1 Introduction and Definitions

Let k be a finite field and K a dimension 1 function field over k (i.e. a field extension K/k of transcendence degree 1). Let $\bar{k}$ be a fixed algberaic closure of k and $\widetilde{K} = K\bar{k}$ the compositum inside a fixed alebgraic closure $\overline{K}$. Let X denote the unique regular projective curve over k with $K(X) = K$. Note that because k is perfect, X is smooth. We assume that X is geometrically integral over k so that $k = \Gamma(X, \mathcal{O}_X)$ is the field of constants, otherwise we replace k by $\Gamma(X, \mathcal{O}_X)$. Throughout we denote $q = p^e = \#k$ where $p = \operatorname{char} k$.

Remark. Indeed, the following are equivalent,

$$X \text{ is geometrically integral over } k \iff \Gamma(X, \mathcal{O}_X) = k \iff K \cap \bar{k} = k \text{ inside } \overline{K}$$

The first is Tag 0FD2 (recall that k is perfect is X is geometrically reduced). The second is as follows. Since X is integral and proper $k' = \Gamma(X, \mathcal{O}_X)$ is a finite k-algebra domain and hence a finite field extension of k. In particular, $k' \subset K \cap \bar{k}$ inside $\overline{K}$ proving one direction. Furthermore, suppose that $f \in K$ is integral over k'. Then for any affine open $U \subset X$ we see that f is integral over $\mathcal{O}_X(U)$ and hence $f \in \mathcal{O}_X(U)$ since X is normal so $\mathcal{O}_X U$ is a normal domain with fraction field K. Hence since $f \in \mathcal{O}_X(U)$ for each affine open $f \in \Gamma(X, \mathcal{O}_X)$ proving that k' is algebraically closed inside K.

1.1 Background Results

Here we collect some results on the class group which we will try to reprove using adelic techniques.

Remark. Because X is smooth we can freely use isomorphisms $\mathrm{Cl}(X) \cong \mathrm{CaCl}(X) \cong \mathrm{Pic}(X)$. Furthermore, there is a degree map, $\deg : \mathrm{Cl}(X) \to \mathbb{Z}$ sending

$$[P] \mapsto [\kappa(P) : k] = \log_q \#\kappa(P)$$

Lemma 1.1.1. $\mathrm{Pic}^0(X)$ is finite and $\mathrm{Pic}(X) \cong \mathrm{Pic}^0(X) \times \mathbb{Z}$ noncanonically.

Proof. Choose some prime divisor D_0 (meaning a point $P \in X$) and let $d = \deg D$. Then the map $D \mapsto D - nD_0$ gives an isomorphism $\mathrm{Pic}^{nd}(X) \xrightarrow{\sim} \mathrm{Pic}^0(X)$ so it suffices to show that $\mathrm{Pic}^{nd}(X)$ is finite for $n \gg 0$. However, by Riemann-Roch, if $\deg D = nd \geq 2g$ then

$$H^0(X, \mathcal{O}_X(D)) = \deg D + 1 - g \geq g + 1 \geq 1$$

so there is an effective divisor $D' \sim D$. Then fixing n large enough so that $nd \geq 2g$ we see that for any $D \in \mathrm{Pic}^{nd}(X)$ there is $D' \sim D$ with D' effective and $\deg D' = nd$ however there are finitely many prime divisors of bounded degree because $X(k')$ is finite for each finite extension k'/k and thus there are finitely many effective divisors with fixed degree so $\mathrm{Pic}^{nd}(X)$ is finite.

There is a canonical exact sequence,

$$0 \longrightarrow \mathrm{Pic}^0(X) \longrightarrow \mathrm{Pic}(X) \longrightarrow \mathbb{Z}$$

