0.1 K-theory of the moduli space of genus zero curves

Definition 0.1.1. $\mathcal{M}_{0,n}$ parametrizes genus zero curves with n labeled distinct points (up to isomorphism fixing the marked points).

Remark. We will always take $n \geq 3$ for stability reasons. Notice that $\mathcal{M}_{0,3} = *$ and $\mathcal{M}_{0,4} = \mathbb{P}^1 \setminus \{0, 1, \infty\}$. We see that $\mathcal{M}_{0,n}$ is noncompact. We will consider a nice compactification.

Definition 0.1.2. $\overline{\mathcal{M}}_{0,n}$ parametrizes stable genus zero curves with n marked points. This is a tree of $\mathbb{P}^1$ with at last 3 nodes or marked points on each component.

Example 0.1.3. $\overline{\mathcal{M}}_{0,3} = *$ and $\overline{\mathcal{M}}_{0,4} = \mathbb{P}^1$ and $\overline{\mathcal{M}}_{0,5}$ is the blowup of $\mathbb{P}^2$ at four general points.

Proposition 0.1.4. $\dim \overline{\mathcal{M}}_{0,n} = n - 3$ and $\mathcal{M}_{0,n} \subset \overline{\mathcal{M}_{0,n}}$ is a dense open.

Remark. $\psi : \overline{\mathcal{M}}_{0,n+1} \to \overline{\mathcal{M}}_{0,n}$ forgetting points and contracting components if necessary induces a birational map then there are sections s_i which induce divisors and we define the ψ_i classes as the conormal bundles of these sections.

Theorem 0.1.5 (Kapranov)**.** The ψ_i are base-point-free line bundle and defines a birational map,

$$\overline{\mathcal{M}}_{0,n} \twoheadrightarrow \mathbb{P}^{n-1}$$

where $\overline{\mathcal{M}}_{0,n}$ maps to

Remark. For $S \subset [1, \dots, n-1]$ with $|S| \geq 3$ we can consider forgetfull maps,

$$\begin{tikzcd}
\overline{\mathcal{M}}_{0,n} \arrow[d] \arrow[dr] & \\
\overline{\mathcal{M}}_{0,k} \arrow[r, "\psi"] & \mathbb{P}^{k-2}
\end{tikzcd}$$

therefore we have,

$$\overline{\mathcal{M}}_{0,n} \hookrightarrow \prod_{\substack{S \subset [n-1] \\ |S| \geq 3}} \mathbb{P}^{|S|-2}$$

Then we let $X_s = C_i(f_* \psi_n)$