EFFECTIVE PARAMETERS OF FERROELECTRIC DIELECTRIC MIXTURES

B. Vial and Y. Hao

School of Electronic Engineering & Computer Science, Queen Mary, University of London

ABSTRACT

We investigate the homogenized parameters of ferroelectric-dielectric composites under a static electric field. A numerical model that takes into account the coupling between the electrostatic problem and the electric field dependent permittivity of the ferroelectric material is used.

Metamaterials consisting of periodic and random arrays of rods are considered for the transverse electric polarization case and we study their effective permittivity, losses, electrically induced anisotropy and tunability by a two scale convergence homogenization method.

CONTEXT

Ferroelectric materials play a crucial role in reconfigurable microwave devices, with typical applications including antenna beam steering, phase shifters, tunable power splitters, filters, voltage controlled oscillators and matching networks [1], and the key requirements are large tunability and low losses. These materials have high permittivity values even at microwave frequencies, which can be an issue in some practical applications. Thus it has been considered to mix ferroelectrics with low-index and low-loss non-tunable dielectrics in order to reduce both permittivity values and losses, or to use porous ceramics [2, 3].

METHOD

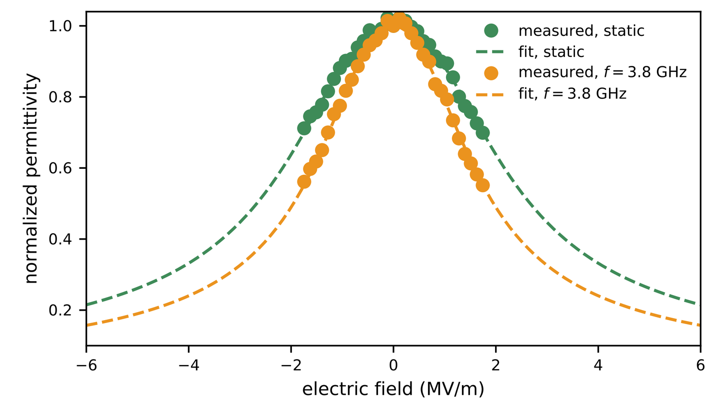
Ferroelectric permittivity

Landau potential given by $F(P,E) = F_0 + \alpha P^2/2 + bP^4/4 + cP^6/6 - EP$, where E is the applied electric field and P is the polarization [4, 5]

$$\varepsilon^{\mathrm{f}}(E) = \left[\frac{\partial^2 F(P,E)}{\partial P^2}\right]^{-1} = \frac{\varepsilon^{\mathrm{f}}(0)}{1+\alpha P_0^2 + \beta P_0^4}, \quad (1)$$

Anisotropy

$$\varepsilon^{\mathrm{f}}(E) = \begin{pmatrix} \varepsilon^{\mathrm{f}}_{xx}(E_x) & 0 & 0 \\ 0 & \varepsilon^{\mathrm{f}}_{yy}(E_y) & 0 \\ 0 & 0 & \varepsilon^{\mathrm{f}}_{zz}(E_z) \end{pmatrix} \quad (2)$$



Variation of the ferroelectric permittivity as a function of the applied electric field.

case	$\varepsilon^{\mathrm{f}}(0)$	α (μm^2/V^2)	β (μm^4/V^4)
static	3050	0.120	0.024
$f = 3.8$ GHz	165	0.240	0.079

Electrostatic model

Gauss' law for the potential V:

$$\nabla\cdot(\varepsilon\nabla V) = 0 \quad (3)$$

Coupled problem

The electric field $E = -\nabla V$ derived from the solution of Eq. (3) depends on the permittivity distribution, which itself depends on the electric field through Eq. (1).

Homogenization

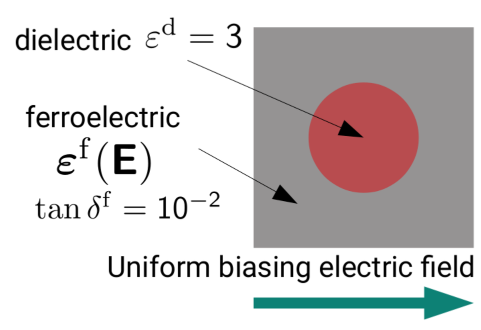
Two scale convergence homogenization [6, 7], TE case: solutions ψ_j of two annex problems $\mathcal{P}_j$, $j = \{1,2\}$:

$$\nabla\cdot\left[\xi\nabla(\psi_j + r_j)\right] = 0, \quad (4)$$

where $\boldsymbol{r} = (x,y)^{\mathrm{T}}$ and $\boldsymbol{\xi} = \boldsymbol{\varepsilon}^{\mathrm{T}}/\det(\boldsymbol{\varepsilon})$.
The homogenized tensor $\tilde{\xi}$ is obtained with:

$$\tilde{\xi} = \langle\xi\rangle + \phi, \quad (5)$$

where $\langle . \rangle$ denotes the mean value over the unit cell.
Correction terms $\phi_{ij} = \langle \boldsymbol{\xi}\nabla\psi_i\rangle_j$.



