NUMERICAL CALIBRATION FOR 3-AXIS ACCELEROMETERS AND MAGNETOMETERS

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ABSTRACT

Magnetometers and accelerometers are sensors that are now integrated in objects of everyday life like automotive applications, mobile phones and so on. Some applications need information of acceleration and attitude with a high accuracy. For example, MEMS magnetometers and accelerometers can be integrated in embedded like mobile phones and GPS receivers. The parameters of such sensors must be precisely estimated to avoid drift and biased values. Thus, calibration is an important step to correctly use these sensors and get the expected measurements. This paper presents the theoretical and experimental steps of a method to compute gains, bias and non orthogonality factors of magnetometer and accelerometer sensors. This method of calibration can be used for automatic calibration in embedded systems. The calibration procedure involves arbitrary rotations of the sensors platform and a visual 2D projection of measurements.

***Index Terms*—** sensor, magnetometer, accelerometer, calibration.

1.INTRODUCTION

In the context of a pedestrian geopositioning project, a GPS receiver is combined to a MEMS based inertial unit in order to improve the precision. The inertial unit, composed of magnetometers, accelerometers and gyrometers, provides information about heading and speed of the pedestrian, by using statistical filtering. The development of such a system requires the calibration of sensors, which is a challenging task [1,2,3,4,5] because the voltage output depends on :

- the local magnetic field or acceleration
- the sensor attitude
- the real gain and bias parameters

In a 3D sensor each axis i has a different gain noted as α_i and a voltage bias noted as β_i .

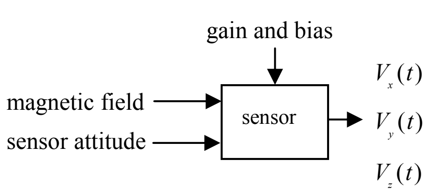

Fig 1 – Representation of sensor

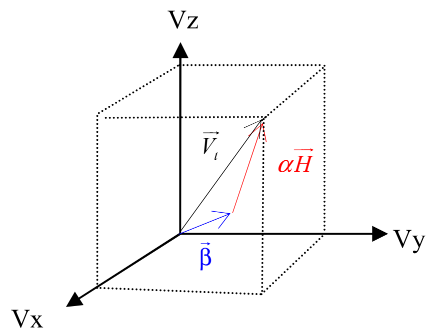

Fig 2 – Magnetic or acceleration vector $\vec{H}$ measured as $\vec{V}(t)$

Fig.1 represents a 3D sensor with offset $\vec{\beta} = (\beta_x, \beta_y, \beta_z)^T$ and gains $\alpha_x, \alpha_y, \alpha_z$. For a magnetometer, each axis measures the projection of the earth's magnetic field $\vec{H}$ on itself. The model of the sensor output is given by (see Fig.2):

$$\begin{pmatrix} V_x(t) \\ V_y(t) \\ V_z(t) \end{pmatrix} = \begin{pmatrix} \alpha_x & 0 & 0 \\ 0 & \alpha_y & 0 \\ 0 & 0 & \alpha_z \end{pmatrix} \begin{pmatrix} h_x(t) \\ h_y(t) \\ h_z(t) \end{pmatrix} + \begin{pmatrix} \beta_x \\ \beta_y \\ \beta_z \end{pmatrix} + \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{pmatrix} \quad (1)$$

With : $V(t)$ in volts, α : gain volt/gauss, $h\ (t)$ in gauss, β : offset in volts, $\varepsilon_x \simeq 0$: residual error (noise)

2. CALIBRATION OF 3D SENSOR

Characterizing this type of sensor is estimating the gain and bias. The norm of input vector $\vec{H} = \left[h_x(t) h_y(t) h_z(t) \right]^T$ is constant. Using this fact and equation (1), we obtain:

$$\forall t \;\; H^2 = h_x^2(t) + h_y^2(t) + h_z^2(t)$$

$$H^2 = \left(\frac{V_x(t) - \beta_x}{\alpha_x} \right)^2 + \left(\frac{V_y(t) - \beta_y}{\alpha_y} \right)^2 + \left(\frac{V_z(t) - \beta_z}{\alpha_z} \right)^2 + e_p(t) \quad (2)$$

Equation (2) is the parametric equation of an ellipsoid [4] with center $\vec{\beta}$ and semi-axes $\alpha_x, \alpha_y, \alpha_z$. Considering the system of equations formed by the various measurements at times *t,* we can estimate the parameters by a standard non-linear least squares minimization of $e_p(t)$.

