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Heating of Milky Way Disk Stars by Dark Matter Fluctuations in Cold Dark Matter and Fuzzy Dark Matter Paradigms

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ABSTRACT

Although highly successful on cosmological scales, Cold Dark Matter (CDM) models predict unobserved over-dense 'cusps' in dwarf galaxies and overestimate their formation. We consider a popular modification to CDM particle physics involving an ultra-light axion-like scalar boson which promises to reduce these observational discrepancies at galactic scales. The model, known as Fuzzy Dark Matter (FDM), avoids cusps, suppresses small-scale power, and delays galaxy formation via macroscopic quantum pressure. We compare the substructure of galactic dark matter halos comprised of ultra-light axions with masses $10^{-24}\,\mathrm{eV} \le m_p \le 10^{-18}\,\mathrm{eV}$ to conventional CDM results. Besides self-gravitating subhalos, FDM includes additional substructure in the form of non-virialized over-dense wavelets formed by quantum interference patterns. Wavelets provide a more efficient source of heating to the galactic disk than subhalos. We provide analytical measures of substructure-induced perturbations and heating to interior test particles responsible for the thickening of galactic disks and tidal streams. FDM models with moderate particle masses produce a cumulative heating of $30\,\mathrm{km\,s^{-1}}$ over an approximate galaxy lifetime of $10\,\mathrm{Gyr}$ corresponds to the velocity dispersion of the Milky Way thick disk. Under the assumption that the dynamics of dark substructure we provide explict calculations of the radial dependent disk velocity dispersion and corresponding scale height. Although the source of thickended disks is unknown, the heating due to perturbations caused by dark substructure cannot exceed the total disk velocity dispersion. Comparing the best theoretical estimates for heating along with best fit parameters from FDM simulations to observations of the velocity dispersion of the thick disk provides a lower bound on the FDM particle mass $m_p > 1.30 \times 10^{-22}\,\mathrm{eV}$.

Key words: cosmology: theory – dark matter – galaxies: structure

1 INTRODUCTION

Over-dense regions of dark matter collapse to roughly localized concentrations of matter called 'halos' which are well-described by spherically symmetric density profiles. However, compositionally identical halos may vary significantly from these profiles in small-scale power i.e. substructure. Simulations suggest that substructure forms clumps which are qualitatively similar to free dark matter halos. We adopt a substructure formalism which tracks subhalos as an estimate of total substructure. The secondary gravitational effects of these subhalos is dependent on the subhalo mass function and the shape of small halo profiles, which is directly dependent on properties of the dark matter physics under consideration. The standard cosmological model (ΛCDM) contains a massive electromagnetically non-interacting component known as Cold Dark Matter (CDM) with density parameter $\Omega_c = 0.259 \pm 0.006$. The properties of this substance are largely unknown and are assumed to be approximately that of a perfect fluid with negligible pressure compared to its energy density i.e. comprised of cold or non-relativistic matter. Constraints obtained from The Sloan Digital Sky Survey measurement of Ly-α forest power spectra and surveys of dwarf galaxies below the free-streaming scale rule out most models of relativistic (hot) matter as the primary component of dark matter (Seljak et al. 2006).

Although successful on cosmological scales, CDM models fail to make accurate predictions when compared with

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observations at distance scales less then 10 kpc. When compared with studies of dwarf galaxy rotation curves, CDM predicts unobserved density 'cusps' in the center of dark matter halos (Hui et al. 2016). This failure is known as the 'cusp-core problem'. Another serious concern is the 'missing satellite problem' (Klypin et al. 1999). The number of satellite galaxies predicted for a milky-way-like galaxy is greater than what we observe by an order of magnitude. This issue is sharpened by the 'too big to fail' problem of galaxy formation that claims some of the predicted satellites are so massive that it is impossible for them to not have any stars (Boylan-Kolchin et al. 2011). Various complex phenomena have been suggested as solutions to these problems such as baryonic feedback mechanisms which redistribute matter or hypothetical dark matter self-interactions.

An alternative model gaining popularity known as Fuzzy Dark Matter (FDM) describes dark matter as comprised of ultra-light($m \approx 10^{-22}$ eV) bosons whose characteristic wavelength is on the order of 1 kpc. Such ultralight particles are possible in various theories beyond the standard model (Marsh 2015). In particular, the class of axion-like particles is a perfect candidate for FDM and therefore, we refer to the mass of the constituent particles of FDM as the axion mass m_p. Macroscopic quantum mechanical wave effects "smear" the density profile on scales less than ~ 1 kpc which removes problematic cusps from the center of large halos. These cusps are replaced by dense self-gravitating quantum states known as soliton cores which may facilitate indirect observation evidence for FDM. Furthermore, the quantum mechanical pressure caused by the large wavelength of FDM suppresses the formation of low-mass halos and entirely eliminates the formation of any halo or subhalo smaller than this wavelength, reducing or eliminating the missing satellites problem (Mocz and Succi 2015). Significant further effort remains in determining further discrepancies between FDM and CDM and whether these can, in conjunction with observational constraints, rule out one or both of the models.

