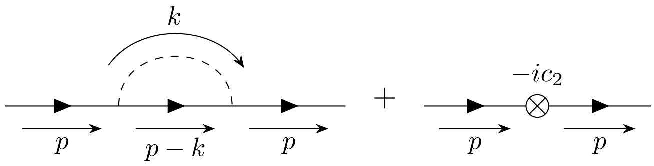
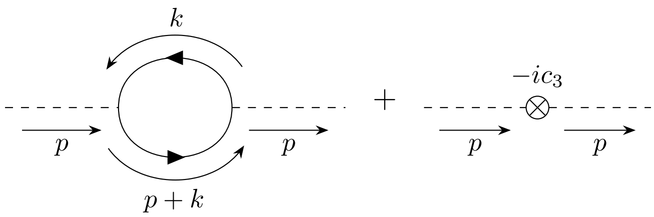
Mass Renormalization

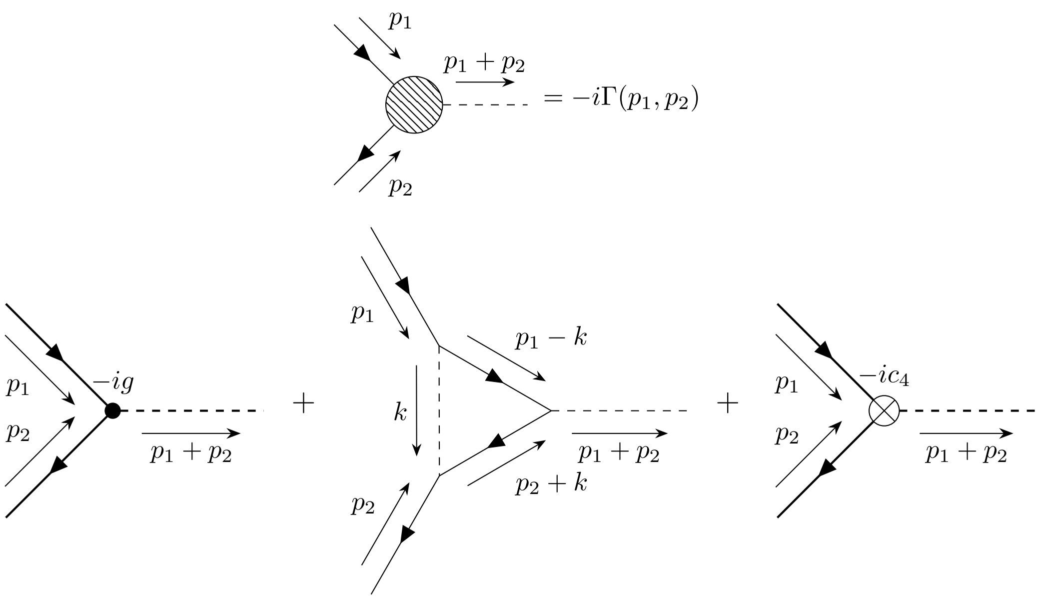
1-Loop Mass ψ



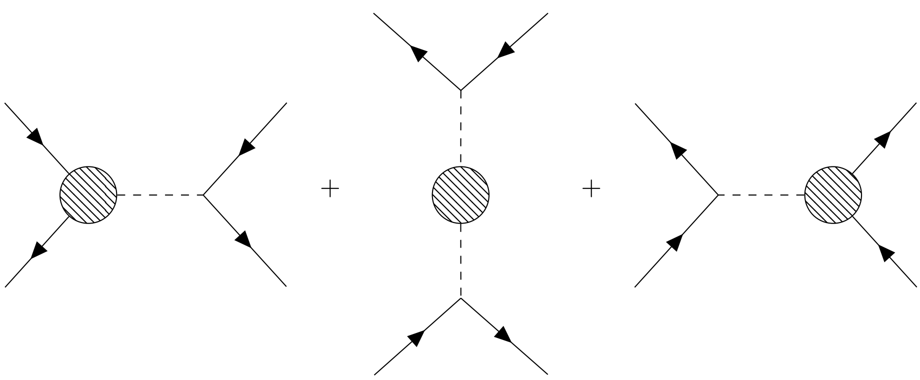
1-Loop Mass ϕ

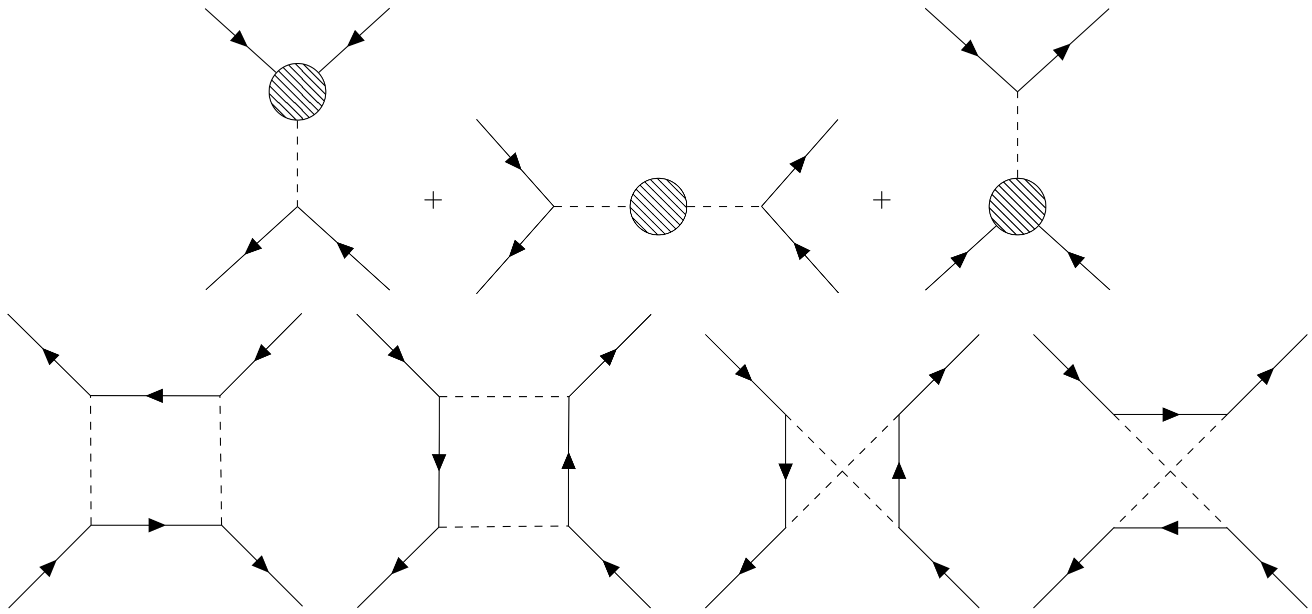


Vertex Function



All the Diagrams





0.1 The Scattering Cross Amplitudes

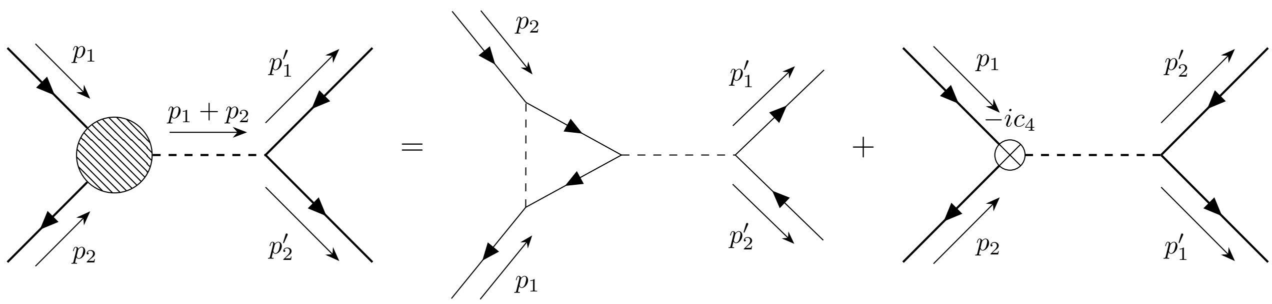
We now have all the tools necessary to compute the scattering cross section for the $\psi\bar{\psi} \to \psi\bar{\psi}$ process at the one-loop level. Following the LSZ reduction formula, we need only consider connected amputated diagrams. The first step is to properly organize the Feynman diagrams to simplify computations. There are two tree-level diagrams, ten one-loop diagrams, and six counterterm diagrams. I will organize these diagrams into classes of diagrams that "look like" each of the schematics,

(PUT IN DIAGRAM LIST)

We must consider each class of diagrams.

