

By zooming in on my plot of $A[x]$, I see that the 1st few zeros are

$$x_1 \approx -2.3$$

$$x_2 \approx -4.1$$

$$x_3 \approx -5.5$$

$$So E_1 = 2.3 \left(\frac{m \hbar^2 g^2}{2} \right)^{1/3}$$

$$E_2 = 4.1 \left(\frac{m \hbar^2 g^2}{2} \right)^{1/3}$$

$$E_3 = 5.5 \left(\frac{m \hbar^2 g^2}{2} \right)^{1/3}$$

Unit check

$$\left(\text{kg} \cdot \text{J}^2 \cdot \text{s}^2 \cdot \frac{\text{m}^2}{\text{s}^4} \right)^{1/3} = \text{J} \checkmark$$

6) see mathematica

$$\psi_1 = A \exp \left[-2.3 + \underbrace{\left(\frac{2m^2g}{\hbar^2} \right)^{1/3} z}_{\equiv z'} \right]$$

The normalization integral is

$$1 = \int_0^\infty \psi_1^2 dz$$

$$z' = k_1^{1/3} z$$

$$dz' = k_1^{1/3} dz$$

$$1 = k_1^{-1/3} \int_0^\infty \psi_1^2 dz'$$

$$1 = k_1^{-1/3} A^2 0.49 \Rightarrow A = 1.4 k_1^{1/6}$$

$$F \langle z \rangle = \int_0^\infty z \psi_1^2 dz = k_1^{-2/3} \int_0^\infty z' \psi_1^2 dz'$$

mathematica $\rightarrow 1.5 A^2$

$$= 1.5 \left(\frac{\hbar^2}{2m^2g} \right)^{1/3} \approx \text{units of length!}$$