The dynamical effects of CDM have been studied in some detail, especially regarding the accretion and tidal stripping of CDM subhalos by large galaxies and their associated dark matter halos. On the other hand, research on the dynamics of FDM distributions is in its infancy. However, dynamical studies have focused on simulations which inherently have a fixed resolution and thus artificially suppress small-scale power. In this paper, we present analytic estimates for substructure dynamical fluctuations which do not suffer from limited resolution. The resolution limit of simulations causes underestimates to subhalo populations and therefore to their dynamical effects.

CDM and FDM models predict significant differences in the distribution of halo substructure. The FDM subhalo mass function is zero below a cutoff scale determined by the axion mass expressed by the Jean's scale at which quantum pressure and gravitational attraction balance:

$$k_J = 66.5 \cdot (1+z)^{1/4} \left(\frac{\Omega_a h^2}{0.12}\right) \left(\frac{m_p}{10^{-22}\,\mathrm{eV}}\right)^{1/2} \mathrm{Mpc}^{-1} \quad (1)$$

as given by Marsh (2015).

For light axions, the subhalo mass function is significantly suppressed though the entire range of substructure for a halo comparable to the Milky Way's. However, FDM subhalos will exhibit solitons which, for moderately-sized subhalos and large axion masses, are very dense and largely unaffected by tidal disruption. In addition, a surprising standing wave phenomenon has been observed in the density profiles produced by small-scale FDM computer simulations (Hu et al. 2000). These standing waves, named wavelets, radically alter the dark matter profile and may through gravitational interactions disturb the baryonic components in directly measurable ways. The primary effect of these wavelets is to introduce time-varying perturbations to the gravitational potential on a much shorter time-scale than the evolution of primary structure after the system has come into virial equilibrium Mocz et al. (2018).

The velocity dispersion of stars in a galactic disk provide a proxy of the gravitational perturbations produced by the dark matter halo substructure. We expect that dispersion increases with age via an approximate power law $\sigma \propto t^{\beta}$. Therefore, the best observational bounds will come from old thick disk stars.

2 GALACTIC DISKS

Current observations sugest that the disks of spiral galaxies follow an exponetial profile in radius and in vertical structure. The vertical scaling is determined by the mass density of the disk and the perpenducular velocity dispersion. However, the velocity dispersion of stars in galactic disks is corrolated with vertical height from the disk. The disks of normal galaxies are divided into a thin disk of relatvely young stars and a thick disk of old stars. The velocity dispersion of the old stars is significantly greater which is why this population is refered to as the thick disk. Thick disk stars show reduced metalicity which suggests they originate from an epoch of star formation than the thin disk stars (Binney and Tremaine 2008).

Studys of the velocity dispersions of spiral galaxies at varying redshits showed that the velocity dispersion of the thick disk is time dependent fit approximately by a power law $\sigma_D^2 \propto t^{\beta}$ where the best estimates for the exponent are $\beta \approx 0.3$. The origin of this twin disk structure is unknown but there are three widely considered explinations. One hypothesis is that the thick disk formed thicker due to the tendancy for self-gravitating disks to flatten over time due to viscous effects of the interstellar medium and dynamical friction of stars and clusters. However, this explination struggles to explain the observed decrease in velocity dispersion with redshift (Brook et al. 2004). An alternative hypothsis is that tidal interactions between galaxy close encounters can inject energy into existing stars which effectively puffs up the disk (Bensby and Feltzing 2009). A final possibility is that the thick disk is formed by continually heating order stars (Steinmetz 2012). The primary source of heating is also unknown. Interactions between spiral modes have been proposed as a mechanism for heating spiral disks. However, here we will consider the possibility that disk heating is caused by dynamical dark substructure. In this case, we will be able to sharply restrict the parameter space of possible dark matter paradigms. Regardless of the origin of thick disks, the heating due to dark substructure can at most account for the total observed velocity dispersion of the thick disk.

Therefore, these results are able to put a lower bound on the mass of the FDM axion which is agnostic to the mechanism behind producing thick disks.

The disks of normal spiral galaxies follow an exponential density profile,

$$\rho(r,z) = \frac{\Sigma_0}{2H(r)} e^{-|z|/H(r)} e^{-r/r_0} \tag{2}$$

where r_0 is the scaling radius and $H(r)$ is the scale height at a given radius. Furthermore, the projection of the mass density onto the galactic plane has the same exponential form. In particular, the surface density has the functional form,

$$\Sigma(r) = \int_{-\infty}^{\infty} \rho(r,z)\,\mathrm{d}z = \Sigma_0 e^{-r/r_0} \tag{3}$$

For the Milky Way, these parameters are best fit by $r_0 = 5\,\mathrm{kpc}$ and $\Sigma_0 = 64\,\mathrm{M}_\odot/\mathrm{pc}^2$. The vertical profile is determined by hydrostatic equilibrium under the gravity from the planar density distribution of the disk. The pressuse supporting the vertical structure is given by the vertical velocity dispersion $P = \rho\sigma_D^2$.

