1 TODO!!

(a) Finish symplectic geometry course

  (a) figure out if symplectic toric is the same as projective toric variety (projectivity needed to come from a polytope and also to be Kahler)
  (b) review coisotropic reduced and write some notes
  (c) hyperkahler reduction examples
  (d) are there examples of noncompact hyperkahlers?
  (e) work out the kinks in notes on hamiltonian actions

(b) review killing homotopy groups columbia lectures and write some notes

(c) figure out those damn jet bundles and connections on principal bundles

  (a) RMK: $\pi^* E$ is NOT trivial for a vector bundle let alone a fiber bundle. it does get equiped with a canonical section but for a vector bundle this is just the trivial section, only for a principal bundle does giving a section trivialize it.
  (b) role of atiyah sequence vs jet bundle sequence
  (c)

(d) spectral sequences for tor and ext in derived category (FIND MY NOTES ON THIS!)

  (a) application to universal coefficient theorem
  (b) Kunneth spectral sequence
  (c) Kunneth formula for smash product?
  (d) why are derived functors triangulated
  (e) derived functors in terms of Kan extensions (NOTES)

(e) write notes on universal morphisms

(f) G-action of X/Y induces map Descent data X/Y to G-equivariant sheaves

  (a) isomorphism when X/Y is a G-cover i.e. $X \to Y$ is a G-torsor
  (b) write down explicit G-equivariant structure on Ω_X
  (c) Galois descent derive explicit form

(g) Weil restriction

  (a) write down trivialization after going back up
  (b) Galois descent in explicit form

(h) notes on Galois actions on schemes

(i) notes on Frobenii

(j) notes on universal constructions in math with examples

(k) fix notes on Tor in category of sheaves and Tor symmetry (do I need symmetry of flat objects a priori?).

(l) Finish stable homotopy theory course.

(m) Finish vector bundles and connections notes (in AG folder)

  (a) Kahler iff $\nabla I = 0$ where ∇ is the Levi-Civita connection
  (b) Ricci tensor and the trace bullshit
  (c) Riemann-Hilbert and existence of flat frames for integrable connections

2 What I Want to Think About

(a) Flat cohomology equal etale cohomology for smooth (affine groups) apply this to that counting rational points things

(b) work out the details for the group fixing $\mathbb{C}$ inside endomorphism group. What does an integrable structure of this kind look like, how close to a complex manifold can we get? In dimension two this should be exactly a conformal (not necessarily orientable) structure.

(c) FINISH CONFORMAL NOTES!

(d) Hilbert Class Field of curves (ASK BRIAN FOR REFERENCE)

(e) Read about Fredholm index and Riemann-Roch

(f) Cohmology and inclusion-exclusion: cohomology for vectorspaces?

3 Some Questions I Have

(a) Reduction of structure group for a scheme.

  (a) what about the algebraic group $\mathrm{SL}^{\pm} = \det^{-1}(\mu)$ what does reduction of structure group give. For a manifold this is supposed to be a pseudo-volume form but obviously that's not right.
  (b) what about $\mathrm{Res}_{\mathbb{R}}^{\mathbb{C}}(\mathbb{G}_m) \hookrightarrow \mathrm{GL}_2$ from the action $\mathbb{G}_m \circlearrowleft \mathbb{A}^1_{\mathbb{C}}$ restricted giving an action $\mathrm{Res}_{\mathbb{R}}^{\mathbb{C}}(\mathbb{G}_m) \circlearrowleft \mathbb{A}^2_{\mathbb{R}}$. I feel like this should give an almost complex structure. What properties does it have? What about for other fields?
  (c) What is an almost complex structure on a scheme look like?

(b) Is my calculation of an "almost almost complex structure" as reduction of structure group to $\langle\sigma\rangle \ltimes \mathrm{GL}(n, \mathbb{C}) \subset \mathrm{GL}(2n, \mathbb{R})$. For the case $n = 1$ this should be the conformal group justfying that I think this should correspond to the non-oriented case of a complex manifold since Riemann surfaces are exactly oriented conformal manifolds.

