ASTR GR6001 Radiative Processes
Assignment # 1

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Problem 1

Consider an ideal blackbody star with specific intensity B_ν constant on its surface and independent of angle in the ourtwards facing hemisphere. Now we may compute,

$$
\begin{aligned}
J_\nu &= \frac{1}{4\pi} \int B_\nu \, \mathrm{d}\Omega \; = \frac{1}{4\pi} \int_0^1 B_\nu \, \mathrm{d}\cos\theta \int_0^{2\pi} \mathrm{d}\phi \; = \tfrac{1}{2} B_\nu \\
F_\nu &= \int B_\nu \cos\theta \, \mathrm{d}\Omega \; = \int_0^1 B_\nu \cos\theta \, \mathrm{d}\cos\theta \int_0^{2\pi} \mathrm{d}\phi \; = \pi B_\nu \\
P_\nu &= \frac{1}{c} \int B_\nu \cos^2\theta \, \mathrm{d}\Omega \; = \frac{1}{c} \int_0^1 B_\nu \cos^2\theta \, \mathrm{d}\cos\theta \int_0^{2\pi} \mathrm{d}\phi \; = \tfrac{2\pi}{3c} B_\nu \\
u_\nu &= \tfrac{4\pi}{c} J_\nu = \tfrac{2\pi}{c} B_\nu
\end{aligned}
$$

Problem 2

Consider a radiation field of the form,

$$
I_\nu(\mu) = a_\nu + b_\nu \mu
$$

where $\mu = \cos\theta$. Then we may compute,

$$
\begin{aligned}
J_\nu &= \frac{1}{4\pi} \int B_\nu \, \mathrm{d}\Omega \; = \frac{1}{4\pi} \int_{-1}^1 [a_\nu + b_\nu \mu] \, \mathrm{d}\mu \int_0^{2\pi} \mathrm{d}\phi \; = a_\nu \\
F_\nu &= \int B_\nu \cos\theta \, \mathrm{d}\Omega \; = \int_{-1}^1 [a_\nu + b_\nu \mu] \mu \, \mathrm{d}\mu \int_0^{2\pi} \mathrm{d}\phi \; = \tfrac{4}{3} \pi b_\nu \\
P_\nu &= \frac{1}{c} \int B_\nu \cos^2\theta \, \mathrm{d}\Omega \; = \frac{1}{c} \int_{-1}^1 [a_\nu + b_\nu \mu] \mu^2 \, \mathrm{d}\mu \int_0^{2\pi} \mathrm{d}\phi \; = \tfrac{4\pi}{3c} a_\nu \\
u_\nu &= \tfrac{4\pi}{c} J_\nu = \tfrac{4\pi}{c} a_\nu
\end{aligned}
$$

Therefore this radiation field has the same energy density and pressure as an isotropic one however has nonzero flux unlike the purely isotropic case.