$$-4\pi G\rho = \frac{\partial^2\phi}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial\rho\sigma_D^2}{\partial z}\right) \tag{4}$$

Therefore, integrating through the disk and approximating σ_D^2 as varying slowly vertically throught the disk we arrive at the following expression relating the scale height and vertical veloicty dispersion of the disk.

$$4\pi G\Sigma(r) = \frac{\sigma_D(r)^2}{H(r)} \tag{5}$$

For stars oscillating through the disk with vertical dispacement large compared to the scale height the period of oscillation is simply given by approximating the the disk gravity as producing a constant magnitude restoring force. Therefore the period for a star with vertical velocity σ_D is given by,

$$P = \frac{2H}{\sigma_D} = \frac{2\sigma_D}{4\pi G\Sigma} \tag{6}$$

The period of disk oscillations determines the adiabatic cutoff for heating due to purturbing interactions. Under the assumption that disk thickening is due primarily to the proposed dark substructure dynamics the dependence on radius of the disk velocity dispersion σ_D can be calculated explicitly. Using the above relations and the radial dependence of σ_D we will be able to predict the radial thickness profile of the galactic disk and compare these results to measurements of the flaring of acutal disks.

We now turn our attention to the Milky Way which has the best sutudied galactic disk. Binney (2009) has performed a comprehensive study of the various distribution functions of the Milky Way and, in particular, has compiled data on the velocity dispersions of the thin and thick disks as a function of vertical distance. Binney (2009) proposes a "pseudo-isothermal" distribution function for the velocity dispersion which has the form,

$$f_z(J_z) = \frac{\left(\Omega_z J_z + V_\gamma^2\right)^{-\gamma}}{2\pi\int_0^\infty \mathrm{d}J_z\ \left(\Omega_z J_z + V_\gamma^2\right)^{-\gamma}} \tag{7}$$

in term of the action variable J_z. For stars in the solar neighborhood with $r \approx R_0 = 8\,\mathrm{kpc}$ the best fit parameters of $\gamma = 2.6$ and $V_\gamma = 18.7\,\mathrm{km\,s^{-1}}$ give essentially perfect agreement with survey data collected by Gilmore and Reid (1983). Using this formalism, the main component of the perpendicular velocity dispersion of the thick disk is $\sigma_D \approx 34\,\mathrm{km\,s^{-1}}$ at the solar neighborhood near the radius R_0.

3 HEATING VIA SUBSTRUCTURE DYNAMICS

3.1 Estimating The Heating Due to Transits

We provide a rough analysis of the heating due to transiting over-dense regions of dark matter. Each transit causes a change in velocity of approximately,

$$v^2 = \frac{M_l G}{b v_l} \tag{8}$$

where M_l is the mass of the region, v_l is its velocity, and b is the distance at closest approach. However, the heating is only effective if these perturbations are adiabatic which approximately corresponds to the constraint,

$$\frac{b}{v_l} < \frac{P}{4} \tag{9}$$

where P is the characteristic period of the objects subjected to perturbation, in this case the period for lateral oscillations through the galactic disk. The total heating is therefore given by the accumulation of all such encounters,

$$\frac{\mathrm{d}\sigma_D^2}{\mathrm{d}t} = \int (2\pi b)\ \mathrm{d}b\, n v_p \left(\frac{M_l G}{b v_p}\right)^2 \tag{10}$$

$$= \frac{M_l^2 G^2}{v_l}\, 2\pi n \ln\frac{b_{\max}}{b_{\min}} \tag{11}$$

where $\ln\Lambda = \ln\frac{b_{max}}{b_{\min}}$ is the Coulomb logarithm. The maximum distance across which the heating is efficient is fixed by 9 and the minimum distance is given by r_l, the characteristic size for the perturbing objects. Furthermore, the period of oscillation out of a thin disk is given by the velocity dispersion and surface density of the disk via,

$$P = \frac{2\sigma_D}{4\pi G\Sigma} \tag{12}$$

Therefore, in terms of the overall one-dimensional velocity dispersion, we derive the time-dependent heating as,

$$\frac{\mathrm{d}\sigma_D^2}{\mathrm{d}t} = \frac{M_l^2 G^2}{\sigma_H}\, 2\pi n \ln\frac{\sigma_H\sigma_D}{8\pi G\Sigma\, r_l} \tag{13}$$

A more detailed calculation detailed in Lacey and Ostriker (1985) gives somewhat alterned numerical factors,

$$\frac{\mathrm{d}\sigma_D^2}{\mathrm{d}t} = \frac{8\pi M_l^2 G^2}{V}\, n\ \ln\frac{\sigma_H\sigma_D}{8\pi G\Sigma\, r_l} \tag{14}$$

where V is the relative velocity between the disk stars and the transiting objects. An alternative approach is given by Hui et al. (2016) which only considers the tidal effects on disk heating. This model assumes that only local dispersion can lead to overall heating. However if processes such as outgoing density waves are an inefficient means for disapating differential motions of the disk then dynamical friction between oscillating disk components will tend to dissipate oscillation energy into disk heating and therefore contribute to the thickening. In this analysis, we will assume that such processes are inefficient and therefore equation 14 gives a more reasonable estimate of the total rate of heating.

