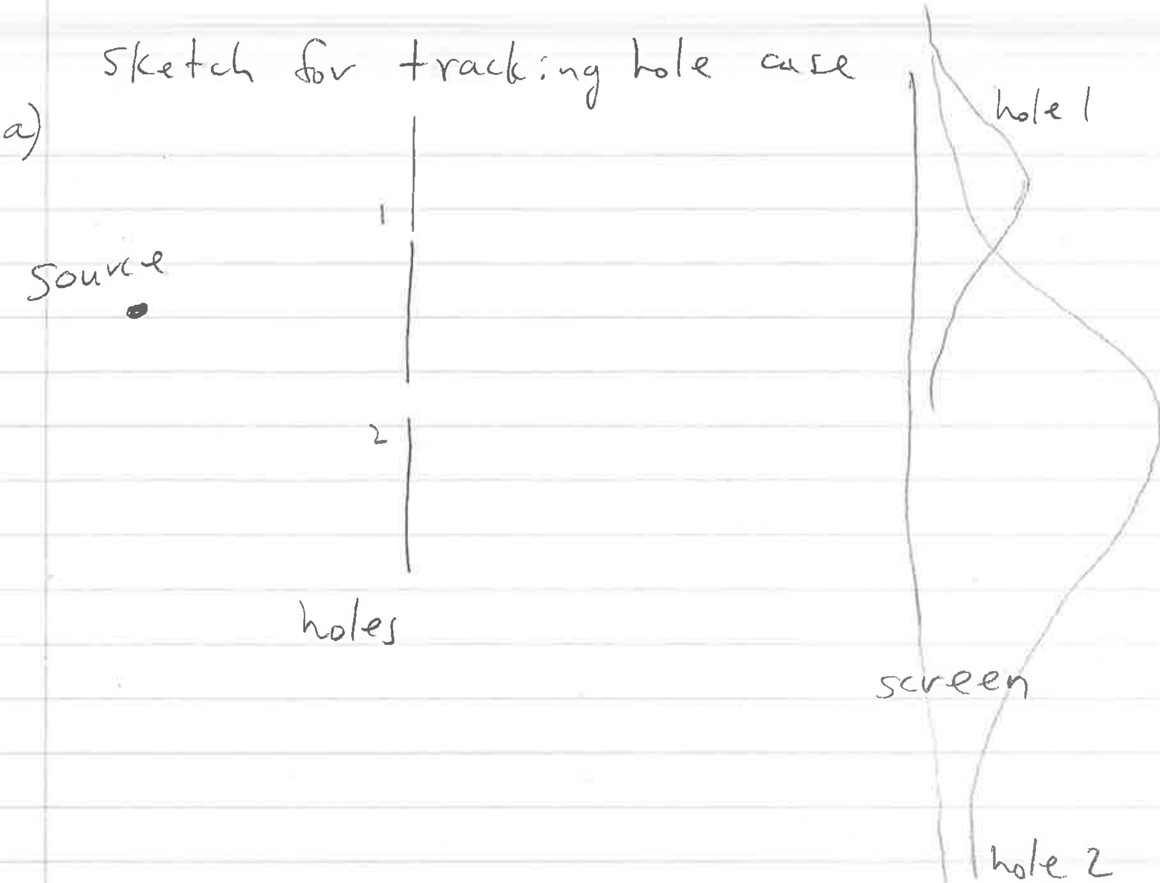
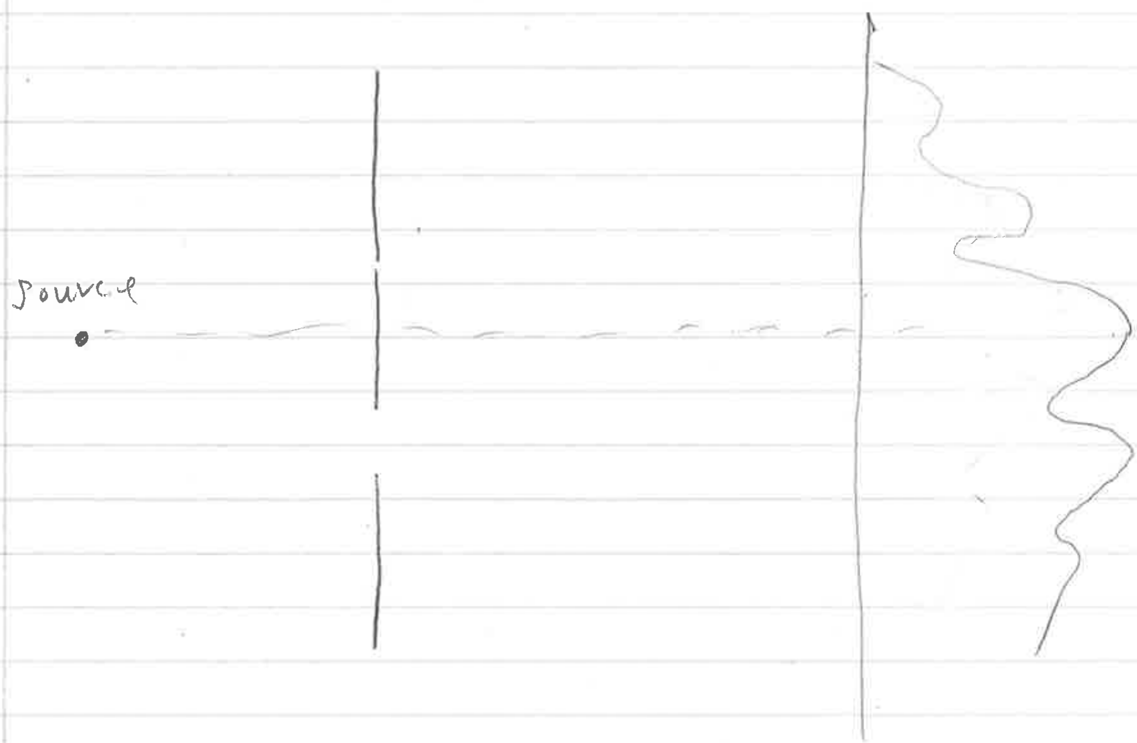
1 a)