Periodic unit cell

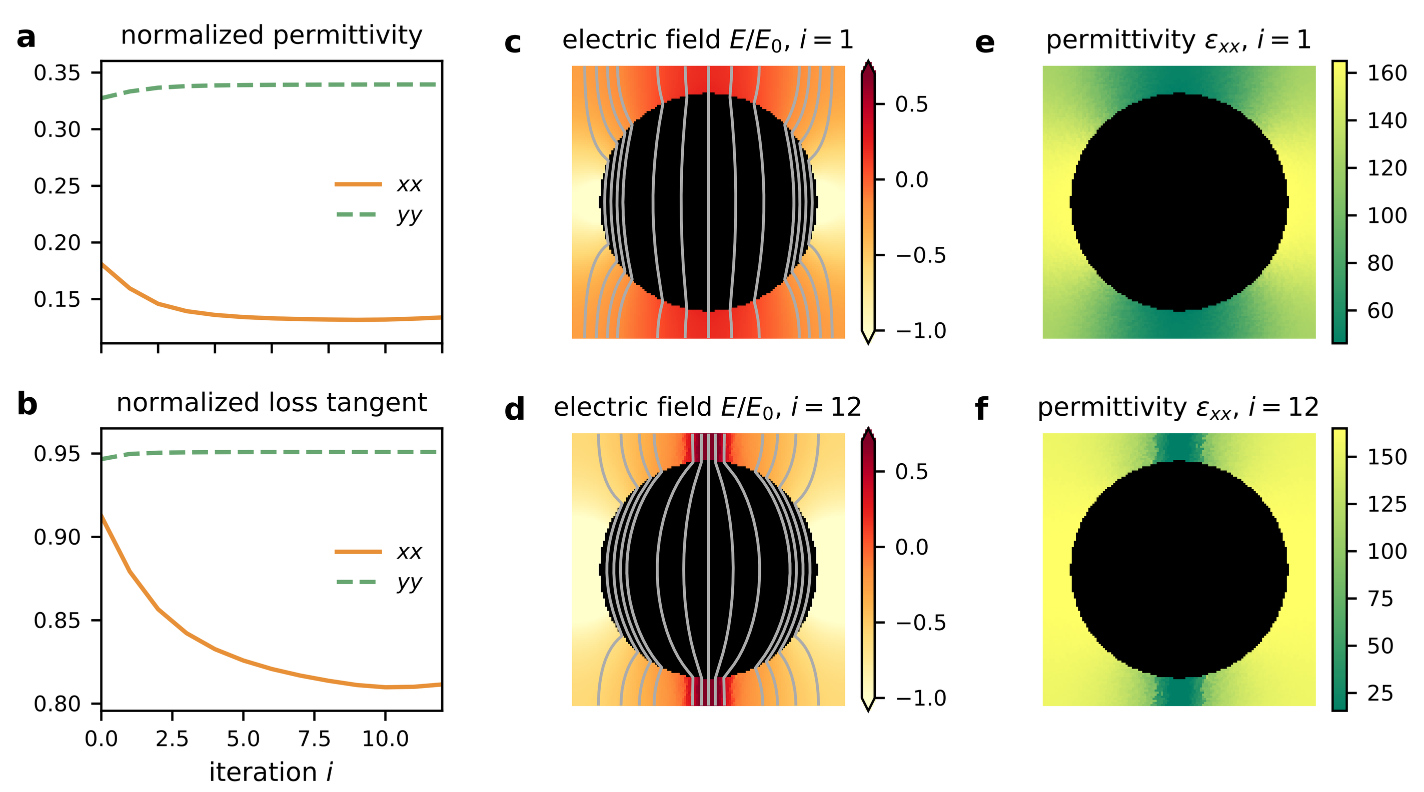
Effective permittivity tensor: $\tilde{\varepsilon} = \tilde{\xi}^{\mathrm{T}}/\det(\tilde{\xi})$.

Open source numerical implementation

Equations (3) and (4) are solved with a Finite Element Method using the libre software Gmsh [8] and GetDP [9] driven by a Python package.

