Name: ________________

PHYS 335 HW 6

Due 2/26 at 5:00PM

The usual instructions

0. (2 pts) Estimate the time it took to do this problem set. Include any comments about the homework that you'd like to share with Jay.

1. *What the well?*

(a) Consider an "even" solution to the finite square well, that is, one for which $A = 0$ in Eq. 6.92. Write the equations that specify continuity of ψ and its derivative at the right edge of the well.

(b) Take the ratio of your equations from part (a) such that you get an equation of the form of a constant (that is proportional to the energy eigenvalue) equal to a tan of some stuff that also involves the energy eigenvalue. The integration constants should be eliminated in this result. If you could solve this equation for E, these would be the energy eigenvalues. The trouble is, this is a transcendental equation (of the sort we met in 231). Beat your equation into the form

$$\tan z = \sqrt{(z_0/z)^2 - 1}. \tag{1}$$

What are z and z_0 in terms of some subset of the energy E, the width of the well a, the mass of the particle m, the height of the well V_0, and Planck's constant $\hbar$?

(c) Pick $\sqrt{\frac{2mV_0}{\hbar^2}} = \frac{16}{a}$. For this choice, how many even energy eigenfucntions are there that describe particles bound in the well? What are the corresponding energy eigenvalues in terms of some subset of a, m, $\hbar$ and V_0? To get this going, you'll need to numerically solve the transcendental equation from (b) somehow. Jay plotted $\tan z$ vs z, and $\sqrt{(z_0/z)^2 - 1}$ vs z on the same graph to extract the (approximate) values of z and hence E that are solutions to our transcendental equation. Include your work in Mathematica or otherwise indicate how you solved this.

(d) Google something like "systems modeled as a square well". As your answer to this question, write a sentence or two describing the application you found and give a citation or web address. It need not be published literature this time, wikipedia or similar is ok.

2. *More applications...you (or someone) asked for it!*
Townsend 6.24

3. *Another one you asked for...*
In class, and in the development leading up to Eq. 6.76, we showed that a gaussian wavepacket spreads in position as a function of time, in other words, Δx increases. In example 6.4, Townsend shows that the momentum uncertainty does not increase. Hence if we minimally satisfied the uncertainty principle at time $t = 0$, we will more safely satisfy it as time increases. Several of you wondered what would happen if you take the spread wavepacket in position space given by Eq. 6.76 and Fourier transform it to momentum space. You should conclude that you get the same probability distribution in momentum space as we had at time $t = 0$, in agreement with the result of example 6.4. Try it.

4. *Keep it moving, i.e. not stationary*

(a) Townsend 6.11

(b) Make some comments about the meaning of your result in part (a). There's nothing super profound, I just want you think about the physics.

(c) If the system starts out in the ground state of the narrow well at time $t = 0$, when the well is expanded, does the probability you calculated in part (a) change with time? Explain.

(d) If the system starts out in the ground state of the narrow well at time $t = 0$, when the well is expanded, does the system remain in the same state over time? Explain.

(e) If the system starts out in the ground state of the narrow well at time $t = 0$, when the well is expanded, does the probability to be found in the original state change over time? Explain.

5. *A matter of some gravity...*

In class I argued that solving for the position space wave function is pretty much the same no matter the potential. Consider a particle of mass m that can move in one dimension z in the gravitational field of the Earth. It is bounded from below by an impenetrable barrier at $z = 0$. So it's a quantum bouncing ball, which can be done with cold neutrons on a mirror.

(a) Sketch the basic setup.

(b) Sketch a potential energy plot and a rough qualitative guess for what the position space wave function solutions look like.

(c) Let's find the energy eigenvalues and eigenfunctions in terms of relevant constants. To get started, write down the Schrödinger Equation. I found it helpful to my subsequent calculations to define $k_1 = 2m^2g/\hbar^2$ and $k_2 = 2mE/\hbar^2$.

(d) The differential equation you need to solve does not have a solution in terms of simple functions. It's known as Airy's Equation having solutions known as Airy Functions, Ai and Bi. You'll either need to look it up in a math handbook or use DSolve in Mathematica to deal with it. Either include a Mathematica printout with this problem or a reference to a published math handbook that you used. (I think you'll actually need Mathematica by the end.) You'll find something of the form

$$\psi(z) = A\mathrm{Ai}(stuff) + B\mathrm{Bi}(stuff), \tag{2}$$

as your initial solution to the Schrödinger Equation where A and B are integration constants. What is the "stuff"? Note that Mathematica calls the Airy functions AiryAi and AiryBi.

(e) To get a sense for what's going on, plot or look up plots of $\mathrm{Ai}(x)$ and $\mathrm{Bi}(x)$. I find that a plot range of -10 to +2 gives me a good look at things. Just by staring at these plots, I claim that you can fix one of your integration constants and understand your eigenfunctions as truncated Airy functions that are shifted down the z axis. Describe this.

(f) By estimating the first few zeros of the Airy function off of your plot, find approximate expressions for the first few energy eigenvalues.

(g) Find ψ_1 in normalized form, the ground state eigenfunction. (Hint: do a numerical integral in mathematica.)

(h) Find the expectation value of the position of the particle when it's in the ground state. (Hint: do another integral in mathematica.) Evidently, the average location of our "bouncing particle" is somewhat above $z = 0$ even in its lowest energy state. Stand in awe of the power of science!

As a side note, you may have heard that no one knows how to do quantum gravity. But we seem to be doing quantum mechanics with gravity here, so what gives? Here we treat the gravitational field as the usual old classical thing. It's the particle that we treat quantum mechanically. We do the same thing with the electric field when we do the hydrogen atom. The thing we don't know how to do for gravity is treat the field itself as made of quantized “gravitons” as we do for the E&M fields with photons.

6. Nothing to submit here, but at some point before submitting this homework, email jay with a project idea. Academic Honesty Statement

____I did not receive help on this problem set. I did all the work on my own.
____I received on this problem set from:
____Jay on problems ______________
____Student Assistants on problems ______________
____Other Students (please name ___________________) on problems ______________
____Other Instructor (please name ___________________) on problems ______________
____A resource other than our text (please name ___________________)
on problems ______________

By way of acknowledgements, several of these problems were adapted from Townsend.