3. CALIBRATION PROCEDURE

3.1. Acquisition of a set of points

The calibration procedure must be described. We need a set of measurements with various attitudes, so that the ellipsoid in equation (2) can be estimated. In the case of a magnetometer, the magnitude of the local magnetic field must be precisely known. In an undisturbed environment, it equals the magnitude of the Earth magnetic field (in our case $\|\vec{H}\| = 0.47\text{gauss}$).

The calibration procedure stops when the measurements cover most of the surface of an ellipsoid [6,7]. The minimum procedure is to rotate the sensor in the reference XYZ coordinate system to measure a set of points which describes a closed solid:

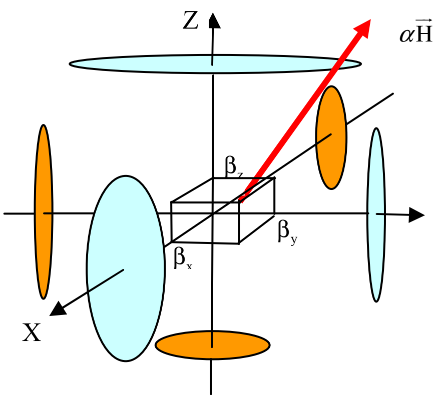

Fig 3 – Minimum calibration procedure of 3D sensor

3.2. 2D projection of measurements

The calibration can be facilitated by a real-time display of measured magnetic fields in spherical coordinates $r(t), \varphi(t), \theta(t)$ (figure 4). This transformation requires a real-time estimation of gain and bias that is obtained from the first measured points :

$$r(t) = \sqrt{h_x(t)^2 + h_y(t)^2 + h_z(t)^2}$$

$$\theta(t) = \arctan\left(\frac{\sqrt{h_x(t)^2 + h_y(t)^2}}{h_z(t)}\right) \quad \varphi(t) = \arctan\left(\frac{h_y(t)}{h_x(t)}\right) \quad (3)$$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2} \text{ and } -\pi < \varphi < \pi$$

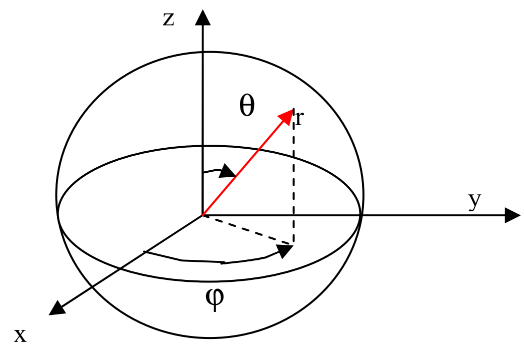

Fig 4 – Spherical coordinates

Each measurement t is represented by a point with coordinates $\varphi(t)$ and $\theta(t)$. In Fig 5, we can see areas without measurements, the projection of data points help us to get a uniform set of measurements:

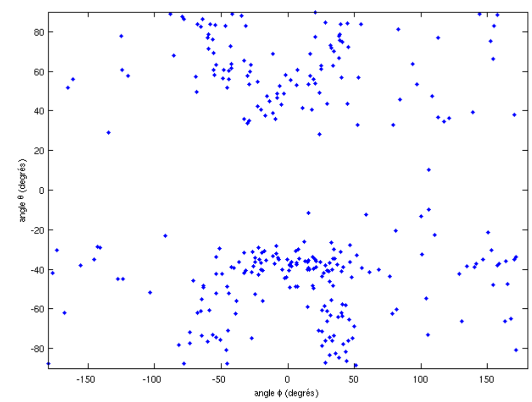

Fig 5 – Measurements of magnetic field in 2D projection

4. ESTIMATION OF PARAMETERS α_i, β_i

This section describes the estimation of α_i, β_i parameters from a set of measurements. The accuracy of the numerical result depends on the uniform repartition of measurements over the ellipsoid, which is visible on the 2D projection (figure 5).

