1 Introduction

1.1 Homotopy Theory

1.2 Abstract Nonsense

Definition 1.2.1. Category enriched in a category.

Definition 1.2.2. Strict 2-category

Example 1.2.3. Category of categories.

So naturally we define a 3-category as a category enriched in 2-categories etc. and so an n-category should be a category enriched in $(n-1)$-categories. Therefore, naturally, an ∞-category which should be a category with morphisms of every order and thus an ∞-category is a category enriched in ∞-categories ... um wait that can't work as a definition. However, it turns out there's a problem already at the level of 2-categories which is that our notion is two strict to handle constructions in homotopy theory. Recall that composition of paths and homotopies is not litterally associative or unital but rather only associative or unital up to homotopy. If we want to retain the homotopies and not just work up to equivalence, we need to relax our notion of a 2-category to alow for associativity and unitality *up to equivalence* where the equivalences must also satisfy some compatibility.

Definition 1.2.4. A *(weak) 2-category* $\mathcal{C}$ consists of the following data

(a) a collection of objects $\mathrm{Ob}(\mathcal{C})$

(b) for each pair $x, y \in \mathrm{Ob}(\mathcal{C})$ a collection of morphisms $\mathcal{C}(x, y)$

(c) for each pair $f, g \in \mathcal{C}(x, y)$ a collection of 2-morphisms $H(f, g)$

(d) a composition law $H(g, h) \times H(f, g) \to H(f, h)$ given $f, g, h \in \mathcal{C}(x, y)$

(e) a bifunctor $* : \mathcal{C}(y, z) \times \mathcal{C}(x, y) \to \mathcal{C}(x, z)$

(f) natural transformations

where

(a) $\mathcal{C}(x, y)$ is a category whose objects are elements $f, g \in \mathcal{C}(x, y)$ and whose morphisms are $H(f, g)$ meaning the composition of 2-morphisms is associative and unital

(b) there is a bifunctor $* : \mathcal{C}(y, z) \times \mathcal{C}(x, y) \to \mathcal{C}(x, z)$

1.3 General Philosophy

2 Simplicial Sets

2.1 Quasi-Categories and ∞-Groupoids

2.2 Kan-Enriched Categories