

From "GRE: Practicing to take the physics test" 3rd edition (ETS: 1997)

1.

Eigenfunctions for a rigid dumbbell rotating about its center have a ϕ dependence of the form $\psi(\phi) = Ae^{im\phi}$, where m is a quantum number and A is a constant. Which of the following values of A will properly normalize the eigenfunction?

- (A) $\sqrt{2\pi}$
- (B) 2π
- (C) $(2\pi)^2$
- (D) $\frac{1}{\sqrt{2\pi}}$
- (E) $\frac{1}{2\pi}$

2.

Which of the following is an eigenfunction of the linear momentum operator $-i\hbar \frac{\partial}{\partial x}$ with a positive eigenvalue $\hbar k$; i.e., an eigenfunction that describes a particle that is moving in free space in the direction of positive x with a precise value of linear momentum?

- (A) $\cos kx$ (B) $\sin kx$ (C) e^{-ikx}
- (D) e^{ikx} (E) e^{-kx}

3.

A system containing two identical particles is described by a wave function of the form

$$\psi = \frac{1}{\sqrt{2}} [\psi_{\alpha}(x_1) \psi_{\beta}(x_2) + \psi_{\beta}(x_1) \psi_{\alpha}(x_2)]$$

where x_1 and x_2 represent the spatial coordinates of the particles and α and β represent all the quantum numbers, including spin, of the states that they occupy. The particles might be

- (A) electrons
- (B) positrons
- (C) protons
- (D) neutrons
- (E) deuterons

4.

The wave function of a particle is $e^{i(kx - \omega t)}$, where x is distance, t is time, and k and ω are positive real numbers. The x -component of the momentum of the particle is

- (A) 0
- (B) $\hbar\omega$
- (C) $\hbar k$
- (D) $\frac{\hbar\omega}{c}$
- (E) $\frac{\hbar k}{\omega}$

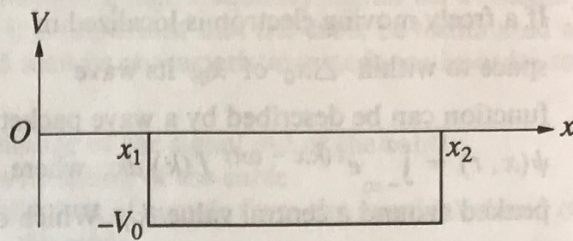
5.

A system is known to be in the normalized state described by the wave function

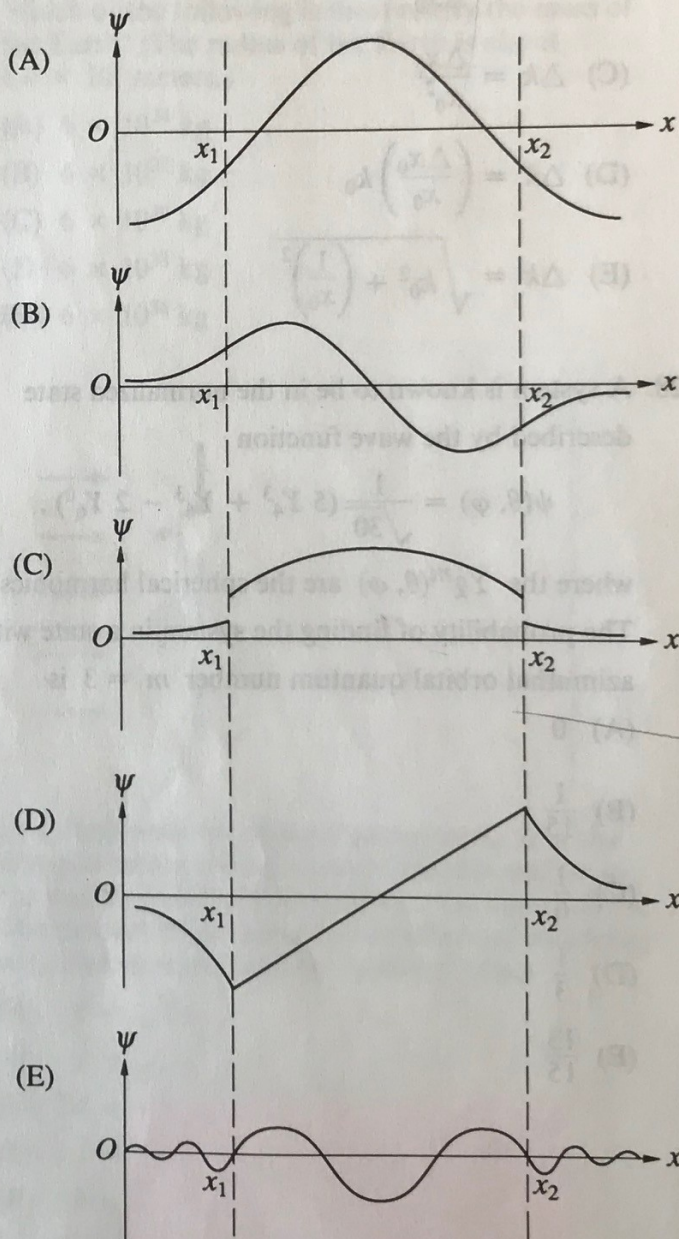
$$\psi(\theta, \phi) = \frac{1}{\sqrt{30}} (5 Y_4^3 + Y_6^3 - 2 Y_6^0),$$

where the $Y_{\ell}^m(\theta, \phi)$ are the spherical harmonics. The probability of finding the system in a state with azimuthal orbital quantum number $m = 3$ is

- (A) 0
- (B) $\frac{1}{15}$
- (C) $\frac{1}{6}$
- (D) $\frac{1}{3}$
- (E) $\frac{13}{15}$



An attractive, one-dimensional square well has depth V_0 as shown above. Which of the following best shows a possible wave function for a bound state?



