Physics C2801, Fall 2013 Prof. Brian Cole

Issued: **Sept. 19**	Problem Set # 3	Due: **Sept. 26**

Problem 1. Kleppner and Kolenkow problem 2.11

Problem 2. Kleppner and Kolenkow problem 2.15

Problem 3. Kleppner and Kolenkow problem 2.17

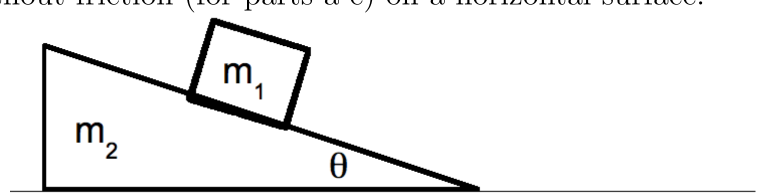
Problem 4. Kleppner and Kolenkow problem 2.19

Problem 5. Kleppner and Kolenkow problem 2.24

Problem 6. Kleppner and Kolenkow problem 2.30

Problem 7. Box on freely sliding ramp

A Box of mass m_1 is placed on a ramp of mass m_2 with inclination angle θ. The ramp slides without friction (for parts a-e) on a horizontal surface.



a. Evaluate the (vector) acceleration of the box and the acceleration of the ramp in the case that there is no friction between the box and the ramp. I suggest that you use the approach at the top of page 74 of Kleppner and Kolenkow.

b. Evaluate the normal force of the horizontal surface on the ramp for the conditions in part a.

c. Now suppose that there is friction between the box and the inclined plane with μ large enough that the box does not slide on the inclined plane. A hypothetical student analyzes this problem and draws the conclusion that while the box is not sliding relative to the ramp, the ramp will slide (accelerate) because of the contact (normal) force of the box on the ramp. Is this analysis correct? If not what error did the student make?

d. In the situation of part c, evaluate the condition on μ at fixed angle θ at which the box will start sliding down the ramp. How is this result different from the case in which the ramp is fixed in position?

e. Now suppose that μ is small enough that the box starts sliding on the ramp. Then $|\vec{F}_f| = \mu'|\vec{F}_N|$ where μ' is a different coefficient of (sliding) friction. Evaluate the acceleration of the box and the ramp as you did in part a.

f. Now suppose that there is no friction between the box and the ramp but there is friction between the ramp and the horizontal surface. For fixed coefficient of friction μ_r (between the ramp and surface), find the condition on the ratio m_1/m_2 at which the ramp will start sliding when the box is sliding down the ramp.

Problem 8. Stretchy pendulum

One of my students a few years asked a good question during lecture when we were working the pendulum problem about how to treat the case when the rope/cable stretches due to the hanging mass and the motion. You have learned enough to treat this problem approximately, so I figured it would make sense to have you do it. So, consider a rope that has length L_0 with no tension applied. We will hang a mass, m from the rope and study its motion under small amplitude displacements from equilibrium. The mass of the rope itself will be sufficiently small compared to m that it can be neglected.

We will write the relationship between the tension in the rope and it's (change in) length, $T = k\Delta L = k(L - L_0)$. But, we need to be careful because a rope can't "push" (unlike a spring). So this expression only makes sense for $\Delta L > 0$.

a. First let's analyze the vertical motion of the mass with the rope hanging straight down. Suppose we represent the vertical position of the mass relative to some (unspecified) reference point with the symbol y. The position of the mass when the rope is unstretched (length L_0) we will write as y_0. Write the equation of motion for the mass in terms of the coordinate y using Newton's second law. Find the equilibrium position of the mass, y_{eq}.

b. The equation of motion you obtained in part a can be transformed to look like a harmonic oscillator equation of motion with a change of coordinates. Show that for an appropriate choice of Δy, the coordinate transformation $y' = y - \Delta y$, produces an equation of motion that takes the form $\ddot{y}' = -\omega^2 y'$. What is the frequency of oscillations of the mass?

c. The equation of motion in part a has a restricted range of validity due to the fact that the rope can't push. What is the range of validity of the harmonic oscillator equation you obtained in part b in terms of y' (give maximum and/or minimum values of y' for which it applies).

