ASTR GR6001 Radiative Processes
Assignment # 6

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1 Problem 1

(a)

If the optical depth is thin along the entire line then we have,

$$W_\lambda = \int_0^\infty \left[1 - e^{-\tau_\nu}\right] \mathrm{d}\lambda \approx \int_0^\infty \tau_\nu \, \mathrm{d}\lambda \approx \frac{\lambda_0^2}{c} \int_0^\infty \tau_\nu \, \mathrm{d}\nu$$

Furthermore, we know that,

$$\tau_\nu = N\sigma(\nu) = \frac{\pi e^2 N}{mc} f_{12} \, \phi(\nu)$$

Then,

$$\int \tau_\nu \, \mathrm{d}\nu = \frac{\pi e^2 N}{mc} f_{12}$$

since the line profile is normalized. Therefore,

$$\frac{W_\lambda}{\lambda_0} = \frac{\lambda_0}{c} \frac{\pi e^2 N}{mc} f_{12} = \pi r_0 (N \lambda_0 f_{12})$$

(b)

We assume that, at the line center, the line profile is dominated by Doppler broadening such that,

$$\phi(\nu_0) = \frac{c}{b \nu_0 \sqrt{\pi}}$$

Therefore,

$$\tau_0 = \frac{\pi e^2 N}{mc} f_{12} \cdot \frac{c}{b \nu_0 \sqrt{\pi}} = \frac{\sqrt{\pi} e^2}{mc^2} \cdot \left(\frac{c}{b}\right) (N \lambda_0 f_{12}) = \sqrt{\pi}\, r_0 \cdot \left(\frac{c}{b}\right) \cdot (N \lambda_0 f_{12})$$

In particular, we see that,

$$\tau_0 = \frac{W_\lambda}{\lambda_0} \cdot \left(\frac{c}{b\sqrt{\pi}}\right)$$

Now, to compute the number density for Lyα we need parameters,

$$\lambda_0 = 1215.7 \text{ Å}$$
$$f_{12} = 0.4216$$

and we have the constant,

$$r_0 = 2.82 \cdot 10^{-13}\ \mathrm{cm}$$

Therefore, putting $\tau_0 = 0.1$ and $b = 6\ \mathrm{km\,s^{-1}}$ we find,

$$N(\mathrm{HI}) = 7.81 \cdot 10^{11}\ \mathrm{cm}^{-2}$$

(c)

Suppose that b is thermal i.e.

$$b = \sqrt{\frac{2k_B T}{m_H}}$$

Then we have,

$$T = \frac{m_H b^2}{2k_B} = 2180\ {}^\circ\mathrm{K}$$

2 Problem 2

As before, assuming that Doppler broadening dominates the line profile at its center we have an expression for the center-line optical depth,

$$\tau_0 = \sqrt{\pi}\, r_0 \cdot \left(\frac{c}{b}\right) \cdot (N\lambda_0 f_{12})$$

Using the same parameters as before but for $\tau_0 = 100$ and $b = 6\ \mathrm{km\,s^{-1}}$ we find,

$$N(\mathrm{HI}) = 7.81 \cdot 10^{14}\ \mathrm{cm}^{-2}$$

Now, since we are the flat regime, we have derived an approximate expression for the line width,

$$\frac{W_\lambda}{\lambda_0} = \frac{2b}{c}\sqrt{\log\left(\tau_0/\log 2\right)}$$

For $\tau_0 = 100$ this gives,

$$\frac{W_\lambda}{\lambda_0} = 9.64 \cdot 10^{-5}$$

3 Problem 3

Now we consider a damped Lyα absorber with column density,

$$N(\mathrm{HI}) = 3 \cdot 10^{20}\ \mathrm{cm}^{-2}$$

3.1 (a)

In the square-root part of the curve of growth we have,

$$\frac{W_\lambda}{\lambda_0} = \sqrt{\frac{\lambda_0^2 N \sigma_{12} \Gamma}{4\pi}}$$

where,

$$\sigma_{12} = \frac{\pi e^2}{mc} f_{12} \qquad\qquad \Gamma = \frac{8\pi^2 e^2}{3mc\lambda_0^2} f_{21}$$

Therefore,

$$\frac{W_\lambda}{\lambda_0} = \frac{\pi e^2}{mc} \left(\frac{2N f_{12} f_{21}}{3} \right)^{\frac{1}{2}} = \pi r_0 f_{12} \sqrt{\frac{2N g_1}{3 g_2}}$$

Therefore, plugging in, we find,

$$\frac{W_\lambda}{\lambda_0} = 3.05 \cdot 10^{-3}$$

3.2 (b)

When the Lyα is damped, the primary contribution to the equivalent width is the wings of the Lorentzian distribution. However, at line-center, the primary contibution to τ_0 is still due to Doppler broadening so,

$$\tau_0 = \sqrt{\pi}\, r_0 \cdot \left(\frac{c}{b} \right) \cdot (N \lambda_0 f_{12})$$

If we set $b = 6\,\mathrm{km\,s^{-1}}$ then we find,

$$\tau_0 = 3.84 \cdot 10^7$$