Sketch for tracking hole case



Sketch for interference case



b) i This part is symmetric between holes 1 and 2 up to issues associated with the slightly different paths to the edges of the wider slit.

ii If the probabilities differ by a factor of 100, then the amplitudes differ by a factor of 10 in magnitude. We can write it as shown if the phase is uneffected

iii Again these trips are symmetric in a similar way to part i

iV The way that the minima arrise is by having the 2 electrons arrise 180° out of phase. They also have slightly different magnitudes as one gets further from the center. That second part is what we approximate away in both approx signs.

c) $$\frac{P_{max}}{P_{min}} = \frac{\left|\cancel{a_{A1}}\, a_{B1}\, a_{C1}(0) + \cancel{a_{A2}}\, a_{B2}\, a_{C2}(0)\right|^2}{\left|\underset{i}{\cancel{a_{A1}}}\, a_{B1}\, a_{C1}(min) + \cancel{a_{A2}}\, a_{B2}\, a_{C2}(min)\right|^2}$$

$$= \frac{\left|\cancel{a_{B1}\, a_{C1}(0)} + \overset{ii}{10}\, \cancel{a_{B1}}\, \overset{iii}{a_{C1}(0)}\right|^2}{\left|\cancel{a_{B1}}\, \underset{iv}{\cancel{a_{C1}(0)}} + \underset{ii}{10}\, \cancel{a_{B1}}\, \underset{iv}{\cancel{a_{C1}(0)}}\, e^{i\pi}\right|^2}$$

This is just -1

$$= \frac{11^2}{9^2}$$

$$= \frac{121}{81}$$ So the "Intensity varies across the pattern by ~50%

2. a) To point at the $+y$ direction, we need $\theta = \frac{\pi}{2}$, $\phi = \pi/2$.

