

ASTR GR6001 Radiative Processes

Assignment # 1

Benjamin Church

February 17, 2020

Problem 1

Consider an ideal blackbody star with specific intensity B_ν constant on its surface and independent of angle in the outwards facing hemisphere. Now we may compute,

$$\begin{aligned}J_\nu &= \frac{1}{4\pi} \int B_\nu d\Omega = \frac{1}{4\pi} \int_0^1 B_\nu d\cos\theta \int_0^{2\pi} d\phi = \frac{1}{2} B_\nu \\F_\nu &= \int B_\nu \cos\theta d\Omega = \int_0^1 B_\nu \cos\theta d\cos\theta \int_0^{2\pi} d\phi = \pi B_\nu \\P_\nu &= \frac{1}{c} \int B_\nu \cos^2\theta d\Omega = \frac{1}{c} \int_0^1 B_\nu \cos^2\theta d\cos\theta \int_0^{2\pi} d\phi = \frac{2\pi}{3c} B_\nu \\u_\nu &= \frac{4\pi}{c} J_\nu = \frac{2\pi}{c} B_\nu\end{aligned}$$

Problem 2

Consider a radiation field of the form,

$$I_\nu(\mu) = a_\nu + b_\nu \mu$$

where $\mu = \cos\theta$. Then we may compute,

$$\begin{aligned}J_\nu &= \frac{1}{4\pi} \int B_\nu d\Omega = \frac{1}{4\pi} \int_{-1}^1 [a_\nu + b_\nu \mu] d\mu \int_0^{2\pi} d\phi = a_\nu \\F_\nu &= \int B_\nu \cos\theta d\Omega = \int_{-1}^1 [a_\nu + b_\nu \mu] \mu d\mu \int_0^{2\pi} d\phi = \frac{4}{3} \pi b_\nu \\P_\nu &= \frac{1}{c} \int B_\nu \cos^2\theta d\Omega = \frac{1}{c} \int_{-1}^1 [a_\nu + b_\nu \mu] \mu^2 d\mu \int_0^{2\pi} d\phi = \frac{4\pi}{3c} a_\nu \\u_\nu &= \frac{4\pi}{c} J_\nu = \frac{4\pi}{c} a_\nu\end{aligned}$$

Therefore this radiation field has the same energy density and pressure as an isotropic one however has nonzero flux unlike the purely isotropic case.