## 0.1 K-theory of the moduli space of genus zero curves

**Definition 0.1.1.**  $\mathcal{M}_{0,n}$  parametrizes genus zero curves with n labeled distinct points (up to isomorphism fixing the marked points).

Remark. We will always take  $n \geq 3$  for stability reasons. Notice that  $\mathcal{M}_{0,3} = *$  and  $\mathcal{M}_{0,4} = \mathbb{P}^1 \setminus \{0,1,\infty\}$ . We see that  $\mathcal{M}_{0,n}$  is noncompact. We will consider a nice compactification.

**Definition 0.1.2.**  $\overline{\mathcal{M}}_{0,n}$  parametrizes stable genus zero curves with n marked points. This is a tree of  $\mathbb{P}^1$  with at last 3 nodes or marked points on each component.

**Example 0.1.3.**  $\overline{\mathcal{M}}_{0,3} = *$  and  $\overline{\mathcal{M}}_{0,4} = \mathbb{P}^1$  and  $\overline{\mathcal{M}}_{0,5}$  is the blowup of  $\mathbb{P}^2$  at four general points.

**Proposition 0.1.4.** dim  $\overline{\mathcal{M}}_{0,n} = n - 3$  and  $\mathcal{M}_{0,n} \subset \overline{\mathcal{M}}_{0,n}$  is a dense open.

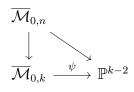
Remark.  $\psi : \overline{\mathcal{M}}_{0,n+1} \to \overline{\mathcal{M}}_{0,n}$  forgetting points and contracting components if necessary induces a birational map then there are sections  $s_i$  which induce divisors and we define the  $\psi_i$  classes as the conormal bundles of these sections.

**Theorem 0.1.5** (Kapranov). The  $\psi_i$  are base-point-free line bundle and defines a birational map,

$$\overline{\mathcal{M}}_{0,n} \to \mathbb{P}^{n-1}$$

where  $\overline{\mathcal{M}}_{0,n}$  maps to

Remark. For  $S \subset [1, \ldots, n-1]$  with  $|S| \geq 3$  we can consider forgetfull maps,



therefore we have,

$$\overline{\mathcal{M}}_{0,n} \hookrightarrow \prod_{\substack{S \subset [n-1]\\|S| > 3}} \mathbb{P}^{|S|-2}$$

Then we let  $X_s = C_i(f_*\psi_n)$