Surjectivity of $\deg : \mathrm{Pic}(X) \to \mathbb{Z}$ is obvious if X has a k-point P because $\deg [P] = 1$. In general, surjectivity is a consequence of X being geometrically integral (otherwise suppose that X is a k'-scheme then every divisor will have residue field containing k' so $\operatorname{im} \deg$ will have index at least $[k' : k]$). Because X is geometrically integral, the Weil conjectures gives,

$$\#X(\mathbb{F}_{q^n}) = 1 + q^n - \sum_{i=1}^{2g} \beta_i^n$$

with $|\beta_i| = q^{\frac{1}{2}}$. Therefore,

$$|\#X(\mathbb{F}_{q^n}) - 1 - q^n| \le 2g\, q^{\frac{n}{2}}$$

and thus for $n \gg 0$ we have $q^n + 1 > 2gq^{\frac{n}{2}}$ so $X(\mathbb{F}_{q^n}) \neq \varnothing$. In paricular there are points $P, Q \in X$ with $\deg P = n$ and $\deg Q = n+1$ so $D = [Q] - [P]$ is a divisor with $\deg D = 1$ proving surjectivity. Then because $\mathbb{Z}$ is projective the sequence splits. □

Remark. The point counting formula requires X to be geometrically integral such that $X_{\bar{k}}$ is an (in particular connected) variety so that $H^0_{\text{ét}}(X_{\bar{k}}, \mathbb{Q}_\ell)$ and $H^1_{\text{ét}}(X_{\bar{k}}, \mathbb{Q}_\ell)$ have the expected dimension and Galois representations. To see what can go wrong, consider $X = \mathbb{P}^1_{\mathbb{F}_{q^2}}$ over $\mathrm{Spec}\,(\mathbb{F}_q)$. Then,

$$X(\mathbb{F}_{q^n}) = \begin{cases} 2(1+q^n) & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

which does not following the counting formula nor does it have a divisor of degree 1.

Remark. We can also give a much fancier proof. There is an exact sequence of group schemes,

$$0 \longrightarrow \mathbf{Pic}^0_{X/k} \longrightarrow \mathbf{Pic}_{X/k} \longrightarrow \underline{\mathbb{Z}} \longrightarrow 0$$

where $\mathbf{Pic}^0_{X/k}$ is finite type over k and thus $\mathrm{Pic}^0\,(X) = \mathbf{Pic}^0_{X/k}(k)$ is finite because k is a finite field. Surjectivity of $\mathbf{Pic}_{X/k} \to \underline{\mathbb{Z}}$ is clear in the étale topology on $\mathrm{Spec}\,(k)$ because X has a degree 1 prime divisor after a finite extension of k (e.g. take the residue field of any closed point). Therefore, we get an exact sequence,

$$0 \longrightarrow \mathbf{Pic}^0_{X/k}(k) \longrightarrow \mathbf{Pic}_{X/k}(k) \longrightarrow \underline{\mathbb{Z}}(k) \longrightarrow H^1(k, \mathbf{Pic}^0_{X/k})$$

However, $\mathbf{Pic}^0_{X/k}$ is an abelian variety so by Lang's theorem on H^1-vanishing, $H^1(k, \mathbf{Pic}^0_{X/k}) = 0$ and therefore,

$$0 \longrightarrow \mathrm{Pic}^0\,(X) \longrightarrow \mathrm{Pic}\,(X) \longrightarrow \mathbb{Z} \longrightarrow 0$$

is exact and $\mathbb{Z}$ is projective so it splits.

Remark. We needed to assume X was geometrically integral over k for representability of the relative Picard functor [FGA V, Thm. 3.1]. In general, there is an abelian variety J called the *Jacobian* but $J(k) \neq \mathrm{Pic}^0\,(X)$ in general (see Poonen 5.7).

Theorem 1.1.2 (Lang)**.** Let A be a smooth connected finite type k-group with k finite. Then,

$$H^1(k, A) = 0$$

Proof. FIND BETTER REFERENCE? Can Look at Chapter VI of Serre's *Algebraic Groups and Class Fields*. Or Poonen Rational Points Thm. 5.12.19. □

Remark. In the case that A is an elliptic curve over k, I have a cheeky proof. A class $X \in H^1(k, A)$ represents an A-torsor on $\mathrm{Spec}\,(k)_{\text{ét}}$ and thus is trivial if and only if X has a k-point (a "global section"). However, X is a form of A and thus,

$$H^i_{\text{ét}}(X_{\bar{k}}, \mathbb{Q}_\ell) = H^i_{\text{ét}}(A_{\bar{k}}, \mathbb{Q}_\ell)$$

so by the Lefschetz trace formula $\#X(k) = 1 + q - \alpha - \bar{\alpha}$ with $|\alpha| = \sqrt{q}$. Thus,

$$\#X(k) \ge 1 + q - 2\sqrt{q} = (\sqrt{q} - 1)^2 > 0$$

so $X(k) \neq \varnothing$. Maybe I can make this work for abelian varieties.