3.2 Heating Due to Subhalos

3.2.1 Spatial Profile

We model substructure as comprised entirely of subhalos which act as distinct massive particles subject only to gravitational attraction and tidal disruption. Following Lee (2003); Han et al. (2015) we adopt a simplified model of CDM subhalo formation and dynamics. The shape of the subhalo mass function is assumed to be spatially invariant, i.e. the position and mass variables are decoupled. The total density of subhalos is assumed to follow the Navarro–Frenk–White (NFW) profile:

$$\rho(r) = \frac{\rho_0}{r/r_c(1+r/r_c)^2} \tag{15}$$

which also describes the density profile of the primary dark matter halo given by Navarro et al. (1995) from CDM simulations. This profile is fairly universal for the halos of massive cold particles which we assume accurately describe subhalos.

3.2.2 The Subhalo Mass Function

The only additional information needed to calculate the perturbations due to subhalos is the distribution of subhalo populations as a function of mass. However, the tidal interactions between the primary halo and its interior subalos complicates the elegant halo mass functions determined by the overdensity correlation function and power specturm. We approximate the subhalo populations by assuming that the unresolved subhalo mass function has the same form as the free halo mass function derived from the extended Press–Schechter formalism. The unresolved mass function is then modified by tidally truncating unresolved halos such that the unresolved halo mass function is shifted twards lower mass and the modified subhalo mass function becomes coupled in subhalo mass and radius inside the primary halo. Therefore, the population of subhalos is entirely determined by two functions, $\frac{\mathrm{d}n}{\mathrm{d}\ln M}$, the unresolved (free) halo mass function, and $T_R(M)$, the truncated mass function at a radius R.

3.2.3 Calculating the Heating

In terms of these functions and using the approximation $V \approx \sqrt{3}\sigma_H$ the rate of heating is given by,

$$\frac{\mathrm{d}\sigma_D^2}{\mathrm{d}t} = \frac{8\pi G^2}{\sqrt{3}\,\sigma_H}\int_0^{M_p} T_R(m)^2 \frac{\mathrm{d}n}{\mathrm{d}m} \ln \frac{\sigma_H \sigma_D}{8\pi G \Sigma\, r_{\mathrm{t}}(m)}\,\mathrm{d}m \tag{16}$$

where $r_{\mathrm{t}}(m)$ is the radius of a subhalo of mass m after truncation at a radius R. If we assume that the primary halo and substructure formed quickly on the scale the lifetime of the disk and that dark substructure dynamics in the form of transiting subhalos alone constitutes the primary source of disk thickening then equation 16 gives the time-dependence of σ_D as a function of time over the entire history of the halo. Furthermore, if we assume that $\sigma_D \propto \sigma_H$ at all times then equation **??** reduces to the differential equation,

$$\frac{\mathrm{d}\sigma_D^2}{\mathrm{d}t} \propto \sigma_D^{-1} \ln \sigma_D^2 \tag{17}$$

which implies that $\beta \approx \frac{1}{3}$ for the time dependence $\sigma_D \propto t^\beta$.

3.3 Heating Due to Quantum Wavelets

In models of wave dark matter, there is an added contribution to the fluctuating substructure due to interference of excited modes which produce time dependence overdensity known as wavelets. We assume that the density of the wavelets is a fixed multiple of the local mean density. Using results from numerical FDM simulation given by Mocz et al. (2017),

$$M_w = A\left(\frac{\lambda}{2}\right)^3 \rho(r) \tag{18}$$

where λ is the local characteristic scale. We also assume that entire mass of the primary FDM halo forms the wavelets which gives, $M_w n = \rho(r)$. However, because these wavelets are inherently produced by quantum mechanical interference, they will approximately saturate the uncertainty relations such that,

$$\lambda m_p \sigma_H \approx \hbar \tag{19}$$

where m_p is the particle mass. A more careful calculation given by Philip Mocz (Mocz et al. 2017) shows that,

$$\lambda = \frac{2\pi\hbar}{m_p \sigma_H \sqrt{\frac{3}{2}}}$$

Therefore, the heating due to wavelets is approximately given by,

$$\frac{\mathrm{d}\sigma^2}{\mathrm{d}t} = \frac{A\pi}{2^{3/2}}\left(\frac{h}{m_p}\right)^3 \frac{(\rho(r)G)^2}{\sigma_H^4} \ln \frac{m_p \sigma_H^2 \sigma_D}{8\pi\hbar G\Sigma} \tag{20}$$

Therefore, using $\Sigma = \Sigma_0 e^{-r/r_0}$ we can express the rate of heating as a function of radius. Under the same assumptions that the considered dynamics are the dominant contribution to disk thickening and that $\sigma_D \propto \sigma_H$ at all times then equation 20 reduces to the differential equation,

$$\frac{\mathrm{d}\sigma_D^2}{\mathrm{d}t} \propto \sigma_D^{-4} \ln \sigma_D^3 \tag{21}$$

which implies that $\beta \approx \frac{1}{6}$ for the time dependence $\sigma_D \propto t^\beta$.

