

## 42. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND $d$ FUNCTIONS

Note: A square-root sign is to be understood over every coefficient, e.g., for  $-8/15$  read  $-\sqrt{8/15}$ .

Notation:

$J$	$J$	...
$M$	$M$	...
$m_1$	$m_2$	
$m_1$	$m_2$	

Coefficients

$1/2 \times 1/2$	$1$
$+1/2 + 1/2$	$1$
$+1/2 - 1/2$	$1$
$-1/2 + 1/2$	$1$
$-1/2 - 1/2$	$1$

$$Y_0^0 = \sqrt{\frac{3}{4\pi}} \cos \theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}} \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}$$

$$Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{2i\phi}$$

$2 \times 1/2$	$5/2$
$+5/2$	$5/2$
$+3/2$	$3/2$
$+2$	$1/2$
$+1$	$0$
$0$	$-1/2$
$-1$	$-3/2$
$-2$	$-5/2$

$1/5$	$4/5$	$5/2$	$3/2$
$4/5 - 1/5$	$1/2$	$1/2$	

$+1 - 1/2$	$2/5$	$3/5$	$5/2$	$3/2$
$0 + 1/2$	$3/5 - 2/5$	$-1/2$	$-1/2$	

$0 - 1/2$	$3/5$	$2/5$	$5/2$	$3/2$
$-1 + 1/2$	$2/5 - 3/5$	$-3/2$	$-3/2$	

$-1 - 1/2$	$4/5$	$1/5$	$5/2$
$-2 + 1/2$	$1/5 - 4/5$	$-5/2$	

$-2 - 1/2$	$1$			
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$+3/2 + 1/2$	$1$	$+1$	$+1$
$+3/2 - 1/2$	$1/4$	$3/4$	$2$
$+1/2 + 1/2$	$3/4 - 1/4$	$0$	$0$

$+1/2 - 1/2$	$1/2$	$1/2$	$2$	$1$
$-1/2 + 1/2$	$1/2 - 1/2$	$-1$	$-1$	

$-1/2 - 1/2$	$3/4$	$1/4$	$2$
$-3/2 + 1/2$	$1/4 - 3/4$	$-2$	

$-3/2 - 1/2$	$1$			
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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

$-3/2 - 1$	$1$			
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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

$-3/2 - 1$	$1$			
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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

$-3/2 - 1$	$1$			
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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

$-3/2 - 1$	$1$			
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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

$-3/2 - 1$	$1$			
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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

$-3/2 - 1$	$1$			
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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

$-3/2 - 1$	$1$			
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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

$-3/2 - 1$	$1$			
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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

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$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

$-3/2 - 1$	$1$			
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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

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$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

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$+1/2 - 1$	$3/10$	$8/15$	$1/6$
$-1/2 0$	$3/5 - 1/5$	$-1/3$	$5/2$
$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

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$-3/2 + 1$	$1/10 - 2/5$	$1/2$	$-3/2 - 3/2$

$-1/2 - 1$	$3/5$	$2/5$	$5/2$
$-3/2 0$	$2/5 - 3/5$	$-5/2$	

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$-1/2 - 1$	$3/5$	$2/5$	$5/2$
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