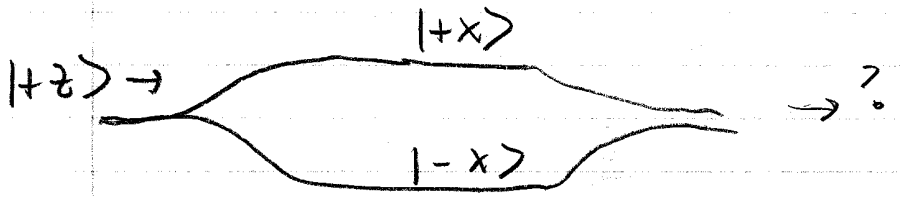
Class 3

Slide 15



The amplitude to split onto the upper track is $\langle +x|+z\rangle$ (we have $|+z\rangle$ need amplitude to be in the $|+x\rangle$ state)

Similarly for the lower track
$\langle -x|+z\rangle$

The amplitude that $|+x\rangle$ is found in the $|+z\rangle$ state at the end is
$\langle +z|+x\rangle$

And similarly for $|-x\rangle$ on the lower track
$\langle +z|-x\rangle$

Puting it together

$$P_{+z} = \left| \underbrace{\langle +x|+z\rangle\langle +z|+x\rangle}_{\text{multiply amplitudes along upper track}} + \underbrace{\langle -x|+z\rangle\langle +z|-x\rangle}_{\text{multiply amplitudes along lower track}} \right|^2$$

$$= \left| \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} \right|^2$$

$$= 1$$

Similar work asking about the probability of $-z$ at the end yields

$$P_{-z} = \left| \langle +x|+z\rangle\langle -z|+x\rangle + \langle -x|+z\rangle\langle -z|-x\rangle \right|^2$$

$$= \left| \frac{1}{2} - \frac{1}{2} \right|^2 = 0$$

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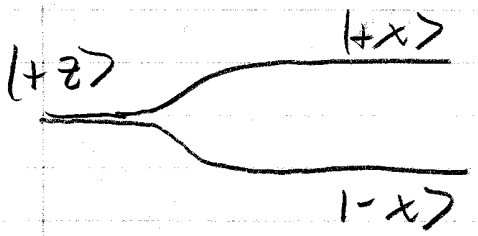
If we look in the middle, we'll find a 50/50 split to $|+x\rangle$ and $|-x\rangle$.



$|+x\rangle$ has a 50% chance of turning into $|+z\rangle$

So the probability that we find $|+x\rangle$ in the middle and $|+z\rangle$ at the end is 1/4.

The probability that we find $|-x\rangle$ in the middle and $|+z\rangle$ at the end is 1/4

So now the probability of $|+z\rangle$ at the end is $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$, or 50%.

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$$P_{max} = |a_{A1}\, a_{B1}\, a_{C1} + a_{A2}\, a_{B2}\, a_{C2}|^2$$

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$$P_{max} = |a_{A1}\, a_{B1}\, a_{C1} + 0.2\, e^{i\pi/2}\, a_{A2}\, a_{B2}\, a_{C2}|^2$$

Now I need the ratio

$$\frac{P_{max}'}{P_{max}} = \frac{|a_{A1}\, a_{B1}\, a_{C1} + 0.2\, i\, a_{A2}\, a_{B2}\, a_{C2}|^2}{|a_{A1}\, a_{B1}\, a_{C1} + a_{A2}\, a_{B2}\, a_{C2}|^2}$$

$$= \frac{(1+0.2i)(1-0.2i)}{4} = \frac{1.04}{4} = 0.26$$