3.4 Heating in the CDM Paradigm

3.4.1 Unresolved Mass Function

As the primary halo forms, it captures subhalos via accretion. The unresolved subhalo mass function refers to the distribution of subhalo masses at the time of accretion before dynamical effects such as tidal stripping have biased the distribution. We assume the unresolved subhalo mass function is very close in shape to the free halo mass function truncated above the primary halo mass. Based on simulations given by Schive et al. (2014a) we take this fraction to be $\sim 10\%$. The CDM halo mass function and corresponding subhalo mass function calculated from simulations (Giocoli et al. 2007; Han et al. 2015) are well fit by a power law:

$$\frac{\mathrm{d}n}{\mathrm{d}\ln m_{acc}} = C\left(\frac{\rho(r)}{M_p}\right)\left(\frac{m_{acc}}{M_p}\right)^{-p} \tag{22}$$

with $p = 0.9$ and where M_p is the mass of the primary or host halo.

Furthermore, we fix the total mass in (accreted) halos as a fraction of the total mass of the primary halo. Therefore,

$$M_{\rm halos} = \int m \frac{{\rm d}n}{{\rm d}(\ln m)}\, {\rm d}(\ln m)\, {\rm d}V \tag{23}$$

$$= C \int \left(\frac{\rho(r)}{M_p}\right)\left(\frac{m}{M_p}\right)^{-p} {\rm d}m\, {\rm d}V \tag{24}$$

$$= \frac{C}{1-p}\left[\frac{(f_2 M_p)^{1-p}}{M_p^{-p}}\right] = f_1 M_p \tag{25}$$

so we fix the constant,

$$C = (1-p)\frac{f_1}{f_2^{1-p}} \tag{26}$$

3.4.2 Tidal Disruption

We adopt a simplistic model of tidal stripping which underestimates the total effect. A tidal radius is calculated by setting the tidal force on a test mass equal to the gravitational attraction of the subhalo. The resulting radius is:

$$R_t = R\left(\frac{m_{\rm acc}}{2M_p(R)}\right)^{1/3} = R_{\rm max}\left(\frac{m_{\rm acc}}{M_p}\right)^{1/3} f_T(R) \tag{27}$$

with $f_T(R) \propto R/R_c(\ln(1+R/R_c) - R/(R+R_c))^{-1/3}$ where R_c is the core radius of the primary halo. This formula simply comes from the functional form of the enclosed mass as a function of radius for an NFW profile. We then suppose that total truncation occurs outside this radius and no disruption occurs within it. Thus,

$$\frac{m}{m_{\rm acc}} = \begin{cases} \frac{\ln(1+R_t/r_c) - R_t/(r_c+R_t)}{\ln(1+c) - c/(1+c)}, & \text{if } R_t < r_{\rm max} \\ 1, & \text{otherwise} \end{cases} \tag{28}$$

However, $m_{acc} = 200\rho_0 \frac{4}{3}\pi c(m)^3 r_c^3$ and $r_{\rm max} = c(m)\, r_c$ so

$$R_t = r_{max} f_T(R) \tag{29}$$

$$\frac{T_R(m_{\rm acc})}{m_{\rm acc}} = \begin{cases} \frac{\ln(1+x) - x/(1+x)}{\ln(1+c) - c/(1+c)}, & \text{if } f_T(R) < 1 \\ 1, & \text{otherwise} \end{cases} \tag{30}$$

where $x = c(m) \cdot f_T(R)$. Since c is a weak function of m, the remaining mass fraction is also a weak function of m. Therefore, even the final resolved subhalo mass function is approximately decoupled in mass and radius, in agreement with the results of Han et al. (2015). Tidal disruption in the FDM case is similar, the density profile is truncated at the tidal radius. However, since the mass and radius of a soliton are inversely related (Marsh and Pop 2015), if the tidal radius is less than the soliton radius a runaway process will completely disrupt the halo. This is because as mass is stripped from the soliton, the soliton grows due to quantum pressure (previously halted by gravitational attraction) which pushes more mass outside the tidal radius. Due to the solitonic cores in FDM, the remaining mass fraction after tidal disruption is strongly dependent on the subhalo mass because the shape of the soliton profile and behavior are strongly mass-dependent.

