Class 1

Slide 8

To be an orthonormal basis we need the vectors to be orthogonal

$\langle \nu_{m_1} | \nu_{m_2} \rangle = 0$ and normal

$\langle \nu_{m_1} | \nu_{m_1} \rangle = \langle \nu_{m_2} | \nu_{m_2} \rangle = 1$

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Probability to find ν_e w/ mass m_1

$$|\langle \nu_{m_1} | \nu_e \rangle|^2$$

$$= |\langle \nu_{m_1} | (\cos\theta \, |\nu_{m_1}\rangle - \sin\theta \, |\nu_{m_2}\rangle)|^2$$

$$= |\cos\theta \cancelto{1}{\langle \nu_{m_1} | \nu_{m_1} \rangle} - \sin\theta \cancelto{0}{\langle \nu_{m_1} | \nu_{m_2} \rangle}|^2$$

$$= \cos^2\theta$$

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$$|\langle \nu_{m_1}(0) | \nu_{m_1}(L) \rangle|^2$$

$$= \left| \exp\left(\frac{-i m_1^2 L}{2E}\right) \cancelto{1}{\langle \nu_{m_1}(0) | \nu_{m_1}(0) \rangle} \right|^2$$

$$= \exp\left(\frac{-i m_1^2 L}{2E}\right) \exp\left(\frac{+i m_1^2 L}{2E}\right)$$

$$= 1$$

This reflects the fact that the states of definite mass don't change as they travel. A state of mass m_1 @ 0 is still a state of mass m_1 @ L w/ 100% probability.

Note that this is also an example of an overall phase that is irrelevant.

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Let's start with a ν_e in the mass basis

(A1) $|\nu_e\rangle = \cos\theta|\nu_{m_1}\rangle - \sin\theta|\nu_{m_2}\rangle$

We know how the mass basis states evolve so at L this becomes

(A2) $|\nu_L\rangle = \cos\theta \exp\left(\frac{-im_1^2L}{2E}\right)|\nu_{m_1}\rangle - \sin\theta \exp\left(\frac{-im_2^2L}{2E}\right)|\nu_{m_2}\rangle$

Now what is the probability that this is still a ν_e? Turn A1 into a bra, dot into A2, keep only non zero parts.

$$|\langle\nu_e|\nu(L)\rangle|^2$$
$$= \left|\cos^2 \exp\left(\frac{-im_1^2L}{2E}\right) + \sin^2\theta \exp\left(\frac{-im_2^2L}{2E}\right)\right|^2$$
$$= \cos^4\theta + \sin^4\theta + \sin^2\theta\cos^2\theta \exp\left(\frac{i(m_1^2-m_2^2)L}{2E}\right)$$
$$+ \sin^2\theta\cos^2\theta \exp\left(\frac{-i(m_1^2-m_2^2)L}{2E}\right)$$

These are the 4 terms we get when we multiply the thing in side of the $|\ \ |$ by it's complex conjugate. The rest is some trig manipulations. For ease of writing, I'll call $\frac{(m_1^2 - m_2^2)L}{2E} = y$

Note
$$\begin{aligned} e^{iy} &= \cos y + i\sin y \\ + e^{-iy} &= \cos y - i\sin y \\ \hline &= 2\cos y \end{aligned}$$

$$= \cos^2\theta(1-\sin^2\theta) + \sin^2\theta(1-\cos^2\theta)$$
$$+ 2\sin^2\theta\cos^2\theta\cos y$$

$= 1 - 2\sin^2\theta\cos^2\theta + 2\sin^2\theta\cos^2\theta\cos y$

$= 1 - 2\sin^2\theta\cos^2\theta\,(1 - \cos y)$

I'll now use the following 2 identities

$1 - \cos 2\theta = 2\sin^2\theta$
$\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

$= 1 - \sin^2 2\theta \sin^2 \frac{y}{2}$

Plugging back in for our definition of y yields the expected result.

It's nice to see that it's ≤ 1.

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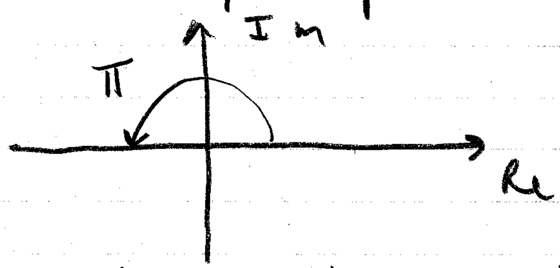
On the complex plane it looks like this



So we're on the negative real axis
So $5e^{i\pi} = -5$.
You could also do

$$5(\underbrace{\cos\pi}_{-1} + i\underbrace{\sin\pi}_{0}) = -5$$