**On the Shioda Conjecture for Diagonal Weighted-Projective Varieties over Finite Fields**

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An algebraic variety is a geometric object given as the solution set to a system of polynomial equations. In this work we study a family of simple weighted-projective algebraic hypersurfaces defined over finite fields known as “diagonal varieties” which are cut out by a single polynomial without cross terms. Andre Weil famously studied such varieties in his seminal 1949 paper *Numbers of Solutions of Equations in Finite Fields* which proved a connection between the number of solutions of a diagonal variety over a finite field and the values of particular Gaussian sums. He also first proposed the Weil conjectures in their modern form. The Weil conjectures (proven in the 1960s) constrain the zeta function for a smooth projective variety defined over the algebraic closure of a finite field to have form of a rational function with degrees giving the dimensions of cohomology and whose roots and poles give the eigenvalues of the Frobenius automorphism on the cohomology which determines how the variety is decomposed in finite subfields by the Galois action. This information together with Weil’s application of Gaussian sums allows us to transfer the problem of determining properties of this Galois action to that of finding the splitting of Gaussian sums in abelian number fields. Specifically, we are investigating two special properties an algebraic variety might have: unirationality and supersingularity. Tetsuji Shioda conjectured that these two notions exactly coincide and, in 1979, proved this conjecture for Fermat surfaces and gave a classification of all supersingular Fermat varieties. Using similar methods, we have demonstrated the existence of three new families of supersingular weighted-projective diagonal varieties one of which we have demonstrated to be rational (and thus unirational) in accordance with the Shioda conjecture.