RESULTS

Periodic case

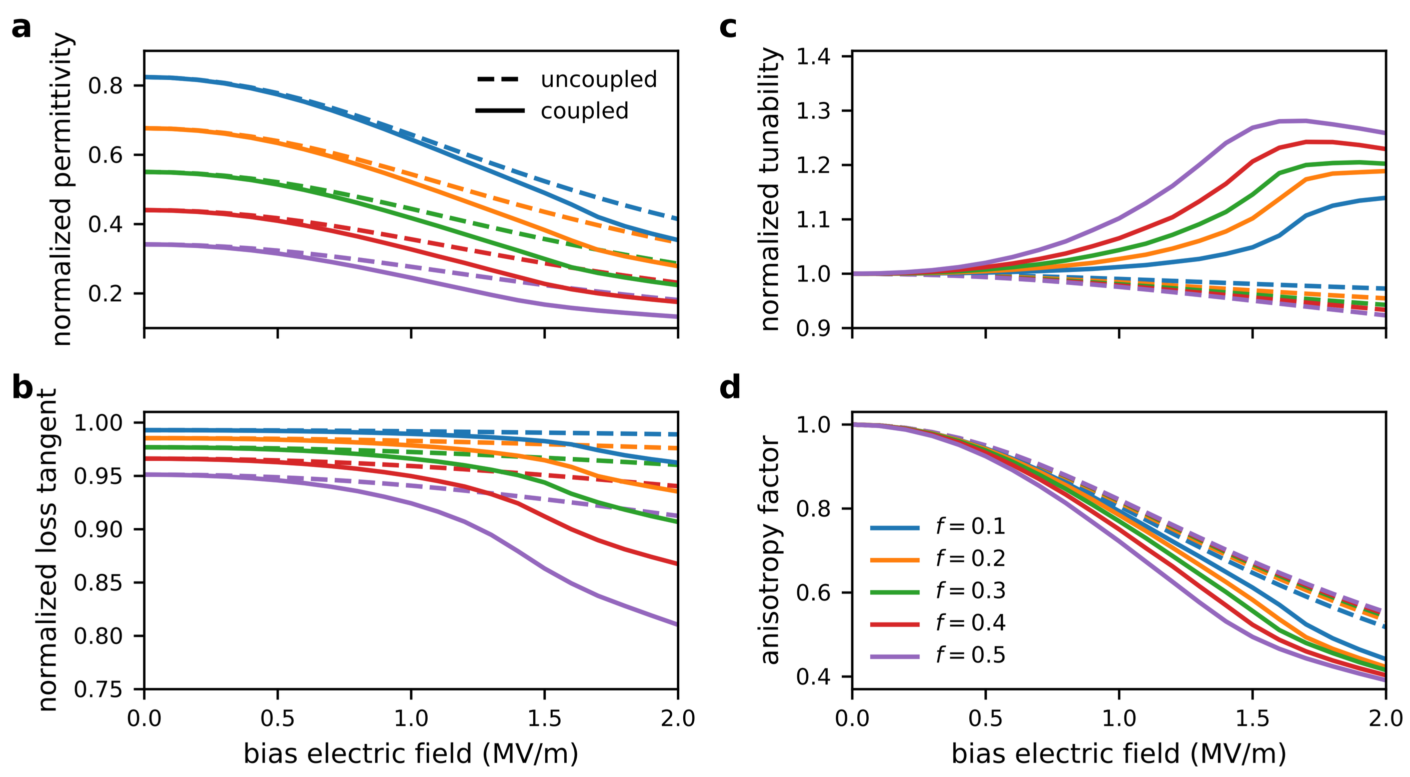


Convergence of the coupled problem. Real part (a) and loss tangent (b) of the components of the homogenized permittivity tensor as a function of iteration step i. The distribution of the normalized electric field (colour map: magnitude in logarithmic scale, lines: equipotential contours) and of the xx component of the permittivity tensor are shown for $i = 1$ (c and d) and $i = 12$ (e and f).