0.1.1 Class (a)

We must consider the class of diagrams,

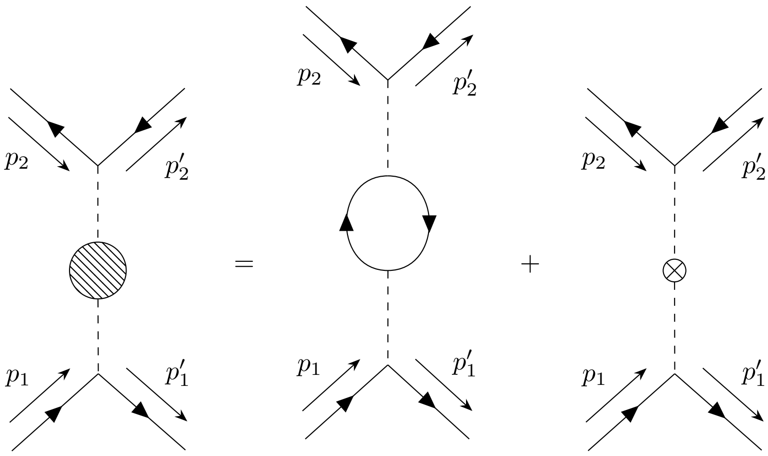


which gives an amplitude,

$$i\mathcal{M}_a = -i\tilde{\Gamma}(p_1, p_2) \cdot \frac{i}{(p_1 + p_2)^2 - m^2}$$

0.1.2 Class (b)

Vertical middle loop diagrams look like,

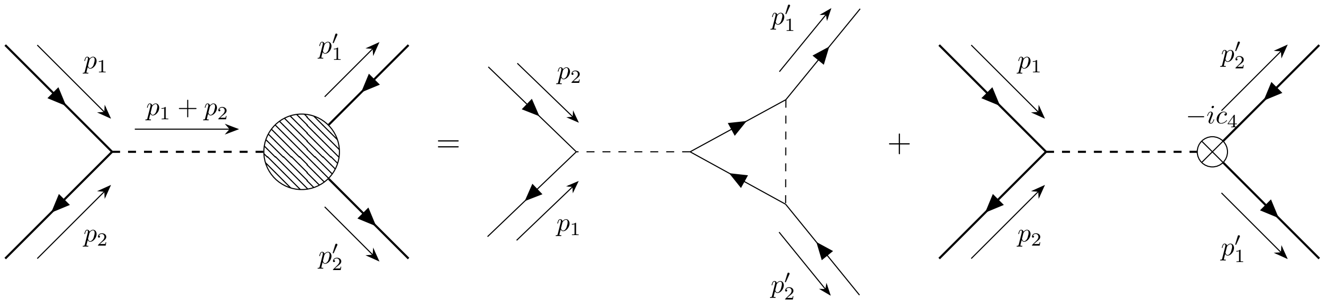


which give an amplitude,

$$i\mathcal{M}_b = \frac{i}{(p_1 - p_1')^2 - m^2} \cdot [-i\Sigma(p_1 - p_1')] \cdot \frac{i}{(p_1 - p_1')^2 - m^2}$$

0.1.3 Class (c)

The next set of diagrams look like,

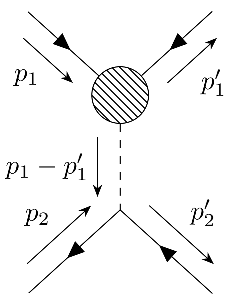


which gives an amplitude,

$$i\mathcal{M}_c = -i\tilde{\Gamma}(p_1', p_2') \cdot \frac{i}{(p_1 + p_2)^2 - m^2}$$

0.1.4 Class (d)

These diagrams are similar,

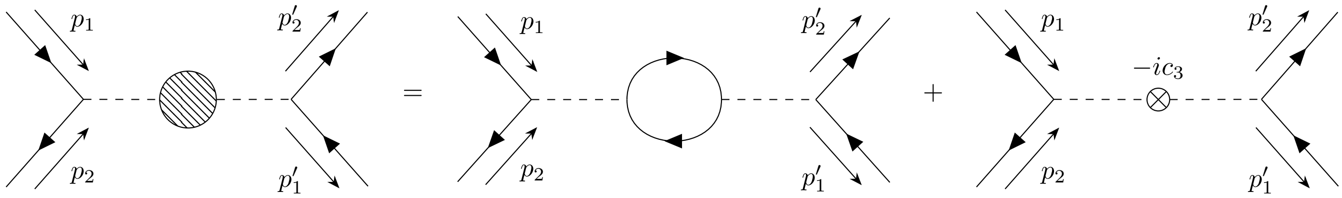


which give an amplitude,

$$i\mathcal{M}_d = -i\tilde{\Gamma}(p_1, p_1') \cdot \frac{i}{(p_1 - p_1')^2 - m^2}$$