1.1.1 The Adèles and Idèles

Lemma 1.1.3. Every valuation ring of K/k is a DVR and is $\mathcal{O}_{X,x}$ for some unique point $x \in K$.

Proof. See [H, Ex. 4.12(a)]. □

Remark. We write v_x for the associated valuation which we normalize so that,

$$v_x(\varpi_x) = \deg x = [\kappa(x) : k] = \log_q \#\kappa(x)$$

This normalization is chosen such that the associated norm $|a|_x = q^{-v_x(a)}$ satisfies $|\varpi_x|_x = (\#\kappa(x))^{-1}$. We also write ord_x for the valuation normalized such that $\mathrm{ord}_x(\varpi) = 1$ so that,

$$\mathrm{div} f = \sum_{x \in X} \mathrm{ord}_x(f)[x]$$

Definition 1.1.4. The adeles and ideles of a function field are,

$$\mathbb{A}_K = \prod_{x \in X}{}' (K_x, \mathcal{O}_{K,x}) \quad \text{and} \quad \mathbb{I}_K = \prod_{x \in X}{}' (K_x^\times, \mathcal{O}_{K,x}^\times)$$

where $\mathcal{O}_{K,x}$ is the completed local ring,

$$\mathcal{O}_{K,x} = \widehat{\mathcal{O}_{X,x}} = \varprojlim_n \mathcal{O}_{X,x}/\mathfrak{m}_x^n$$

and $K_x = \mathrm{Frac}\,(\mathcal{O}_{K,x})$ is the local field at $x \in X$. The valuations and norms extend to $v_x : K_x^\times \to \mathbb{Z}$ making it a non-archimedean local field with discrete valuation ring $\mathcal{O}_{K,x}$.

Remark. Unlike the number field case, all the local fields K_x are isomorphic to $k'((t))$ because X is regular so $\widehat{\mathcal{O}_{X,x}} \cong k'[[t]]$ for $k' = \kappa(s)$. We require k to be finite in order that K_x is a local field, in particular so that K_x is locally compact. Indeed, a fundamental system of neighborhoods of $0 \in k((t))$ are given by groups isomorphic to $k[[t]]$ which is compact if and only if k is finite. Indeed, $k[[t]] \twoheadrightarrow k[t]/(t^n)$ so if $k[[t]]$ is compact then its image $k[t]/(t^n)$ is compact but also discrete and thus finite. Conversely, if k is finite then $k[t]/(t^n)$ is finite and thus discrete so Tychonoff's theorem shows that.

$$k[[t]] = \varprojlim_n k[t]/(t^n)$$

is compact as well. Therefore, it is essential that we restric to function fields over *finite* fields if we want to have a good local theory.

Definition 1.1.5. The idèle class group is,

$$C_K = \mathbb{I}_K / K^\times$$

where $K^\times \hookrightarrow C_K$ via the diagonal embedding $K^\times \hookrightarrow K_x^\times$. This makes sense because each $f \in K$ has only finitely many poles meaning $f \in \mathcal{O}_{X,x}$ and thus $f \in \widehat{\mathcal{O}_{X,x}}$ for all but finitely many $x \in X$.

Definition 1.1.6. There is a degree map $\deg : C_K \to \mathbb{Z}$ defined by taking,

$$\deg(a_v) = \sum_v v(a_v)$$

which is well-defined because $a_v \in \mathcal{O}_{K,v}$ so $v(a_v) = 0$ for all but finitely many v and a norm,

$$|a| = \prod_v |a_v|_v$$

Now define the open subgroup $C_K^0 = \ker \deg = \mathbb{I}^1/K^\times$ where

$$\mathbb{I}_K^1 = \left\{ (a_v) \,\middle|\, a_v \in K_v \text{ and } a_v \in \mathcal{O}_{K,v} \text{ for all but finitely many } v \text{ and } \prod_v |a_v|_v = 1 \right\}$$

There is another open subgroup,

$$U_K = \left(\prod_v \mathcal{O}_{K,v} \right) / K^\times$$

Proposition 1.1.7. There is a surjection $C_K \twoheadrightarrow \mathrm{Pic}\,(X)$ with kernel U such that the diagram,

$$\begin{tikzcd} C_K \arrow[r, two heads] \arrow[dr, "\mathrm{ord}"'] & \mathrm{Pic}\,(X) \arrow[d, two heads, "\deg"] \\ & \mathbb{Z} \end{tikzcd}$$

commutes giving isomorphisms $C_K/C_K^0 \xrightarrow{\sim} \mathbb{Z}$ and $C_K^0/U_K \xrightarrow{\sim} \mathrm{Pic}^0\,(X)$.

