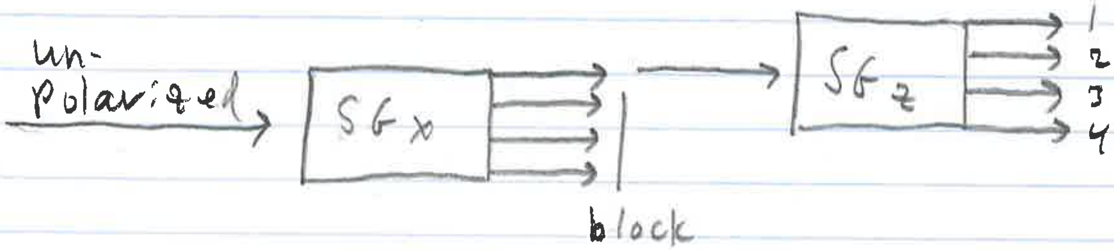
A general spin question

An unpolarized beam enters an SG_x, the maximally deflected beam in the $+x$ direction is sent through an SG_z.

A) what is s for these particles?

B) what are the probabilities to find particles coming exiting the final SG in each port!



A) If we see 4 beams exis the 1st SG, we have $s = 3/2$ since spin $3/2$ particles admit 4 spin states separated by $\hbar/2$.

B) The states exiting through ports 1-4 are named as follows

$1 \to |\tfrac{3}{2}\,\tfrac{3}{2}\rangle \xrightarrow[\text{z basis}]{} \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $\qquad 2 \to |\tfrac{3}{2}\,\tfrac{1}{2}\rangle \to \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$

$3 \to |\tfrac{3}{2}\,-\tfrac{1}{2}\rangle \longrightarrow \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ $\qquad 4 \to |\tfrac{3}{2}\,-\tfrac{3}{2}\rangle \to \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

* What we need to compute to answer B are $|\langle \tfrac{3}{2}\,\tfrac{3}{2}|\tfrac{3}{2}\,\tfrac{3}{2}\rangle_x|^2$, $|\langle \tfrac{3}{2}\,\tfrac{1}{2}|\tfrac{3}{2}\,\tfrac{3}{2}\rangle_x|^2$, $|\langle \tfrac{3}{2}\,-\tfrac{1}{2}|\tfrac{3}{2}\,\tfrac{3}{2}\rangle_x|^2$, and $|\langle \tfrac{3}{2}\,-\tfrac{3}{2}|\tfrac{3}{2}\,\tfrac{3}{2}\rangle_x|^2$

To do this, I need to ind $|\tfrac{3}{2}\,\tfrac{3}{2}\rangle_x$ in the z basis.

This state is an eigenstate of $\hat{S}_x$.

Example 3.4 calculates $\hat{S}_x$ in the z basis. You could use their methods to get any $\hat{S}_j$ in any basis. Here they find

$$\hat{S}_x \xrightarrow[S_z \text{ basis}]{} \frac{\hbar}{2}\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

You could find the eigenvectors any way you want. I used the posted Mathematica notebook and found

$$_x\langle \tfrac{3}{2}\,\tfrac{3}{2}| \xrightarrow[S_z \text{ basis}]{} (1, \sqrt{3}, \sqrt{3}, 1) \xrightarrow[\text{Normalize}]{} \frac{1}{2\sqrt{2}}(1, \sqrt{3}, \sqrt{3}, 1)$$

Lets's check

$$\frac{\hbar}{2}\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{2} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}\begin{pmatrix} 1 \\ \sqrt{3} \\ \sqrt{3} \\ 1 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 3 \\ \sqrt{3}+2\sqrt{3} \\ \sqrt{3}+2\sqrt{3} \\ 3 \end{pmatrix} = 3\frac{\hbar}{2}\begin{pmatrix} 1 \\ \sqrt{3} \\ \sqrt{3} \\ 1 \end{pmatrix}$$

So indeed $(1\ \sqrt{3}\ \sqrt{3}\ 1)$ is an eigenstate of our $\hat{S}_x$ matrix in the z basis with eigenvalue $\frac{3}{2}\hbar$.

We can now work out the probabilities at the

Port 1: $\left|\frac{1}{2\sqrt{2}}(1\ \sqrt{3}\ \sqrt{3}\ 1)\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}\right|^2 = \frac{1}{8}$

Port 2 : $\left| \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & \sqrt{3} & \sqrt{3} & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right|^2 = \frac{3}{8}$

By symmetry, Port 3 : $3/8$
Port 4 : $1/8$