Q1 $\frac{d^2u}{d\rho^2} - \frac{u}{4} = 0$ The ρ independent part dominates over $\rho^{-1} + \rho^{-2}$ at large ρ

Q2 $\frac{du}{d\rho} = -\frac{A}{2} e^{-\rho/2} + \frac{B}{2} e^{\rho/2}$

$\frac{d^2u}{d\rho^2} = \frac{A}{4} e^{-\rho/2} + \frac{B}{4} e^{\rho/2} = \frac{1}{4} u$ ✓

Q3 It blows up at $\rho \to \infty$ and is hence not normalizable.

Q4 $\frac{du}{d\rho} = C(\ell+1)\rho^{\ell} - D\ell\rho^{-(\ell+1)}$

$\frac{d^2u}{d\rho^2} = C\ell(\ell+1)\rho^{\ell-1} + D\ell(\ell+1)\rho^{-(\ell+2)}$

Now consider

$\frac{\ell(\ell+1)u}{\rho^2} = C\ell(\ell+1)\rho^{\ell-1} + D\ell(\ell+1)\rho^{-(\ell+2)}$

match!

Q5 $\underbrace{\frac{d^2F}{d\rho^2}}_{\text{Term 1}} = \frac{d}{d\rho}\sum_{k=0}^{\infty} C_k k \rho^{k-1} = \sum_{k=0}^{\infty} C_k k(k-1)\rho^{k-2}$ can start @ $k=0$

$\underbrace{\frac{dF}{d\rho}\left(\frac{2\ell+2}{\rho}\right)}_{\text{Term 2}} = \sum_{k=0} C_k k \rho^{k-1}\left(\frac{2\ell+2}{\rho}\right) = \sum_{k=1} C_k k(2\ell+2)\rho^{k-2}$

Term 3 will be all ρ^{k-1} stuff, one from $dF/d\rho$, the others from F/ρ.

Q6 Since stuff involving ρ^2 can't cancel stuff involving ρ, the stuff in the { } must be zero.

So $\quad k(k+1) + (2\ell+2)(k+1)\,C_{k+1} = -[-k+\lambda-(\ell+1)]\,C_k$

So $\quad C_{k+1} = \dfrac{k+\ell+1-\lambda}{(k+1)(k+2\ell+2)}\,C_k$

Q7 When $k = n_r$ the numerator will be zero so C_{k+1} will be zero allong w/ all higher C_k.

Q8 The need for a normalizable series sol'n.

Q9 Looking at the plots of the radial wave f'ns $r^2|R_{2,1}|^2$ peaks at $\approx 4a_0$. So that's $n=2$, $\ell=1$

So $E_2 = \dfrac{\mu_0 Z^2 e^4}{2\hbar^2 \cdot 4}$. To get m, we need to look

at the angular wave f'ns for the $\ell=1$ case. We see that it's $m=0$ that is never at $\theta = \pi/2$.