RESULTS

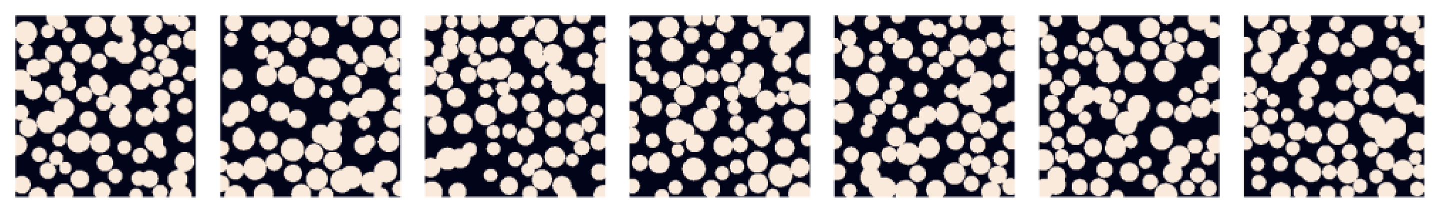
Effective parameters

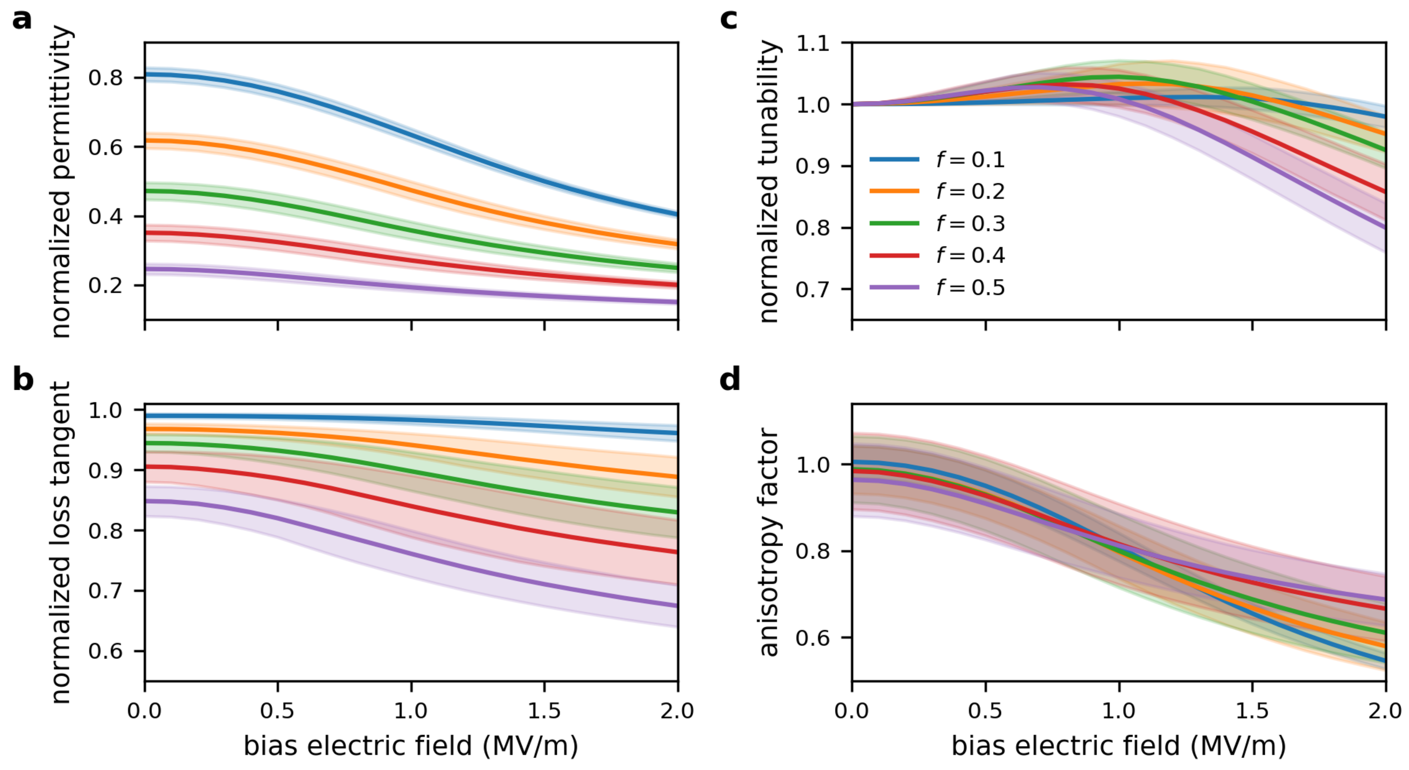
Tunability $n = \varepsilon_{xx}(0)/\varepsilon_{xx}(E)$, anisotropy factor $a = \varepsilon_{xx}(E)/\varepsilon_{yy}(E)$



Effective parameters of the 2D metamaterials as a function of the applied electric field for various filling fraction of dielectric. (a): normalized permittivity, (b): normalized loss tangent, (c): normalized tunability and (d): anisotropy factor. The solid lines correspond to the coupled model and the dashed lines to the uncoupled model. Values are normalized with respect to the bulk properties.

Pseudo-random case





Effective parameters of the pseudo-random composites as a function of the applied electric field for various filling fraction of dielectric, when the coupling is taken into account. (a): normalized permittivity, (b): normalized loss tangent, (c): normalized tunability and (d): anisotropy factor. The solid lines represent the average values over the 21 samples and the lighter error bands show a confidence interval corresponding to the standard deviation.

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CONTACT & LINKS

Corresponding author: Benjamin Vial | b.vial@qmul.ac.uk | bvial.info
Github | github.com/benvial/ferromtm
Antennas & Electromagnetics Research Group | antennas.eecs.qmul.ac.uk