So we have $|+n\rangle \rightarrow \cos\frac{\pi}{4}|+z\rangle + e^{i\pi/2}\sin\frac{\pi}{4}|-z\rangle$

Since $e^{i\pi/2}$ places us on the $+$ Im axis

$$= \cos\frac{\pi}{4}|+z\rangle + i\sin\frac{\pi}{4}|-z\rangle$$

$$= \frac{1}{\sqrt{2}}|+z\rangle + \frac{i}{\sqrt{2}}|-z\rangle$$

$= |y\rangle$ as given by Eq 1.30

b) $P[S_z = \hbar/2]$

$$= |\langle +z|+n\rangle|^2$$

$$= |\cos\frac{\theta}{2}\xcancel{\langle +z|+z\rangle}^{1} + e^{i\phi}\sin\frac{\theta}{2}\xcancel{\langle +z|-z\rangle}^{0}|^2$$

$$= \cos^2\frac{\theta}{2}$$

$P[S_z = -\hbar/2]$

$$= |\langle -z|+n\rangle|^2$$

$$= |\cos\theta/2\,\xcancel{\langle -z|+z\rangle}^{0} + e^{i\phi}\sin\theta/2\,\xcancel{\langle -z|-z\rangle}^{1}|^2$$

$$= (e^{i\phi}\sin\theta/2)(e^{-i\phi}\sin\theta/2)$$

$$= \sin^2\theta/2$$

The nifty check is that we must get either $\hbar/2$ or $-\hbar/2$ so

$$P[S_z = \hbar/2] + P[S_z = -\hbar/2] = 1$$

and $\cos^2\frac{\theta}{2} + \sin^2\theta/2$ indeed is 1.

c) The definition of uncertainty is Eq 1.21

$$\Delta S_z^2 = \langle S_z^2 \rangle - \langle S_z \rangle^2$$

The expectation value is the outcome times the probability summed over outcomes.

So

$$\langle S_z \rangle = \frac{\hbar}{2}\cos^2\frac{\theta}{2} + -\frac{\hbar}{2}\sin^2\frac{\theta}{2}$$

$$= \frac{\hbar}{2}\cos\theta$$

here I used $\cos 2A = \cos^2 A - \sin^2 A$

$$\langle S_z^2 \rangle = \frac{\hbar^2}{4}\cos^2\frac{\theta}{2} + \frac{\hbar^2}{4}\sin^2\theta/2$$

Note the outcome gets $\square$ed, the probability stays the same

$$= \frac{\hbar^2}{4}$$

So

$$\Delta S_z^2 = \frac{\hbar^2}{4} - \frac{\hbar^2}{4}\cos^2\theta$$

$$= \frac{\hbar^2}{4}\sin^2\theta$$

So $\Delta S_z = \frac{\hbar}{2}|\sin\theta|$

The uncertainty is 0 when $\theta = 0$, since $|+n\rangle$ is $|+z\rangle$. Hence a S_z measurement will always yield $\hbar/2$ and there will be no uncertainty. Similarly for $\theta = \pi$, where it will be $|-z\rangle$ and we'll get $-\hbar/2$, again with No uncertainty. This would also be true for any multiple of π, In between, $|+n\rangle$ will be a mix of $|+z\rangle$ and $|-z\rangle$ and we won't get a single outcome with probability 1.

3. a) <u>Amplitude to be found in $|+y\rangle$</u>

$$\langle +y|+n\rangle = \cos\frac{\theta}{2}\langle +y|+z\rangle + e^{i\phi}\sin\frac{\theta}{2}\langle +y|-z\rangle$$

From page 21 we have

$$\langle +y| = \frac{1}{\sqrt{2}}\langle +z| - \frac{i}{\sqrt{2}}\langle -z|$$

$$= \cos\frac{\theta}{2}\left(\frac{1}{\sqrt{2}}\cancel{\langle +z|+z\rangle}^{1} - \frac{i}{\sqrt{2}}\cancel{\langle -z|+z\rangle}^{0}\right)$$

$$+ e^{i\phi}\sin\frac{\theta}{2}\left(\frac{1}{\sqrt{2}}\cancel{\langle +z|-z\rangle}^{0} - \frac{i}{\sqrt{2}}\cancel{\langle -z|-z\rangle}^{1}\right)$$

$$= \frac{1}{\sqrt{2}}\cos\frac{\theta}{2} - \frac{i}{\sqrt{2}}e^{i\phi}\sin\frac{\theta}{2}$$

