1 TODO!!

(a) Finish symplectic geometry course

   (a) figure out if symplectic toric is the same as projective toric variety (projectivity needed to come from a polytope and also to be Kahler)
   (b) review coisotropic reduced and write some notes
   (c) hyperkahler reduction examples
   (d) are there examples of noncompact hyperkahlers?
   (e) work out the kinks in notes on hamiltonian actions

(b) review killing homotopy groups columbia lectures and write some notes

(c) figure out those damn jet bundles and connections on principal bundles

   (a) RMK: π^*E is NOT trivial for a vector bundle let alone a fiber bundle. it does get equiped with a canonical section but for a vector bundle this is just the trivial section, only for a principal bundle does giving a section trivialize it.
   (b) role of atiyah sequence vs jet bundle sequence
   (c)

(d) spectral sequences for tor and ext in derived category

   (a) application to universal coefficient theorem
   (b) Kunneth spectral sequence
   (c) Kunneth formula for smash product?
   (d) why are derived functors triangulated
   (e) derived functors in terms of Kan extensions (NOTES)

(e) write notes on universal morphisms

(f) G-action of X/Y induces map Descent data X/Y to G-equivariant sheaves

   (a) isomorphism when X/Y is a G-cover i.e. $X \to Y$ is a G-torsor
   (b) write down explicit G-equivariant structure on Ω_X
   (c) Galois descent derive explicit form

(g) Weil restriction

   (a) write down trivialization after going back up
   (b) Galois descent in explicit form

(h) notes on Galois actions on schemes

(i) notes on Frobenii

(j) notes on universal constructions in math with examples

2 Some Connection Musings

Definition 2.0.1. Let $f : E \to X$ be a smooth surjection (in the smooth category, what should it be in the algebraic category?) then an *Erhesmann connection* is a splitting of the sequence of vector bundles,

$$0 \longrightarrow \ker \mathrm{d}f \longrightarrow TE \longrightarrow \pi^* TX \longrightarrow 0$$

where we usually call $V = \ker df$ the vertical bundle. In algebraic language, V is the dual of the relative differentials so the connection corresponds to a splitting of,

$$0 \longrightarrow f^*\Omega_X \longrightarrow \Omega_E \longrightarrow \Omega_{E/X} \longrightarrow 0$$

Remark. Such splittings are supposed to correspond to smooth sections of the map $J^1(E) \to E$. We now explain how this works. Unfortunately, I don't know a good unified language to describe the jet bundles so I will give the algebraic and smooth definitions.

Definition 2.0.2. Given a smooth surjection $f : E \to X$, consider the n-th thickened diagonal, $X \to X_n \to X \times_S X$. Then we consider the functor sending $T \to S$ to pairs of maps $T \to X$ and $T \times_X X_n \to E$ such that the diagram,

$$\begin{tikzcd} E \arrow[r, "f"] & X \\ T \times_X X_n \arrow[u] \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & X_n \arrow[u, "\pi_1"'] \arrow[d, "\pi_2"] \\ T \arrow[r] & X \end{tikzcd}$$

commutes. Then the jet scheme $J_n(E/X)$ with maps $J_n(E/X) \to X$ and $J^n(E/X) \times_X X_n \to E$ represents this functor.

3 Counterexamples In Geometry

Example 3.0.1. The Hopf surface is the compact complex surface $H = \mathcal{C}^2 \setminus \{0\}/\mathbb{Z}$ where $\mathbb{Z} \circlearrowright \mathcal{C}^2$ via $(z_1, z_2) \mapsto (\lambda z_1, \lambda z_2)$ for $0 < \lambda < 1$. This surface has $h^{1,0} = 1$ but $h^{0,1} = 0$. Furthermore, H is diffeomorphic to $S^3 \times S^1$. This provides:

(a) a compact complex manifold that is not Kähler

(b) a compact complex manifold without Hodge symmetry

(c) a compact complex manifold that is not symplectic ($H^2(H, \mathbb{Z}) = 0$)

Remark. From the exponential exact sequences,

$$0 \longrightarrow \mathbb{Z} \longrightarrow \mathcal{O}_X \xrightarrow{\exp} \mathcal{O}_X^\times \longrightarrow 0$$

we have that,

4 Two Capacitors

Let $q_1(0) = Q$ and $q_2(0) = 0$ and $I = \dot{q}_1 = \dot{q}_2$ so $q = q_1 - q_2$ is constant so $q_1 = Q + q_2$. Now,

$$R\dot{q}_2 + \frac{q_1}{C_1} + \frac{q_2}{C_2} = 0$$

Therefore,

$$\dot{q}_2 = \frac{Q}{RC_1} + q_2\left(\frac{1}{RC_1} + \frac{1}{RC_2}\right)$$