Let $p = \left[\alpha'_x \alpha'_y \alpha'_z \beta_x \beta_y \beta_z\right]^t$ denote the vector of parameters to estimate, where $\alpha'_x = \left(\frac{1}{\alpha_x}\right), \alpha'_y = \left(\frac{1}{\alpha_y}\right), \alpha'_z = \left(\frac{1}{\alpha_z}\right)$

The problem can be formulated as the minimization of

$S(p) = \sum_t (e_p(t))^2$ with respect to parameter p where $e_p(t)$ is defined from equation 2 :

$$e_p(t) = \alpha_x'^2 (V_x(t) - \beta_x)^2 + \alpha_y'^2 (V_y(t) - \beta_y)^2 + \alpha_z'^2 (V_z(t) - \beta_z)^2 - H^2$$

The cost function $S(p)$ is quadratic, and is minimized iteratively by the Levenberg-Marquardt algorithm (LMA) [8], a numerical method based on the linearization of $e_p(t)$.

Starting with initial guess p_0 obtained from the specifications of sensor, we update the estimation at step $k+1$ by considering the substitution $q = p_{k+1} - p_k$

The linearization of $e_{p_{k+1}}(t)$ can be written using its *Jacobian* $J_t(p_k)$ at p_k (q is the variable)

$$e_{p_{k+1}}(t) = e_{p_k+q}(t) = e_{p_k}(t) + J_t(p_k)q \quad (4)$$

With $J_t(p_k) = \left[\frac{\partial e_{p_k}(t)}{\partial \alpha_x^{'}} \ \frac{\partial e_{p_k}(t)}{\partial \alpha_y^{'}} \cdots \frac{\partial e_{p_k}(t)}{\partial \beta_z} \right]$ (5)

Which leads to
$$J_t(p_k) = \begin{bmatrix} 2\alpha_x^{'}(V_x(t) - \beta_x)^2 \\ 2\alpha_y^{'}(V_y(t) - \beta_y)^2 \\ 2\alpha_z^{'}(V_z(t) - \beta_z)^2 \\ -2\alpha_x^{'}(V_x(t) - \beta_x)^2 \\ -2\alpha_y^{'}(V_y(t) - \beta_y)^2 \\ -2\alpha_z^{'}(V_z(t) - \beta_z)^2 \end{bmatrix}^t$$

Let $\overline{e}_{p_{k+1}} = \begin{bmatrix} e_{p_{k+1}}(1) \\ e_{p_{k+1}}(2) \\ \cdots \\ e_{p_{k+1}}(N) \end{bmatrix}$ and $J_{p_k} = \begin{bmatrix} J_1(p_k) \\ J_2(p_k) \\ \cdots \\ J_N(p_k) \end{bmatrix}$

Thus $\overline{e}_{p_{k+1}} = \overline{e}_{p_k} + J_{p_k} q$

Cost function $S_{p_{k+1}}$ can be written from the linearized form (4) :

$$S_{p_{k+1}} = \overline{e}_{p_{k+1}}^{\ t}.\overline{e}_{p_{k+1}} = \overline{e}_{p_k}^{\ t}.\overline{e}_{p_k} + \overline{e}_{p_k}^{\ t} J_{p_k} q + q^t J_{p_k}^t \overline{e}_{p_k} + q^t J_{p_k}^t J_{p_k} q$$

$S_{p_{k+1}}$ is minimized when $\frac{\partial S_{p_{k+1}}(t)}{\partial q} = 0 \Leftrightarrow J_{pk}^T J_{pk} q = J_{pk} \overline{e}_{pk}$

The Levenberg-Marquardt algorithm adds a damping factor λ to the equation: $(J_{pk}^T J_{pk} + \lambda) q = J_{pk} \overline{e}_{pk}$

5. APPLICATION

We used a modified version of a Levenberg-Marquardt algorithm implementation lmfit [9,10] that proposes default estimation for one-dimensional problems. lmfit can easily be integrated into a software suite for embedded systems.

The initial condition p_0 is defined from the manufacturer's datasheet and from additional bias and gains in the electronic system in question.

6. RESULTS

This section gives some calibration results for a 3D accelerometer [11] and a magnetometer [12] sensor. For each sensor, the calibration procedure has been performed three times successively, according to section 3. Each time, 600 data points are used for the estimation of parameters. The obtained gains and bias are then compared in the following tables. The estimation algorithm lmfit performs the alpha beta computation instantly, with about 100 iterations.

6.1. Magnetometers

Tab. 1 presents the calibration parameters obtained for the magnetometer. The obtained results show that the calibration method produces stable parameters, with a precision of about 10^{-2} V for biases and 10^{-2} V/gauss for gains.

