Name: _____

PHYS 335 HW 7

Due 3/4 at 5:00PM

The usual instructions

0. (2 pts) Estimate the time it took to do this problem set. Include any comments about the homework that you'd like to share with Jay.

1. *Eigenlore of lower eigen...*

Townsend 7.1. It should go pretty quick. If you find yourself thinking you need to do length contortions, talk to someone first.

2. *What would you call your friend Erin's birthday party?*

(a) A particle of mass m in the 1D harmonic oscillator is in a state for which a measurement of the energy yields $\hbar\omega/2$ or $3\hbar\omega/2$, each with 50% probability. I claim that the most general state (without adding an irrelevant overall phase) that gives the right probabilities is

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\delta}}{\sqrt{2}}|1\rangle\,. \tag{1}$$

Explain why this is the most general state. Address questions like the following. Does it give the right probabilities? Does the exponential factor need to go on the second term? etc.

(b) Suppose that the expectation value of momentum for the above state is $\sqrt{m\omega\hbar/2}$. This places a constraint on δ. What is it? Write the state that has this property without any exponentials. When you work out the expectation value, writing the momentum operator in terms of raising and lowering operators is helpful

(c) What is the state of the system after a time t? What is the expectation value of the momentum at time t? If you are clever about how handle the time evolution, you may not need to work out the new expectation value from scratch.

(d) This problem provides a nifty chance to explore Ehrenfest's theorem. Use Eq. 6.33 to find $\langle x\rangle$ for some choice (you pick) of initial conditions. Then check that your results satisfy Eq. 6.34 ... the quantum Newton's second law-ish thing.

3. *What did the parrot say when it went on a diet?*

(a) Have a look at Eq. 7.45. Write some words about where the bits of this equation come from. Then turn it into dimensionless form. Since the dimensionful wave function has units of $1/\sqrt{\text{length}}$, we can define a dimensionless wave function $\psi = \psi\sqrt{x_c}$. When you have the equation in dimensionless form, the only variables it should contain are $\psi, \mathcal{X}, n$, where again curly x, $\mathcal{X}$, is the dimensionless position. Once you have your dimensionless equation, use it to find find ψ_3. No Mathematica allowed.

(b) Construct ψ_3 in terms of a single unknown constant, a_1 using the recursion formula. You could then normalize to find the constant, but you don't have to. Up to identifying this constant, it should match your answer to part (a).

(c) Look up the Hermite polynomial (poly-no-meals) H_3 and identify it in your work.

4. *The quantized version of doofenshmirtz*

(a) Consider a pair of particles of equal mass a distance r_0 apart rotating around their common center of mass. Use you knowledge from classical physics to argue that the kinetic energy for the system can be written

$$T = \frac{L^2}{2I}, \tag{2}$$

where L is the magnitude of the angular momentum of the system and I is the total moment of inertia of the system. If the system is free, this is also the hamiltonian, and if we want to make it quantum, we can put a hat on the L.

(b) If an oxygen molecule (so the dumbbell above is made out of 2 oxygen atoms, mass 16 amu) absorbs a photon of frequency 1×10^{11}Hz and makes a purely rotational transition from the $l = 0$ to the $l = 1$ state, what is the spacing between the atoms? Note that the eigenstates of L^2 are also eigenstates of the hamiltonian, find the relevant energy eigenvalues and ... Also note that 1 amu$= 1.67 \times 10^{-24}$ g.

5. Nothing to submit with the problem set, but your project is due by 5pm on Sunday March 6. I will email you a number under which to submit your project (so that it can be anonymously peer reviewed). Submit your project to Jay via email with "335project" (exactly like that without the quotes) in the subject line. Submit your project as a single pdf attached to your email. Name your pdf with the number I give you followed by a short, descriptive name for your topic. If my assigned number was 57 and my project was to find the energy levels of the gravitational potential, as we did in HW 6, then I would name my pdf "57_gravity_levels.pdf". Put this same name at the top of the first page of your work. Do not include your name anywhere in your submitted pdf. If a failure to follow these instructions results in significant administrative work for Jay, he may bill you for his time, report you to the project-eating dragon, assign extra homework problems, or dream up some other unpleasant consequence at his discretion.

Academic Honesty Statement

___I did not receive help on this problem set. I did all the work on my own.
___I received on this problem set from:
- ___Jay on problems _____
- ___Student Assistants on problems _____
- ___Other Students (please name _____) on problems _____
- ___Other Instructor (please name _____) on problems _____
- ___A resource other than our text (please name _____)
  on problems _____

By way of acknowledgements, several of these problems were adapted from Townsend.