The 3 axis Bell test

A) The hidden variable prediction

Townsend develops a Bell inequality by considering the statement Eq 5.50 about the relative populations of possible outcomes

$$N_3 + N_4 \le (N_2 + N_4) + (N_3 + N_7)$$

Q1 Unless this is obvious to you (It was not to me) do a little algebra to convince yourself it's true, I did this by subtracting N_3 + N_4 from both sides.

Next Townsend turns this statement into a statement about probabilities. This will help us compare this hidden variable prediction with QM.

Q2 Convince your partners that

$$\frac{N_3 + N_4}{\underbrace{\Sigma_i N_i}_{\text{total populations}}} = \underbrace{P(+a\,;+b)}_{\text{Probability of finding +a for 1 + +b for 2}}$$

As Townsend shows in Eq 5.51.

Q3 Do the same for Eq 5.52 and 5.53.

5.52 $\quad \dfrac{N_2 + N_4}{\Sigma_i N_i} = P(+a, +c)$

5.53 $\quad \dfrac{N_3 + N_7}{\Sigma_i N_i} = P(+c, +b)$

Q4 Now to test yourself (it's irrelevant here) write the probability $P(+a, -b)$ as a ratio of populations

You can now re-write 5.50 as

(1) $P(+a;+b) \leq P(+a;+c) + P(+c;+b)$

This is a prediction of our hidden variable model, and within that model, I don't find it very profound. What is profound is that QM makes a <u>different prediction</u>. Thus we can experimentally distinguish the 2 models.

B) The QM prediction.

Our 2 particles are emitted in the $|0,0\rangle$ state. So $P(+a,+b) = |\langle +a,+b|0,0\rangle|^2$

At this point our states in this inner product are not even written with the same notation.

Q5. What do the characters $+a$, $+b$, 0, 0 above mean? In other words, who's eigenvalues are they?

Lets write $|0,0\rangle$ in terms of the eigenstates of $\hat{S}_{1a}$ and $\hat{S}_{2a}$ (component of spin along the a axis, and work out the probability from there.

Q6. Fill in the blanks

$$P(+a,+b) = \left|\langle +a,+b|\left(\tfrac{1}{\sqrt{2}}|+a,-a\rangle - \_\_\_\_\_\_\right)\right|^2$$

$$= \left|\tfrac{1}{\sqrt{2}}\left({}_1\langle +a|\,{}_2\langle +b|\right)\left(|+a\rangle_1 \_\_\_\_\_\_\right)\right|^2$$

$$= \left|\tfrac{1}{\sqrt{2}}\, \overset{1}{\cancel{{}_1\langle +a|+a\rangle_1}}\; {}_2\langle +b|\,\_\_\_\_\_\_\right|^2$$

$$= \tfrac{1}{2}\left|\langle +b|-a\rangle\right|^2$$

We know $|+n\rangle = \cos\frac{\theta}{2}|+z\rangle + e^{i\phi}\sin\frac{\theta}{2}|-z\rangle$
If we imagine aligning $+z$ with $+a$ and we let the general direction $+n$ be $+b$, Then you can finish finding $P(+a, +b)$.

Q7 Do it! (That is, find $P(+a, +b)$) Note that when you do this, the angle θ above can be understood as the angle between the $+a$ and $+b$ directions in the plane perpendicular to the beam, so call it θ_{ab}

(2) $\quad P(+a; +b) =$ __________

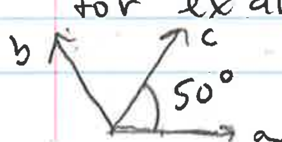
To finish out our QM prediction for the terms in the Bell inequality (1), we need

(3) $\quad P(+a; +c) =$ __________

(4) $\quad P(+c; +b) =$ __________

Q8 Fill in the blanks above. It's easy Since the direction we called $+b$ in the derivation of (2) could easily be named $+c$, so these results can be found by a simple renaming from (2).

Q9
Plug (2), (3), (4) into the Bell inequality (1). Recall that the inequality sign is a <u>prediction of Hidden Variable theory</u>. Show that for at least some value of the angles between the measurement directions $+a$, $+b$, $+c$, the QM prediction is at odds with HV theory. Try, for example $\theta_{ab} = 100°$ $\theta_{ac} = \theta_{cb} = 50°$



A meta-cognition question:
Did you understand this derivation when you read it from Townsend? Did working through these notes help? Could you imagine making a set of notes like this for yourself?

Try developing a few sentence explanation of what Bell's inequalities are that you could give to an A&N student in an elevator. No math!

If you're done with all of this, you could either

A) Find another Bell inequality in the 3 axis experiment or

B) Get a jump on Friday by working through example 5.2 together