0.1.5 Class (e)

The horizontal middle loop diagrams,

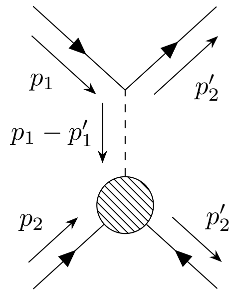


give an amplitude,

$$i\mathcal{M}_e = \frac{i}{(p_1+p_2)^2 - m^2}\left[-i\Sigma(p_1+p_2)\right]\frac{i}{(p_1+p_2)^2 - m^2}$$

0.1.6 Class (f)

And finally,

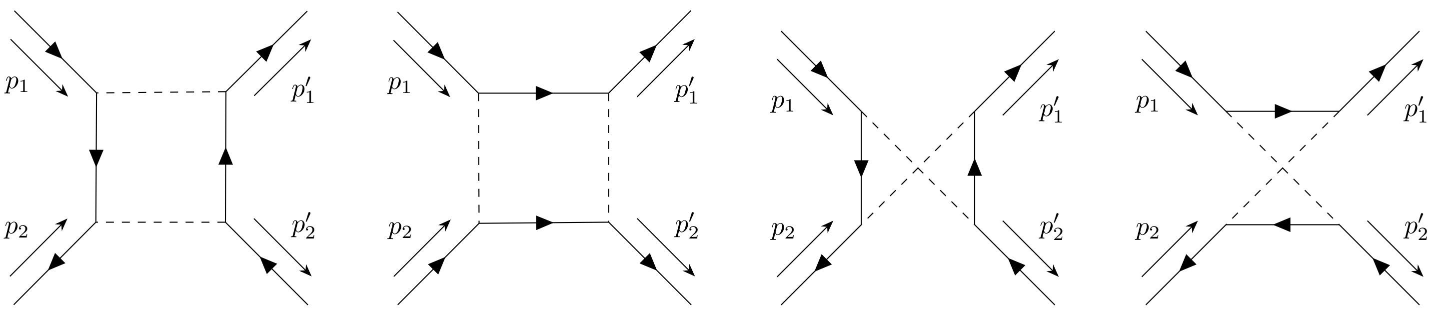


which give an amplitude,

$$i\mathcal{M}_f = -i\tilde{\Gamma}(p_2, p_2') \cdot \frac{i}{(p_1 - p_1')^2 - m^2}$$

0.1.7 Box Diagrams

There are four Box diagrams at the one-loop level,



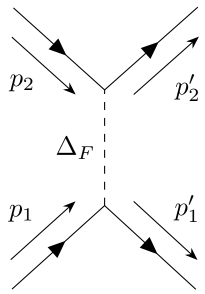
The amplitudes for each of these terms can be transformed into one another by swapping the values of the incoming/outgoing momenta and shifting the value of the undetermined momentum. It can be shown either by writing down the integrals explicitly or diagram chasing that the amplitude for these diagrams are,

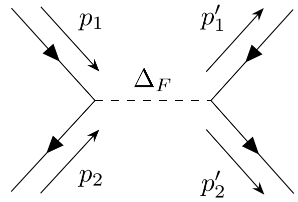
$$i\mathcal{M}_{box} = i\mathcal{M}_B(p_1, p_2, p_1', p_2') + i\mathcal{M}_B(p_1, -p_1', -p_2, p_2') + i\mathcal{M}_B(p_1, p_2, p_2', p_1') + i\mathcal{M}_B(p_1, -p_2', p_1', -p_2)$$

where each term corresponds to the respective box diagram shown above.

0.1.8 Tree-Level Diagrams

As before, we have the two tree-level diagrams,





with total amplitude,

$$i\mathcal{M}_{tree} = \frac{i}{(p_1 - p_1')^2 - m^2} + \frac{i}{(p_1 + p_2)^2 - m^2}$$