Now for the <u>probability</u>:

$$|\langle +y|+n\rangle|^2 = \left(\frac{1}{\sqrt{2}}\cos\frac{\theta}{2} - \frac{i}{\sqrt{2}}e^{i\phi}\sin\frac{\theta}{2}\right)\left(\frac{1}{\sqrt{2}}\cos\frac{\theta}{2} + \frac{i}{\sqrt{2}}e^{-i\phi}\sin\frac{\theta}{2}\right)$$

$$= \frac{1}{2}\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} - i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\left(e^{i\phi} - e^{-i\phi}\right)\right)$$

$\to 1$

Almost Euler's identity

$$e^{iy} = \cos y + i\sin y$$

$$-e^{-iy} = -\cos y - i\sin(-y)$$

$$e^{iy} - e^{-iy} = 0 + 2i\sin y$$

$$= \frac{1}{2}\left(1 + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\phi\right)$$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$= \frac{1}{2}(1 + \sin\theta\sin\phi)$$

b) Check: If $\theta = \frac{\pi}{2}$ and $\phi = \frac{\pi}{2}$, $|+n\rangle \rightarrow |+y\rangle$ and our answer should be 1. Lets see if it works...

$$|\langle +y|+n\rangle|^2 \rightarrow \frac{1}{2}\left(1 + \sin\frac{\pi}{2}\sin\frac{\pi}{2}\right)$$

$$= \frac{1}{2}(1+1)$$

$$= 1 \checkmark$$

c) So here we need to compute $\langle +n|+y\rangle$ and $|\langle +n|+y\rangle|^2$

The way to be tricky is to note that $\langle +n|+y\rangle = \langle +y|+n\rangle^*$ so we just need to complex conjugate the thing we got in part a, at $*$.

So $\langle +n|+y\rangle = \frac{1}{\sqrt{2}}\cos\frac{\theta}{2} + \frac{i}{\sqrt{2}} e^{-i\phi}\sin\frac{\theta}{2}$

Now when it comes to the probability we need to multiply this by its complex conjugate, which is exactly what we did to find $|\langle +y|+n\rangle|^2$ in a. So the result for $|\langle +n|+y\rangle|^2$ will be identical.

$$|\langle +n|+y\rangle|^2 = |\langle +y|+n\rangle|^2 = \frac{1}{2}(1+\sin\theta\sin\phi)$$

d) We expect $\langle +n|-n\rangle = 0$ (orthogonal)

Start by complex conjugating $|+n\rangle$ to get

$\langle +n| = \cos\frac{\theta}{2}\langle +z| + e^{-i\phi}\sin\frac{\theta}{2}\langle -z|$

$\langle +n|-n\rangle = \sin\frac{\theta}{2}\cos\frac{\theta}{2}\langle +z|+z\rangle - e^{i\phi}e^{-i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\langle -z|-z\rangle$

(dropping $\langle +z|-z\rangle = 0$ terms)

$$= \sin\frac{\theta}{2}\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$= 0$$

Now for normalized. We expect $\langle -n | -n \rangle = 1$
Start by C-C ing $|-n\rangle$

$$\langle -n| = \sin\frac{\theta}{2} \langle +z| - e^{-i\phi} \cos\frac{\theta}{2} \langle -z|$$

$$\langle -n | -n \rangle = \sin^2\frac{\theta}{2} \langle +z | +z \rangle + e^{i\phi} e^{-i\phi} \cos^2\frac{\theta}{2} \langle -z | -z \rangle$$
$+ 0$ for $\langle +z | -z \rangle$ terms

$$= \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}$$

$$= 1 \quad \checkmark$$

4. a) Lets call the probability to make it out of SG_n P_2

$$P_2 = |\langle +n | +z \rangle|^2 = \cos^2 \frac{\theta}{2}$$

↑ computed back in 2b.

The particles coming out of SG_n are in the $|+n\rangle$ state.

Lets call the probability to make it out of the last SG_z P_3

$$P_3 = |\langle -z | +n \rangle|^2 = \sin^2 \frac{\theta}{2}$$

↑ computed back in 2b

These are successive events where we have looked at the answer so we multiply the probabilities

$$P_2 \cdot P_3 = \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2} = \frac{1}{4} \sin^2 \theta$$

$$\sin\theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

b) Our probability in (a) is maximized for $\theta = \pi/2$, in which case the fraction of transmitted particles is $1/4$

c) Zero particles are transmitted from the final SG is the middle SG is removed. The particles entering the final SG would be in the $|+z\rangle$ state and would have zero amplitude to be in the $|-z\rangle$ state.

d) Fay's change has no effect.