3.4.3 Calculating the Heating

For a Milky Way-like halo with $M_p = 10^{12}\, M_\odot$, we can calculate heating as a function of radius. We find that the rate of heating is quite weak in the core of the halo where tidal disruption is the dominant effect. However, as seen in figure 1, subhalo heating becomes quite efficient near the solar neighborhood $R_0 = 8\,$kpc. In fact, at R_0 the rate of heating is $\frac{{\rm d}\sigma}{{\rm d}t} = 1.98\,{\rm km\,s^{-1}\,Gyr^{-1}}$ which does accumulate over the lifetime of the galaxy to about 10% of the observed velocity dispersion of $34\,{\rm km\,s^{-1}}$ at R_0. However, slightly beyond the solar neighborhood at $r = 34\,$kpc the rate of heating jumps up to $\frac{{\rm d}\sigma}{{\rm d}t} = 37.6\,{\rm km\,s^{-1}\,Gyr^{-1}}$ which is slightly in excess of the maximum rate allowed by the total velocity dispersion of the thick disk. Therefore, it is marginal if the level of heating predicted by CDM subhalos is in conflict with observational bounds.

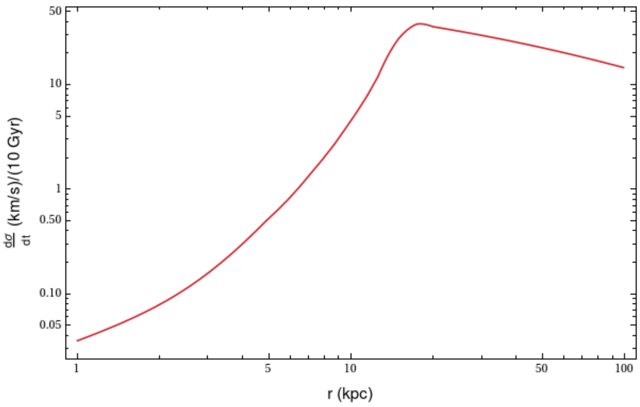

Figure 1. Rate of heating due to dynamical CDM subhalos as a function of radius for a Milky Way-like halo with $M_p = 10^{12}\, M_\odot$ and $\Sigma_0 = 64\, M_\odot\,/{\rm pc}^2$.

3.5 Heating in the FDM Paradigm

3.5.1 The Subhalo Mass Function

The fluctuations due to subhalos dynamics are reduced in FDM compared to CDM because the halo mass function is suppressed at low mass due to the quantum Jeans scale. The FDM halo mass function is calculated numerically from the Press–Schechter formalism (Du et al. 2016; Marsh and Silk 2013) and the soliton profile is fitted from simulations by Schive et al. (2014b).

3.5.2 Tidal Disruption

We employ an identical method for trucating suhalos due to the tidal stripping of the primary halo. However, in FDM, light subhalos are more easily tidally disrupted via a runaway soliton reformation effect. If the tidal radius of a subbalo lies within its soliton core then mass from the soliton will be stripped away which, due to the self-gravitating quantum condition,

$$R \approx \frac{\hbar^2}{m_p^2 M G} \tag{31}$$

forces the soliton to grow in size causing a runaway effect which completely destroys the subhalo. This only requires us to modify our truncation function by setting $T_R(m) = 0$ if the tidal radius lies within the soliton radius.

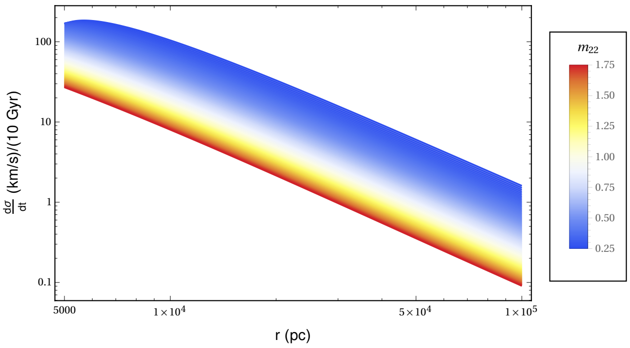

Figure 2. Rate of heating due to wavelets as a function of radius for axion masses m_a in the range $0.25 \times 10^{-22}\,\mathrm{eV}$ to $1.75 \times 10^{-22}\,\mathrm{eV}$. for a Milky Way-like halo with $M_p = 10^{12}\,M_\odot$ and $\Sigma_0 = 64\,M_\odot\,/\mathrm{pc}^2$.

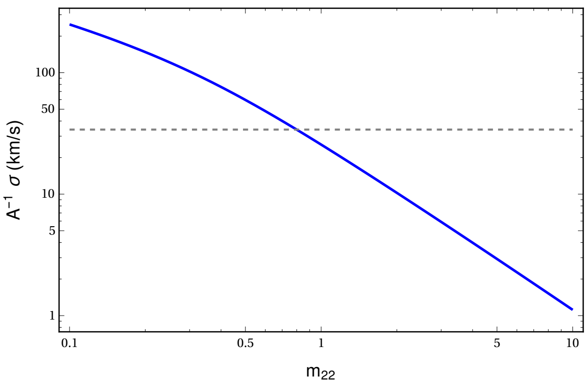

Figure 3. Rate of heating due to wavelets at $r = 8\,\mathrm{kpc}$ as a function of axion masses m_a in the range $0.1 \times 10^{-22}\,\mathrm{eV}$ to $10 \times 10^{-22}\,\mathrm{eV}$. for a Milky Way-like halo with $M_p = 10^{12}\,M_\odot$ and $\Sigma_0 = 64\,M_\odot\,/\mathrm{pc}^2$. The horizontal line is fixed at $34\,\mathrm{km\,s^{-1}}$, the observational maximum on velocity dispersion of the Milky Way thick disk.