Proof. For $(a_x) \in \mathbb{I}_K$ we know $\mathrm{ord}_x(a_x) = 0$ for all by finitely many x so there is a map,

$$(a_x) \mapsto \sum_{x \in X} \mathrm{ord}_x(a_x)[x]$$

which is well-defined because $f \mapsto \mathrm{div} f$ for $f \in K^\times$. This is surjective since divisors are finite sums and $(\varpi_{x_0}) \mapsto [x_0]$. Furthermore,

$$\deg \left(\sum_{x \in X} \mathrm{ord}_x(a_x)[x] \right) = \sum_{x \in X} \mathrm{ord}_x(a_x) \deg x = \sum_{x \in X} v_x(a_x) = \deg\,(a_v)$$

By definition, $C_K^0 = \ker \mathrm{ord}$ giving the first isomorphism. Then $C_K^0 \to \mathrm{Pic}\,(X)$ surjects onto $\mathrm{Pic}^0\,(X) = \ker\,(\mathrm{Pic}\,(X) \to \mathbb{Z})$ and $\ker\,(C_K^0 \to \mathrm{Pic}\,(X)) = U_K$ because if $(a_v) \mapsto D$ and $D = \mathrm{div} f$ then $v(a_v/f) = 0$ so $a_v/f \in \mathcal{O}_{K,v}$ proving that $(a_v) \in U_K$. ☐

Theorem 1.1.8. C_K^0 is compact.

Proof. DO THIS!!! ☐

Corollary 1.1.9. $\mathrm{Pic}^0\,(X)$ is finite. Indeed, because C_K^0 is compact we see that $\mathrm{Pic}^0\,(X)$ is compact. Furthermore, $U_K \subset C_K^0$ is open so $C_K^0/U \xrightarrow{\sim} \mathrm{Pic}^0\,(X)$ is also discrete and thus finite.

2 The First Inequality

(WHAT THE HELL IS CURLY H)

We first recall some facts about the Herbrand quotient. Define,

$$h^i(G, M) = \dim_k H^i(G, M)$$

then the Herbrand quotient is,

$$h_{2/1}(G, A) = h^2(G, A)/h^1(G, A)$$

(DO I NEED G TO BE CYCLIC HERE!!)

Proposition 2.0.1. The index $h_{2/1}$ is multiplicative. Given an exact sequence,

$$0 \longrightarrow M_1 \longrightarrow M_2 \longrightarrow M_3 \longrightarrow 0$$

of G-modules then,

$$h_{2/1}(M_2) = h_{2/1}(M_1) h_{2/1}(M_3)$$

Proposition 2.0.2. If A is finite then $h_{2/1}(A) = 1$.

Proposition 2.0.3. $h_{2/1}(\mathbb{Z}) = |G|$ where $\mathbb{Z}$ has a trivial $\mathbb{Z}$-action.

Proposition 2.0.4. Let L/K be an extension of local fields then,

$$h_1(\mathrm{Gal}\,(L/K)\,, U) = h_2(\mathrm{Gal}\,(L/K)\,, U) = e(L/K)$$

where $U \subset L$ are the units of the ring of integers.

Definition 2.0.5. Let L/K be a finite cyclic extension of order n. Then,

$$h_{2/1}(G, C_L) = n$$

Theorem 2.0.6. Let L/K be a cyclic extension of degree n with Galois group G. Then,

$$h_{2/1}(G, C_L) = n$$

Proof. We have,

$$h_{2/1}(C_L) = h_{2/1}(C_L/C_L^0) h_{2/1}(C_L^0/U) h_{2/1}(U)$$

First, $C_L/C_L^0 \xrightarrow{\sim} \mathbb{Z}$ and thus $h_{2/1}(C_L/C_L^0) = n$ and $h_{2/1}(C_L^0/U_L) \xrightarrow{\sim} \mathrm{Pic}^0\,(X)$ which is finite so $h_{2/1}(C_L^0/U_L) = 1$. Now,

$$h_{2/1}(U_L) = h_{2/1}(W) h_{2/1}(L^\times \cap W)^{-1}$$

where,

$$W = \prod_w \mathcal{O}_{L,w}^\times = \prod_v \left(\prod_{w|v} \mathcal{O}_{L,w}^\times \right)$$

