1 Homework 1

Chapter I

(a) 1.1

(b) 1.2

(c) 1.3

(d) 1.5

(e) 1.6

(f) 1.7

(g) 1.10

(h) 1.11

(i) 1.12

2 Homework 2

Chapter II

(a) 3.2

(b) 3.3

(c) 5.1

(d) 5.2

(e) 5.10

2.1 A

2.2 B

2.3 C

3 Homework 3

3.1 1

3.2 2

3.3 3

4 Homework 4

Chapter II

(a) 1.6

(b) 1.7

(c) 1.8

(d) 1.9

(e) 1.10

(f) 1.11

(g) 1.12

(h) 1.15

(i) 1.16

(j) 1.18

(k) 1.22

4.1 A

5 Homework 5

Chapter II

(a) 2.3

(b) 2.4

(c) 2.5

(d) 2.6

(e) 2.7

(f) 2.8

(g) 2.9

(h) 2.11

(i) 2.13

(j) 2.17

(k) 2.18

(l) 2.19

See homework for some hints.

5.1 A

5.2 B

6 Homework 6

TODO

(a) 3.8

(b) 3.11

(c) 3.17

(d) 3.18

(e) 3.19

(f) 3.20

6.1 A

6.2 B

7 Homework 7

7.1 A

7.2 B

7.3 C

7.4 D

(ASK BRIAN ABOUT THIS TOMORROW HOW TO NOT USE ETALENESS) (WHY CANT WE MAKE THE FOLLOWING ARGUMENT:

we require that A is a separable algebra and then every quotient is a separable algebra so in particular the residue fields are separable and thus each closed point should have residue field contained in separable closure by nullstellensatz (its a finite extension of groud field) and thus the k^{sep}-points are dense because the closed points are dense for any locally finite type k-scheme.

Let X be a scheme locally of finite type over a separably closed field k such that X is *geometrically* integral over k. It suffices to show that every affine open $\text{Spec}\,(A) \subset X$ contains a k-point. Since A is geometrically irreducible we see that $K = \text{Frac}\,(A)$ satisfies $K \otimes_k k^{\frac{1}{p}}$ is reduced and thus by [Mat, Thm. 26.2] admits a separating transcendence basis.

Suppose that X is only required to be integral. Then we may take $k = \mathbb{F}_p(t)^{\text{sep}}$ and $X = \text{Spec}\left(\bar{k}\right)$ then X has no k-points because $\bar{k}$ is a strict extension of k but is clearly integral.

Suppose X is required to be geometrically integral but k is not separably closed. Then we may take $k = \mathbb{R}$ and $X = V(x^2 + y^2 + z^2) \subset \mathbb{P}^2_{\mathbb{R}}$ which is a conic with no $\mathbb{R}$-points such that $X_{\mathbb{C}} \cong \mathbb{P}^1_{\mathbb{C}}$.