The theoretical parameters defined by the sensor and embedded system specification are used to set the initial conditions. The theoretical gains are 8 and biases are 0, thus:

p_0 = {1/8, 1/8, 1/8, 0.0, 0.0, 0.0}

Table 1 – Computed β_i in volts and α_i in V/gauss

Test	*1*	*2*	*3*
α_x	7.2043	7.1866	7.1809
α_y	7.4074	7.3758	7.3772
α_z	7.6276	7.6276	7.6222

Test	*1*	*2*	*3*
β_x	0.1849	0.1919	0.1850
β_y	0.2269	0.2217	0.2206
β_z	0.2417	0.2423	0.2403

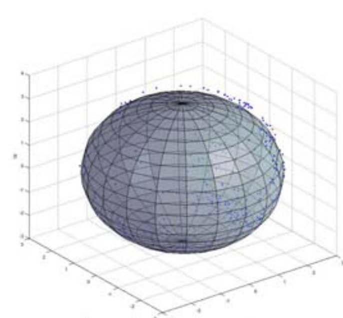

Fig 6 – Ellipsoid of 3D magnetometer sensor

Fig.6 shows that the estimated parameters define an ellipsoid according to the model equation (2) that fits the measurement.

6.2. Accelerometers

In Table 2 we can see the calibration results for a 3D accelerometer sensor. The calibration procedure for accelerometers is quite different from magnetometers. The sensor must only measure the static acceleration of gravity, for which the norm is constant. We need to exclude measurements where the sensor is not static, for which dynamic acceleration is present, by observing the temporal variance of the voltage signal. In static position the accelerometer has a low variance. A threshold to exclude measurements with high variance is experimentally set at $T = 0.0016\ \mathrm{V}^2$. The obtained accuracy is about 10^{-2} V for biases and 10^{-2} V/g for gains.

The theoretical parameters defined by the sensor and embedded system specification are used to set the initial conditions. The theoretical gains are 1.8 and biases are 0, thus:

$$p_0 = \{\ 1/1.8,\ 1/1.8,\ 1/1.8,\ 0.0,\ 0.0,\ 0.0\ \}$$

Table 2 – Computed α, β

Test	*1*	*2*	*3*
α_x	1.7604	1.8100	1.7295
α_y	1.8100	1.8104	1.8111
α_z	1.7295	1.7310	1.7283

Test	*1*	*2*	*3*
β_x	-0.1066	-0.1117	-0.1044
β_y	0.022001	0.01310	0.02511
β_z	-0.1507	-0.1565	-0.1529

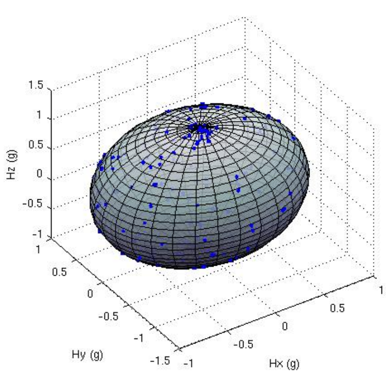

Fig 7 – Ellipsoid of 3D accelerometer sensor

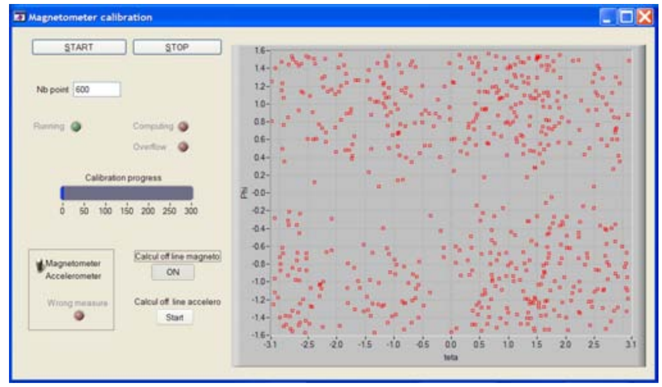

Fig 8 – Calibration software for magnetometer and accelerometer

Fig.8 illustrates the graphical user interface of the developed calibration software, interfaced with a National Instruments acquisition device.