d. Suppose that Prof. Cole is performing a demonstration of this problem for purely vertical motion. For $t < 0$, Prof. Cole holds the mass at rest with the rope unstretched. He releases the mass at $t = 0$. Find a complete expression for the motion of the mass, $y(t)$, for $t > 0$ specifying all constants in the function in terms of other parameters of the problem and/or the initial conditions. Make a sketch of $y(t)$ for a few cycles of oscillation.

e. Now, let's consider the situation where the mass is swinging while it also stretches the rope. Write out Newton's second law and project onto the $\hat{r}$ and $\hat{\theta}$ directions expressing the tension as a function of r. Write out the two resulting equations of motion for $\ddot{r}$ and $\ddot{\theta}$.

f. We will analyze the motion of the pendulum for small angles and small radial displacements of the mass (from the equilibrium length). In lecture, we used small-angle approximations of the non-stretchy pendulum keeping only terms to first order (power) in θ. But now we have terms involving both θ and r and their derivatives. Start with the radial equation of motion, and make the small angle approximation. Show that a change of variables $r' = r - \Delta y$ with Δy the same as in part b simplifies the radial equation of motion **except for the centripetal term**. Under certain conditions the centripetal term is small enough to be neglected. Show that if you drop the centripetal term, you obtain a harmonic oscillator equation of motion for radial motion of the mass. What is the angular frequency of the motion?

g. Now let's analyze the angular motion of the mass. Write out the *angular* equation of motion substituting $r = r' + \Delta y$ and making the small-angle approximation for the force term. We can use the small angle and small radial displacement approximation to simplify the angular equation of motion. To be consistent, we will keep terms to linear power in small quantities. Terms of higher power in small quantities should be dropped. So, for example, you will have a term like $\dot{\theta}\dot{r}'$ which can be dropped if $\dot{\theta}$ and $\dot{r}'$ are (sufficiently small). Since we've already assumed that the displacements in the radial direction and the angular direction are small, the only question is whether the time variation of the quantities is fast enough that the derivatives become non-negligible. We will check (below) whether our solutions to the approximated equations yield time derivatives that satisfy the requirement that $\dot{r}'$ and $\dot{\theta}$ are small. Drop all terms in the angular equation that are higher power (beyond linear) in r', θ or derivatives of these two quantities. You should obtain a simple harmonic oscillator equation of motion. What is the angular frequency of the resulting motion?

h. The approximations that we made above "decoupled" the equations of motion. This means that we were able to write two independent equations of motion – the equation for r' did not involve θ or its derivatives and the equation for θ did not involve r' or its derivatives. *note: the angular equation initially did involve r, but by writing $r = r' + \Delta y$ we we able to express the angular equation in terms of the equilibrium radial stretch of the rope and neglect the radial oscillation of the mass.* Assume solutions to the approximate, decoupled equations,

$$r' = A\cos\left(\omega_r t - \phi_r\right) \tag{1}$$
$$\theta = B\cos\left(\omega_\theta t - \phi_\theta\right) \tag{2}$$

We can use these approximate solutions to evaluate the conditions on the amplitudes and parameters of the problem (expressed through ω_r and ω_θ) under which our approximations are valid. We will start with the centripetal term which, when written in terms of r', is actually the sum of two terms. For the purposes of this analysis, we will only consider and compare the coefficients of the cosine (and cosine squared) functions. By comparing the coefficients of the two terms resulting from the centripetal acceleration to the other terms in the radial equation, find conditions under which our approximation in part *f* is justified. Write the conditions in the form $\{expression\} << 1$ where $\{expression\}$ is a combination of the amplitudes, A and B, and the other parameters of the problem: L, m, k, and g.

i. Using the same approach as in part *h*, evaluate the conditions under which our approximations to the angular equation of motion are justified.

Problem 9. Whirling rope with mass at the end

A rope of length L has has a mass per unit length λ. An object of mass m is attached to the rope at one end. The other end of the rope is held fixed to a pivot. The rope is whirled in the horizontal plane with angular frequency ω such that the object attached to its end moves in a circle of radius L. Find the tension in the rope as a function of radial distance r from the fixed end over the range $0 \leq r \leq L$.