4 Project

5 Foliations

5.1 Riemannian Foliations

Remark. Given a transverse metric $g_{\mathscr{F}}$ to a foliation $\mathscr{F} \subset TM$ we can always choose a metric g on M such that g is bundle-like. Indeed, just choose a splitting $TM \cong \mathscr{F} \oplus Q$ and choose any metric on $\mathscr{F}$ then take the sum.

5.2 Holonomy

5.3 Linearization

HOW IS THIS RELATED TO THE NORMAL BUNDLE?

6 Finding A Leaf

Recall that A is the cokernel of $\mathrm{Alb}_F \to \mathrm{Alb}_X$.

(a) $\dim F \geq \dim \mathrm{Alb}_F$ by [Kawamata, Thm. 1]

(b) $\dim A \geq \dim \mathrm{Alb}_X - \dim \mathrm{Alb}_F$ by definition

(c) hence, $\dim A \geq \dim \mathrm{Alb}_X - \dim F$

(d) $\dim A + \dim W \leq \dim \mathrm{Alb}_X$ by Popa-Schnell

(e) hence we conclude $\dim W \leq \dim \mathrm{Alb}_X - \dim A \leq \dim F$.

However, $\dim W = \dim F$ by assumption. Therefore, every one of the above inequalities is an equality.

Therefore, there is an exact sequence,

$$0 \longrightarrow H^0(Y, \Omega^1_Y) \longrightarrow H^0(X, \Omega^1_X) \longrightarrow H^0(F, \Omega^1_F) \longrightarrow 0$$

The first map is injective by generic smoothness of $X' \to Y$ and the fact that hodge numbers are birational invariants. The second map is surjection by (in)equality (b) and then the sequence is exact by (in)equality (c).

Therefore, the image $W|_F \subset H^0(F, \Omega_F)$ is everything. But recall that $F \to \mathrm{Alb}_F$ is dominant by Kawamata. Therefore generically the global forms on F span the cotangent space meaning the forms $W|_F$ are a generic coframe of F. This implies,

$$X \to \mathrm{Alb}_X \to \mathrm{Alb}_X^\vee \to \mathrm{Alb}_F^\vee$$

is surjective because $\mathrm{Alb}_F \to \mathrm{Alb}_F^\vee$ is finite and any section of $H^0(X, \Omega^1_X) \twoheadrightarrow H^0(F, \Omega^1_F)$ must generically span a $\dim F$ subspace of the cotangent space since these forms span F at its generic point.

7 Contracting Fibers

Proposition 7.0.1. Let X, Y, A be varities over $\mathbb{C}$. Suppose there is a diagram,

$$\begin{tikzcd} X \arrow[r, "g"] \arrow[d, "f"] & A \\ Y \end{tikzcd}$$

where A is an abelian variety and f, g are proper, f is $\mathcal{O}$-connected, and the general fiber of f is an abelian variety which maps finitely onto its image under g. Then g is a smooth isotrivial abelian fibration. (WHAT ABOUT IF Y IS SINGULAR)

Proof. Let $y \in Y$ be a point with a singular fiber. Then choose a map $\mathrm{Spec}\,(R) \to Y$ from a DVR whose closed point hits y. Let $K = \mathrm{Frac}\,(R)$ then X_K is a torsor over an abelian K-scheme so after some extension of DVRs $R' \supset R$ is becomes an abelian variety with semistable reduction. Let $\mathcal{A}$ be the Neron model of $X_{K'}$ and consider the diagram,

$$\begin{tikzcd} \mathcal{A}^\circ \arrow[r, dashed] \arrow[dr] & X_{R'} \arrow[r] \arrow[d] & X_R \arrow[r] \arrow[d] & X \arrow[r] \arrow[d] & A \\ & \mathrm{Spec}\,(R') \arrow[r] & \mathrm{Spec}\,(R) \arrow[r] & Y \end{tikzcd}$$

□