7. ORTHOGONALITY ERROR

The previous sections are based on a model of the 3D sensors with three gains and three biases, equation (1). By engineering, the 3D sensor axes are not orthogonal, so we can extend the previous model with non orthogonality factors as follows:

$$\overbrace{\begin{pmatrix} V_x(t) \\ V_y(t) \\ V_z(t) \end{pmatrix}}^{V} = \overbrace{\begin{pmatrix} \alpha_x & s_{xy} & s_{xz} \\ s_{xy} & \alpha_y & s_{yz} \\ s_{xz} & s_{yz} & \alpha_z \end{pmatrix}}^{\alpha} \overbrace{\begin{pmatrix} h_x(t) \\ h_y(t) \\ h_z(t) \end{pmatrix}}^{\vec{H}} + \overbrace{\begin{pmatrix} \beta_x \\ \beta_y \\ \beta_z \end{pmatrix}}^{\vec{\beta}} + \overbrace{\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{pmatrix}}^{\varepsilon} \quad (6)$$

$s_{\#\#}$ (in $v/gauss$) represents the orthogonality errors between sensor axes. The matrix α is considered symmetrical [3]. To estimate the matrix α and the vector $\vec{\beta}$, we can minimize the following error function :

$$S(p) = \sum_t (e_p(t))^2 \quad (7)$$

with :

$$e_p(t) = \left\| \vec{H} \right\|^2 - (V(t) - \vec{\beta})^T (\alpha^{-1})^2 (V(t) - \vec{\beta})$$

$$\alpha^{-1} \text{ parameterized by } \alpha^{-1} = \begin{pmatrix} \alpha'_x & s'_{xy} & s'_{xz} \\ s'_{xy} & \alpha'_y & s'_{yz} \\ s'_{xz} & s'_{yz} & \alpha'_z \end{pmatrix}$$

and $p = \left[\alpha'_x \alpha'_y \alpha'_z s'_{xy} s'_{xz} s'_{yz} \beta_x \beta_y \beta_z \right]^t$ the new parameters vector to compute using LMA.

These calibrations with detection of non orthogonality for a 3D sensor are based on the same set of measurements as in section 6. Then we can compare the result between these two methods.

For the magnetometer the theoretical gains are 1.8, biases are 0 and non orthogonality factors 0, thus:

$$p_0 = \{\ 1./8.,\ 1./8.,\ 1./8.,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0\ \}$$

Table 3 – Computed α matrix and bias $\vec{\beta}$ for magnetometer

Test	*1*	*2*	*3*
α_x	7.2019	7.1802	7.1791
α_y	7.4019	7.3859	7.3849
α_z	7.6285	7.62608	7.6243
S_{xy}	0.01711	0.0219	0.0201
S_{xz}	0.1421	0.1456	0.14505
S_{yz}	-0.0494	-0.04826	-0.04672
β_x	0.19002	0.1897	0.1892
β_y	0.2237	0.2209	0.2212
β_z	0.2347	0.2317	0.2317

For the accelerometer the theoretical gains are 1.8, biases are 0 and non orthogonality factors 0, thus:

$$p_0 = \{\ 1./1.8,\ 1./1.8,\ 1./1.8,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0\ \}$$

Table 4 – Computed α matrix and bias $\vec{\beta}$ for accelerometer

Test	*1*	*2*	*3*
α_x	1.76107	1.7644	1.7604
α_y	1.8092	1.8105	1.8108
α_z	1.7294	1.7313	1.7283
S_{xy}	0.0005633	0.00026504	-0.0004774
S_{xz}	-0.001812	-0.001145	-0.00150033
S_{yz}	0.0023202	0.002832	0.001572
β_x	-0.1066	-0.1118	-0.10431
β_y	0.02186	0.01328	0.02533
β_z	-0.1510	-0.1567	-0.1529

The calibration results in tables 3-4 show that the gains and bias are stable and close to theoretical values, the orthogonality of sensors is very good. In some applications the orthogonality errors cannot be taken into account because of the low impact on the measurements.

9. CONCLUSION

The numerical calibration of 3D sensors such as magnetometers and accelerometers is not an expensive solution and can be implemented with no mechanical means (in the case of a non industrial production). The calibration procedure can be performed after the integration phase of electronics to take into account the drift and uncertainty of the component itself, all the gains, bias and geometry. The calibration parameters of magnetometers and accelerometers are accurate at 10^{-2}. This method of calibration has been successfully used for the preparation of MEMS inertial units, in a visually impaired pedestrian tracking system.

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