The state of the particles entering the final SG would be

$$e^{i\delta}|+n\rangle = e^{i\delta}\cos\frac{\theta}{2}|z\rangle + e^{i\delta}e^{i\phi}\sin\frac{\theta}{2}|-z\rangle$$

The probability to make it through the last SG, the new P_3' will be

$$P_3' = |\langle -z| e^{i\delta} |+n\rangle|^2$$

$$= e^{-i\delta}e^{i\delta}|\langle -z|+n\rangle|^2$$

$$= |\langle -z|+n\rangle|^2$$

An overall phase has no effect.

5. As Iona suggests lets start with the generic form of a state

$$|\psi\rangle = a_1|+z\rangle + a_2|-z\rangle$$

Iona says

$$|\langle +z|\psi\rangle|^2 = 0.36$$

$\Rightarrow |a_1|^2 = 0.36$ so $a_1 = 0.6\, e^{i\delta_1}$
(See a_1 has a magnitude and a phase, 2 pieces of info. This condition fixes just one of them)

$|\langle -z|\psi\rangle|^2 = 0.64$ since they must add to 100%

$$\Rightarrow |a_2|^2 = 0.64 \Rightarrow a_2 = 0.8\, e^{i\delta_2}$$

$$|\langle +x|\psi\rangle|^2 = 0.5$$

$$\Rightarrow \left|\left(\tfrac{1}{\sqrt{2}}\langle +z| + \tfrac{1}{\sqrt{2}}\langle -z|\right)\left(a_1|+z\rangle + a_2|-z\rangle\right)\right|^2 = 0.5$$

$$\left|\tfrac{1}{\sqrt{2}}a_1 + \tfrac{1}{\sqrt{2}}a_2\right|^2 = 0.5$$

$$\tfrac{1}{2}\left(a_1 a_1^* + a_2 a_2^* + a_1 a_2^* + a_1^* a_2\right) = 0.5$$

$$\cancel{0.36 + 0.64} + 0.48\left(e^{i\delta_1}e^{-i\delta_2} + e^{-i\delta_1}e^{i\delta_2}\right) = \cancel{1}$$

$$e^{i(\delta_1-\delta_2)} + e^{-i(\delta_1-\delta_2)} = 0$$

Again Euler

$$2\cos(\delta_1 - \delta_2) = 0$$

So $\delta_1 - \delta_2 = \frac{\pi}{2}$ or actually $(n+\frac{1}{2})\pi$ for integer n

So our state is

$$|\psi\rangle = 0.6\, e^{i\delta_1} |+z\rangle + 0.8\, e^{i\delta_2} |-z\rangle$$

That's 2 unknown parameters, but we can eliminate one of them using the last thing we learned above. There are a lot of ways to do that but I'll write it as follows:

$$|\psi\rangle = e^{i\delta_2}\left(0.6\, e^{i(\delta_1-\delta_2)} |+z\rangle + 0.8 |-z\rangle\right)$$

$$= e^{i\delta_2}\left(0.6\, e^{i(n+\frac{1}{2}\pi)} |+z\rangle + 0.8 |-z\rangle\right)$$

↑

for odd n, this is $-i$
for even n, this is 1

$$= e^{i\delta_2}\left(\frac{3i}{5}(-1)^n |+z\rangle + \frac{4}{5} |-z\rangle\right)$$

We also know that
$|\langle -x|\psi\rangle|^2 = 0.5$ this does not provide more information, but it is a nice check

$$\langle -x| = \frac{1}{\sqrt{2}}\langle +z| - \frac{1}{\sqrt{2}}\langle -z|$$

$$|\langle -x|\psi\rangle|^2 = \left|\frac{3i}{\sqrt{2}\,5}(-1)^n - \frac{4}{\sqrt{2}\,5}\right|^2$$

$$= \frac{1}{2}\left(\frac{9}{25} + \frac{16}{25}\right)$$

$$= \frac{1}{2} \checkmark$$