3.5.3 *Fluctuations due to Wavelets*

Although the fluctuations due to subhalo transits are reduced in the FDM paradigm in comparision to CDM, for a moderately light particle mass ($m_p \approx 10^{-22}\,\mathrm{eV}$) the dominant effect is due to wavelets, the interference fringes produced by standing FDM waves. In the limit of large particle mass, FDM recreates the results of CDM since the subhalos act identically and the power in wavelets is strongly supressed by a factor of m_{22}^{-3}. For very low particle masses, this power grows rapidly and easily exceeds observational bounds for the velocity dispersion of the Milky Way thick disk. We are primarily interested in the intermediate region in which the transition from wavelet dominated fluctuations to subhalo dominated fluctuations occurs. (INCOMPLETE DUE TO NOT HAVING GOOD FDM SUBHALO RESULTS YET)

3.6 Scaling with Circular Orbit Velocity?

3.7 Comparison with WDM?

4 CONCLUSION

4.1 Disk Destruction

Given a dark matter paradigm, we have shown how to explicitly calculate the heating of the disk caused by dynamical substructure. If we make the further assuption that dark substructure is the dominant disk thickening effect then we have a method of predicting the actual velocity dispersion of a disk as a function of radius. However, in a galaxy with a known surface density profile, there is a simple relationship between the velocity dispersion and the scale height of a disk given by equation 5. Therefore, we can compare the disk shapes predicted by CDM and FDM shown in figures 4 and 5 respectively. Subhalos alone cause very little thickening out to a radius of approximately 15 kpc at which point hte disk flares exponentially. This rapid growth in thickness will cause the outer disk to become unstable and lead to disk destruction beyond a certain radius. (NOT SURE HOW TO CALCULATE THIS). FDM wavelets on the otherhand predict significat thickening at all radii. For example, at the solar neighborhood R_0 an axion of mass $m_p = 1.3 \times 10^{-22}\,\mathrm{eV}$ will lead to a scale height of 1.6 kpc. Wavelets will also give rise to exponentially flaring disks. However, because wavelets produce their maximum power at small radii and subhalos at large radii, the disk flaring caused by wavelets is significatly weaker than the CDM counterpart. Furthermore, the transition point from an approximatly flat disk to a flaring disk is highly sensitive to particle mass. Therefore, the radius at which disk destruction takes place is quite sensitive to m_{22} although it will be at a larger radii than then comperable CDM number. (IN PROGRESS)

4.2 Constraints on Particle Mass

The Milky Way disk has two components, a recently formed thin disk with low velocity dispersion and a old thick disk with large relatively large velocity dispersion. The thick disk has velocity dispersion of approximately $34\,\mathrm{km\,s^{-1}}$ in its thickest parts (Binney 2009). Therefore, the velocity dispersion caused by substructure heating cannot exceed this value. We find that CDM does not produce sufficient heating to be constrained by observational measurements of the Milky Way disk. However, the heating due to wavelets in FDM is strongly dependent on particle mass and far more efficient than other substructure. In the low-mass limit, the heating may vastly exceed observational bounds. Using the results quoted for the velocity dispersion of the thick disk as a strict cutoff, we derive a lower bound on the mass of the FDM particle,

$$m_p > 0.80 A^{1/3} \times 10^{-22}\,\mathrm{eV} \tag{32}$$

Numerical simulations give the best estimate $A \approx 3.6$ (CITATION?). Using this value, we obtain the most probable bound,

$$m_p > 1.30 \times 10^{-22}\,\mathrm{eV}$$

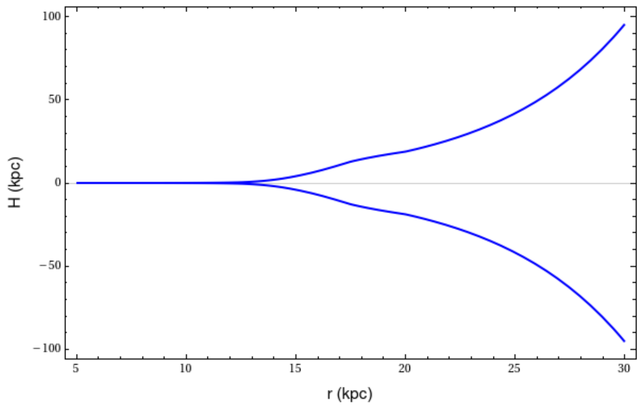

Figure 4. Profile of the galactic disk thickness as a function of radius for a Milky Way-like halo with $M_p = 10^{12}\, M_\odot$ and $\Sigma_0 = 64\, M_\odot\,/\mathrm{pc}^2$. The velocity dispersion of the disk is assumed to be entirely produced by heating due to CDM subhalos.

