1 Introduction

Assume that X is a connected n-dimensional smooth manifold. And $A, B \subset X$ oriented closed submanifolds with $\dim A + \dim B = \dim X$. Assume that A, B intersect transversally menaing,

$$\forall x \in A \cap B : T_x X = T_x A \oplus T_x B$$

This requires the orientations.

If V is a $\mathbb{C}$-vectorspace then V has a canonical orientation. Therefore if X is a complex manifold and A, B are complex submanifolds then,

$$\text{or}_x(X, A, B) = 1$$

Therefore, intersection numbers are allways positive. This is very useful. For example:

Proposition 1.0.1. Let C be an integral curve in $\mathbb{CP}^n$ and $[C] = [L]$ for L a line. Then C is a line.

Proof. Choose two points on C and consider hyperplane H through those two points. Then,

$$\deg\left([C] \cdot [H]\right) = 1$$

becuase $[C] = [L]$. If H is transverse to C then this would be a contradiction. Therefore H is not transverse to C. Therefore $C \subset H$. Therefore C is contained in all hyperplanes through those two points and thus C is a line. $\square$

Proposition 1.0.2. Let X be smooth and $Y \subset X$ is smooth and $Z \subset X$ is CM and $Y \cap Z$ is pure of the expected dimension. Then,

$$[Y] \smile [Z] = \sum_{C \subset Y \cap Z} a_C [C]$$

where $C \subset Y \cap Z$ are components and $a_C > 0$.

1.1 Main Theorems

Let X be a variety (not necessarily smooth) over k (not necessarily $k = \bar{k}$). Then we define Chow,

$$A_i(X) = \mathbb{Z}[\text{prime divisors of dim} = i]/\text{rational equivalence}$$

Given a proper map $f : X \to Y$ there is a pushforward $f_* : A_i(X) \to A_i(Y)$. If $f : X \to Y$ is flat and X, Y are integral then there is a pullback $f^* : A_{\dim X - i}(Y) \to A_{\dim X - i}(X)$.