1. Hugh Rowe-Tate wishes to rotate the state $|+x\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$ by an angle ϕ. Show that the probability of finding Hugh's particle in the $|+x\rangle$ state after the rotation is

$$P_{+x} = \cos^2\frac{\phi}{2}.$$

2. May Tricks wants the matrix representation of $\hat{J}_z$ in the $|a\rangle, |b\rangle$ basis, where $|a\rangle$ and $|b\rangle$ are as follows,

$$|a\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$$

$$|b\rangle = \frac{-1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$$

3. When $\hat{J}_z$ acts on $|+x\rangle$, can the answer be written in the form of a complex number times $|+x\rangle$? Guess the answer, then show it with math.