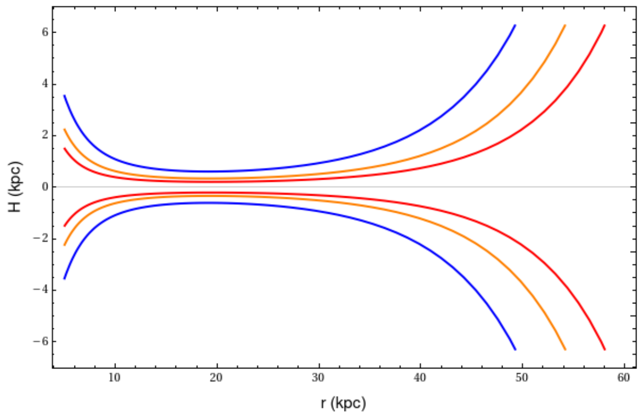

Figure 5. Profile of the galactic disk thickness as a function of radius for a Milky Way-like halo with $M_p = 10^{12}\, M_\odot$ and $\Sigma_0 = 64\, M_\odot\,/\mathrm{pc}^2$. The velocity dispersion of the disk is assumed to be entirely produced by heating due to FDM wavelets. The three cases are for different values of the particle mass: $m_{22} = 1.3$ (blue), $m_{22} = 1.6$ (orange), and $m_{22} = 1.9$ (red).

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REFERENCES

Bensby, T. and Feltzing, S. (2009). The galactic thin and thick discs in the context of galaxy formation.

Binney, J. (2009). Distribution functions for the milky way.

Binney, J. and Tremaine, S. (2008). *Galactic dynamics*. Princeton University Press.

Boylan-Kolchin, M., Bullock, J. S., and Kaplinghat, M. (2011). Too big to fail? the puzzling darkness of massive milky way subhaloes. *Monthly Notices of the Royal Astronomical Society: Letters*, 415(1):L40–L44.

Brook, C. B., Kawata, D., Gibson, B. K., and Freeman, K. C. (2004). The emergence of the thick disk in a cdm universe.

Du, X., Behrens, C., and Niemeyer, J. C. (2016). Substructure of fuzzy dark matter haloes.

Gilmore, G. and Reid, N. (1983). New light on faint stars – iii. galactic structure towards the south pole and the galactic thick disc. *Monthly Notices of the Royal Astronomical Society*, 202(4):1025–1047.

Giocoli, C., Tormen, G., and van den Bosch, F. C. (2007). The population of dark matter subhaloes: Mass functions and average mass loss rates.

Han, J., Cole, S., Frenk, C. S., and Jing, Y. (2015). A unified model for the spatial and mass distribution of subhaloes.

Hu, W., Barkana, R., and Gruzinov, A. (2000). Cold and fuzzy dark matter.

Hui, L., Ostriker, J. P., Tremaine, S., and Witten, E. (2016). Ultralight scalars as cosmological dark matter.

Klypin, A., Kravtsov, A. V., Valenzuela, O., and Prada, F. (1999). Where Are the Missing Galactic Satellites? *ApJ*, 522:82–92.

Lacey, C. G. and Ostriker, J. P. (1985). Massive black holes in galactic halos? *ApJ*, 299:633–652.

Lee, J. (2003). Formalism for the subhalo mass function in the tidal-limit approximation.

Marsh, D. J. E. (2015). Axion cosmology.

Marsh, D. J. E. and Pop, A.-R. (2015). Axion dark matter, solitons, and the cusp-core problem.

Marsh, D. J. E. and Silk, J. (2013). A model for halo formation with axion mixed dark matter.

Mocz, P., Lancaster, L., Fialkov, A., Becerra, F., and Chavanis, P.-H. (2018). On the schrodinger-poisson–vlasov-poisson correspondence.

Mocz, P. and Succi, S. (2015). Numerical solution of the nonlinear schrodinger equation using smoothed-particle hydrodynamics.

Mocz, P., Vogelsberger, M., Robles, V., Zavala, J., Boylan-Kolchin, M., Fialkov, A., and Hernquist, L. (2017). Galaxy formation with becdm: I. turbulence and relaxation of idealised haloes.

Navarro, J. F., Frenk, C. S., and White, S. D. M. (1995). The structure of cold dark matter halos.

Schive, H.-Y., Chiueh, T., and Broadhurst, T. (2014a). Cosmic structure as the quantum interference of a coherent dark wave.

Schive, H.-Y., Liao, M.-H., Woo, T.-P., Wong, S.-K., Chiueh, T., Broadhurst, T., and Hwang, W.-Y. P. (2014b). Understanding the core-halo relation of quantum wave dark matter, ψdm, from 3d simulations.

Seljak, U., Makarov, A., McDonald, P., and Trac, H. (2006). Can sterile neutrinos be the dark matter?

Steinmetz, M. (2012). The galactic thin and thick disk.