Now,

$$H^r(G, W) = \prod_v H^r \left(G, \prod_{w|v} \mathcal{O}_{L,w}^\times \right) = \prod_v H^r(G_\nu, \mathcal{O}_{L_\nu}^\times)$$

by Shapiro's lemma since,

$$\prod_{w|v} \mathcal{O}_{L,w}^\times = \mathrm{Ind}_{G_\nu}^G \left(\mathcal{O}_{L_v}^\times \right)$$

By the local theory,

$$h_2(G_\nu, \mathcal{O}_{L_v}^\times) = h_1(G_\nu, \mathcal{O}_{L_v}^\times) = e_\nu$$

and since $e_\nu = 1$ all but finitely often we see that,

$$h^1(G, W) = h^2(G, W) = \prod_v e_v$$

and therefore $h_{2/1}(W) = 1$. Finally, $L^\times \cap W$ is the field of constants which is finite (this is where we're using the function field setting! otherwise we need to do more work) so $h_{2/1}(L^\times \cap W) = 1$ proving that,

$$h_{2/1}(C_L) = n \cdot 1 \cdot 1 \cdot 1 = n$$

□

3 The Second Inequality

4 The Existence Theorem

Definition 4.0.1. Let L/K be a finite extension and $f : X' \to X$ the corresponding finite map of nonsingular curves. Then L_w/K_v is a finite extension so there is a local norm $\mathrm{N}_{L_w/K_v} : L_w^\times \to K_v^\times$. Then we define the norm,

$$\mathrm{N}_{L/K} : C_L \to C_K \quad (a_w) \mapsto (b_v) \quad \text{where} \quad b_v = \prod_{w \mapsto v} \mathrm{N}_{L_w/K_v}(a_w)$$

Definition 4.0.2. What is ω??????????????

Theorem 4.0.3. Let $N \subset C_K$ be a finite index open subgroup. Then there exists a finite abelian extension L/K such that $\mathrm{N}_{L/K}(C_L) = N$ and K is the fixed field of $\omega(N)$.

Theorem 4.0.4. Let L/K be a finite abelian extension with $N = \mathrm{N}_{L/K}(C_L)$. Then $x \in X$ is uniramified if and only if $\mathcal{O}_{K,x}^\times \subset N$ and x splits completely if and only if $K_x^\times \subset N$.

5 The Hilbert Class Field

5.1 Review of the Number Field Case

Consider the open subgroup $U_K = (K^\times \cdot \mathbb{I}_{K,S_\infty})/K^\times$ with $C_K/U_K \xrightarrow{\sim} \mathrm{Cl}\,(K)$. Then by the global existence theorem there is a finite abelian extension H_K/K with $\mathrm{N}_{H_K/K}(C_{H_K}) = U_K$. Therefore, we see that H_K is unramified everywhere because $\mathcal{O}_{K,\nu}^\times \subset U_K$ and also for any L/K such that L/K is everywhere unramified then $\mathcal{O}_{X,\nu}^\times \subset \mathrm{N}_{L/K}(C_L)$ and therefore $U_K \subset \mathrm{N}_{L/K}(C_L)$ which implies that $L \subset H_K$ so H_K is the maximal abelian unramified extension of K.

Remark. In this context, it is probably more correct to say that that, in the Hilbert class field, we require the places at infnity to be *totally split* rather than *unramified* although these mean the same thing. This is because at infinity unramified corresponds to $K_\nu^\times \subset N$ which for finite places is the condition of being totally split and indeed $K_\nu^\times \subset N$ for all $\nu \in S_\infty$.

Proposition 5.1.1. Let K be a number field and H_K/K be its Hilbert class field. Then $\mathfrak{p}$ is prinicpal iff $\mathfrak{p}$ splits completely in H_K.

