Definition 0.0.1. Let X be a compact irreducible hyperkahler manifold: X a compact KAhler manifold and $\pi_1(X) = 0$ and $H^0(X, \Omega_X^2) = \mathbb{C}\sigma$ where σ is nowhere degenerate.

Example 0.0.2. (a) if $\dim X = 2$ then X is just a K3 surface

(b) $S^{[g]}$ the Hilbert scheme of g points on a K3

(c) $M_v(S)$ is the moduli space sof stable sheaves on S with the pairing,

$$\omega : \operatorname{Ext}^1_S(E, E) \times \operatorname{Ext}^1_S(E, E) \to \mathbb{C}$$

gives a holomorphic symplectic form on the tangent space

(d) If H is an able divisor on S and $\mathcal{C}/|H|$ then $\overline{\operatorname{Pic}}^d{}_{\mathcal{C}/|H|}$

(e) noncompact: T^*C and $M_v(T^*C)$ is the moduli space of Higgs bundles.

Theorem 0.0.3 (Matsushita)**.** Let X be compact irreducible HK manifold $\dim X = 2g$. The only fibrations X can have are $f : X \to B$ with σ-Lagrangian fibers which are generically Lagrangian g-dim abelian varieties.

Example 0.0.4. For S is a K3 surface then every fibration is of the form $S \to \mathbb{P}^1$ making S an elliptic surface. Then either,

(a) isotrivial (constant j-invariant) for example $\operatorname{Kum}(E_1 \times E_2) \to E_2/\pm 1 = \mathbb{P}^1$

(b) fibers vary in moduli

Then we get $S^{[g]} \to \mathbb{P}^g$ which is fibered in abelian varieties which are products of elliptic curves. In this case either,

(a) $S \to \mathbb{P}^1$ is isotrivial then $S^{[g]} \to \mathbb{P}^g$ is isotrivial

(b) $S^{[g]} \to \mathbb{P}^g$ has g-dimensional moduli variation (since the fiber is $E_{z_1} \times \cdots \times E_{z_g}$ have moduli varying independently).