Phun with QM

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why QM?

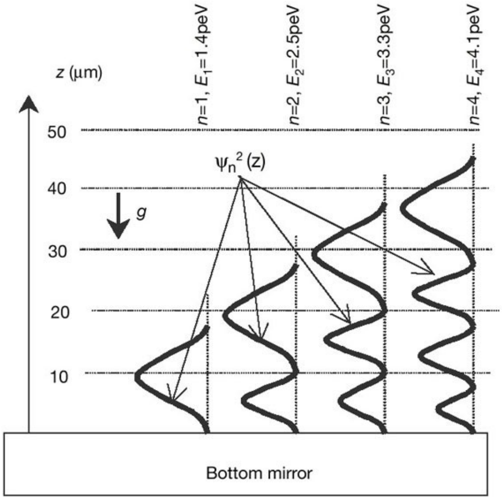
- laser
- the transistor (and thus the microchip)
- the electron microscope
- magnetic resonance imaging
- superconductivity
- superfluidity
- much of chemistry
- quantum computation
- squeezed light in the LIGO experiment
- ...

not just atoms, weird atoms

- antihydrogen
- muonic hydrogen
- positronium
- mononium
- ...

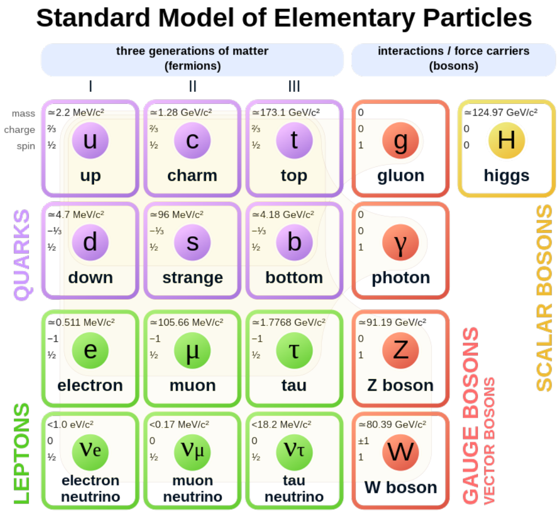
not just weird atoms, weird bound states

gravitational bound states of the neutron



nature

Neutrino Oscillations

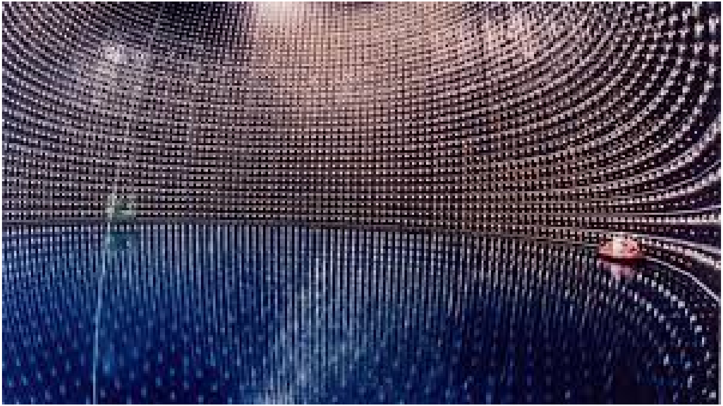


wikipedia

Neutrino Oscillations

- originally thought to be massless (travel at speed of light)
- participate in reactions involving the weak nuclear interaction
- sun involves lots of nuclear reactions
- 1960s plan: measure solar neutrinos, test solar models
- result: $1/3$ of expected number of neutrinos seen
- interpretation? experiment wrong, solar model wrong, **Standard Model of Particle physics wrong**
- current highly favored solution: neutrinos with mass, through the magic of QM cause neutrinos to change flavor as they travel to us
- in other words ν_e is produced, but some ν_μ arrives
- in still other words, if Sun makes a ν_e there is an amplitude and hence a probability for a ν_μ to arrive

Super Kamiokande



symmetry magazine

magic of QM (+ lin-al review)

- simplified 2 neutrino model
- neutrinos produced in interactions (ν_e, ν_μ) do not have well defined masses
- neutrinos with well defined masses (ν_{m1}, ν_{m2}) do not interact in well defined ways
- set up some linear algebra to capture this feature:
  - use a 2 dimensional complex vector space
  - let vectors representing the states of definite mass $(|\nu_{m1}\rangle, |\nu_{m2}\rangle)$ form an orthonormal basis
  - if these states are an orthonormal basis, what is true of them?

magic of QM

Let states of definite interaction $(|\nu_e\rangle, |\nu_\mu\rangle)$ be a "rotated" basis for the same space

$$
\begin{aligned}
|\nu_e\rangle &= \cos\theta\,|\nu_{m1}\rangle - \sin\theta\,|\nu_{m2}\rangle && (1)\\
|\nu_\mu\rangle &= \sin\theta\,|\nu_{m1}\rangle + \cos\theta\,|\nu_{m2}\rangle && (2)
\end{aligned}
$$

or

$$
\begin{aligned}
|\nu_{m1}\rangle &= \cos\theta\,|\nu_e\rangle + \sin\theta\,|\nu_\mu\rangle && (3)\\
|\nu_{m2}\rangle &= -\sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\mu\rangle && (4)
\end{aligned}
$$

What is the probability of finding an electron neutrino with mass $m1$?
You could also check that $(|\nu_e\rangle, |\nu_\mu\rangle)$ form an orthonormal basis for the same space.

magic of QM

States of well-defined mass evolve as they travel through space in a well defined way (that we'll later learn how to calculate)

$$|\nu_{m1}(L)\rangle = \exp\left(\frac{-im1^2L}{2E}\right)|\nu_{m1}(0)\rangle$$

Evaluate $|\langle\nu_{m1}(0)|\nu_{m1}(L)\rangle|^2$ what does the result mean?

magic of QM

Now suppose that an electron neutrino is created and propagates a distance L. Write down the sate of this particle when it's at position L. Let's just call this state $|\nu(L)\rangle$.

Then calculate the probability that $|\nu(L)\rangle$ is still an electron neutrino.

[The answer can be written $1 - \sin^2 2\theta \sin^2\left(\frac{(m1^2 - m2^2)L}{4E}\right)$ see if you can get it]

One more exercise...

Here is another exercise with complex numbers that I couldn't work into the above context...

Write $5e^{i\pi}$ in the form $a + ib$ for some a and b. Draw a picture to go with your answer.