are complex!

6.
Spin is a huge deal in physics. Read an article that is intended for a scientific audience that is not a journal article about spin. I'm thinking of publications like *Physics Today* (go here `https://physicstoday.scitation.org/action/doSearch?AllField=spin&ConceptID=`) or *Physics* (go here `https://physics.aps.org/browse/?page=1&per_page=10&sort=relevance&q=spin`). Provide a reference to the article and write a paragraph about what you learned from reading it.

Academic Honesty Statement

___I did not receive help on this problem set. I did all the work on my own.
___I received on this problem set from:
   ___Jay on problems __________
   ___Student Assistants on problems __________
   ___Other Students (please name ______________) on problems __________
   ___Other Instructor (please name ______________) on problems __________
   ___A resource other than our text (please name ______________)
   on problems __________

By way of acknowledgements, problem 1 was adapted from one written by Mike Snow at Indiana University, problems 2-5 were adapted from Townsend, and problems 0 and 6 were inspired by Bill Titus's work.