Questions 51-53

A particle of mass m is confined to an infinitely deep square-well potential:

$$V(x) = \infty, x \leq 0, x \geq a$$

$$V(x) = 0, 0 < x < a.$$

The normalized eigenfunctions, labeled by the quantum number n , are $\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$.

51. For any state n , the expectation value of the momentum of the particle is

- (A) 0
- (B) $\frac{\hbar n\pi}{a}$
- (C) $\frac{2\hbar n\pi}{a}$
- (D) $\frac{\hbar n\pi}{a} (\cos n\pi - 1)$
- (E) $\frac{-i\hbar n\pi}{a} (\cos n\pi - 1)$

52. The eigenfunctions satisfy the condition

$$\int_0^a \psi_n^*(x) \psi_\ell(x) dx = \delta_{n\ell}, \delta_{n\ell} = 1 \text{ if } n = \ell, \text{ otherwise } \delta_{n\ell} = 0.$$

This is a statement that the eigenfunctions are

- (A) solutions to the Schrödinger equation
- (B) orthonormal
- (C) bounded
- (D) linearly dependent
- (E) symmetric

53. A measurement of energy E will always satisfy which of the following relationships?

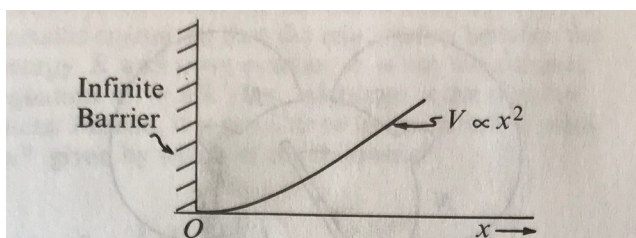
- (A) $E \leq \frac{\pi^2 \hbar^2}{8ma^2}$
- (B) $E \geq \frac{\pi^2 \hbar^2}{2ma^2}$
- (C) $E = \frac{\pi^2 \hbar^2}{8ma^2}$
- (D) $E = \frac{n^2 \pi^2 \hbar^2}{8ma^2}$
- (E) $E = \frac{\pi^2 \hbar^2}{2ma^2}$

10

The configuration of three electrons $1s^2 2p^3$ has which of the following as the value of its maximum possible total angular momentum quantum number?

- (A) $\frac{7}{2}$
- (B) 3
- (C) $\frac{5}{2}$
- (D) 2
- (E) $\frac{3}{2}$

11



The energy levels for the one-dimensional harmonic oscillator are $h\nu\left(n + \frac{1}{2}\right)$, $n = 0, 1, 2, \dots$. How will the energy levels for the potential shown in the graph above differ from those for the harmonic oscillator?

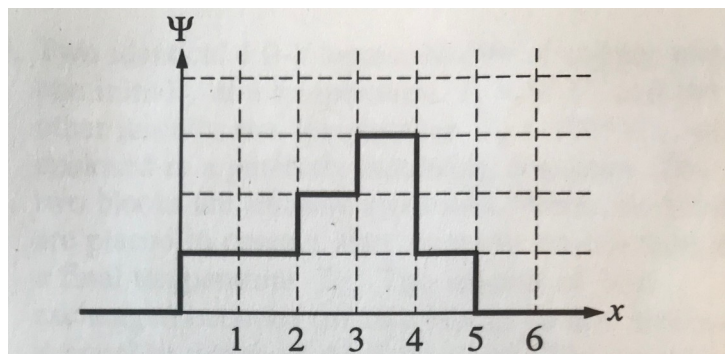
- (A) The term $\frac{1}{2}$ will be changed to $\frac{3}{2}$.
- (B) The energy of each level will be doubled.
- (C) The energy of each level will be halved.
- (D) Only those for even values of n will be present.
- (E) Only those for odd values of n will be present.

12

A diatomic molecule is initially in the state $\Psi(\Theta, \Phi) = (5Y_1^1 + 3Y_5^1 + 2Y_5^{-1})/(38)^{1/2}$, where Y_l^m is a spherical harmonic. If measurements are made of the total angular momentum quantum number l and of the azimuthal angular momentum quantum number m , what is the probability of obtaining the result $l = 5$?

- (A) $36/1444$
- (B) $9/38$
- (C) $13/38$
- (D) $5/(38)^{1/2}$
- (E) $34/38$

13



The wave function for a particle constrained to move in one dimension is shown in the graph above ($\Psi = 0$ for $x \leq 0$ and $x \geq 5$). What is the probability that the particle would be found between $x = 2$ and $x = 4$?

- (A) $17/64$
- (B) $25/64$
- (C) $5/8$
- (D) $\sqrt{5/8}$
- (E) $13/16$