Proof. The isomorphism $C_K/\mathrm{N}_{H_K/K}(H_K) \xrightarrow{\sim} \mathrm{Gal}\,(H_K/K)$ and $C_K/\mathrm{N}_{H_K/K}(H_K) \xrightarrow{\sim} \mathrm{Cl}\,(K)$ send the uniformizer $\varpi_\mathfrak{p}$ to $\mathrm{Frob}_\mathfrak{p}$ and $[\mathfrak{p}]$ respectively. Therefore, $[\mathfrak{p}] = [0]$ iff $\mathrm{Frob}_\mathfrak{p}$ is trivial iff $\mathfrak{p}$ splits completely in H_K/K. ☐

5.2 The Function Field Case

Unlike for number fields, in the function field case we have,

$$C_K/U_K \xrightarrow{\sim} \mathrm{Pic}\,(X) \cong \mathrm{Pic}^0\,(X) \times \mathbb{Z}$$

which is not finite. Therefore, we cannot directly apply the global existence theorem. There are two possible resolutions to this issue. First, we can consider all finite quotients of $\mathrm{Pic}\,(X)$ to immediately conclude using the existence theorem and the inclusion-reversing correspondence,

$$\mathrm{Gal}\left(K^{\text{un, ab}}/K\right) = \varprojlim_{L/K} \mathrm{Gal}\,(L/K) \xrightarrow{\sim} \varprojlim_{H \subset \mathrm{Pic}(X)} \mathrm{Pic}\,(X)\,/H = \widehat{\mathrm{Pic}\,(X)}$$

where we take the limit over all abelian unramified extensions L/K and all finite index subgroups $H \subset \mathrm{Pic}(X)$ which correspond because an abelian extension L/K is unramified if and only if $\mathrm{N}_{L/K}(C_L) \supset U_K$ and finite index subgroups $H \supset U_K$ are open (it is a union of translates of U_K) and hence by the existence theorem correspond to some abelian unramified extension L_H/K. Alternatively, to get an analogue of the Hilbert Class field we need to choose a different appropriate open subgroup that does have finite index.

The issue is essentially due to extensions of the constant field k which are all abelian and unramified. This should somehow correspond to the factor $\mathbb{Z}$ in $\mathrm{Pic}(X)$ which should relate to $\mathrm{Gal}\left(\bar{k}/k\right) \cong \hat{\mathbb{Z}}$. We will now make these correspondences precise.

Definition 5.2.1. We say that L/K is *regular* if L does not contain a nontrivial algebraic extension of k. Explicitly, this means that $L \cap \bar{k} = k$ inside $\overline{K}$ or equivalently $\Gamma(C_L, \mathcal{O}_{C_L}) = k$.

Remark. Based on the above heuristic, we expect there to be finitely many regular abelian unramified extensions and perhaps even a maximal one in some sense.

Example 5.2.2. Let E be an elliptic curve over k and consider the maps $[n] : E \to E$ which seem to be a sequence of regular abelian unramified extensions of unbounded degree. However, these are usually not actually abelian. Indeed, abelian extensions are, by definition, Galois. The cover $[n]$ is an $E[n]$-torsor but $E[n]$ is a split group only finitely often (since E has finitely many rational points) and hence is usually not a Galois cover. It is weird because this is a torsor for an abelian group so it seems like it should be an abelian extension. However, the torsor's full automorphism group (the Aut scheme given by $E[n]$) does not appear over k and hence these automorphisms do not all fix the field $K(E)$ but rather the compositum with some finite extension k'/k of the ground field. Now the Galois closure of the field extension $[n] : K(E) \hookrightarrow K(E)$ will include the ground field extension k'/k splitting $E[n]$ and thus will not be regular.

Proposition 5.2.3. There are only finitely many regular abelian unramified extensions. However, there does not exist a maximal regular abelian unramified extension.

(EXTENSIONS WHERE SOME FIXED DIVISOR STAYS SPLIT CORRESPOND TO SPLITTINGS OF THE Z PART OF Pic)

We saw that the Hilbert class field is more correctly understood as being the maximal abelian extension which is unramified everywhere and totally split at infinity. In the number field case, there is a canonical notion of places at infinity. The function field case does not afford such a luxury requiring us to make some arbitrary choices of a "divisor at infinity".

(I THINK THERE ARE FINITELY MANY REGULAR EXTNS BUT NOT A MAXIMAL GUY SHOW THIS)