DeGroot Learning

Marty Rudolf

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Suppose there is a vote matrix, V, of size c x p, where c is the number of choices made and p is the number of people making each of these choices. Every choice, c_i, made by person, p_j, amounts to a vote v_{ij}. Where $\forall(v_{ij} \in \{-1, 0, 1\})$ and $0 \leq i \leq c, 0 \leq j \leq p$:

$$V = \begin{bmatrix} v_{00} & \cdots & v_{0c} \\ \vdots & v_{ij} & \vdots \\ v_{p0} & \cdots & v_{pc} \end{bmatrix} \quad (1)$$

To find the similarity, s, between any two people, say l and k where $0 \leq l, k \leq p$, we compute:

$$s_{lk} = \mid \bar{p}_l \bar{p}_k \mid \quad (2)$$

for all $l, k \mid l \neq k$. Such that,

$$S = \begin{bmatrix} s_{00} & \cdots & s_{0p} \\ \vdots & s_{lk} & \vdots \\ s_{p0} & \cdots & s_{pp} \end{bmatrix} \quad (3)$$

To find the trust, t, between any two people, say l and k where $0 \leq l, k \leq p$, we compute:

$$t_{lk} = \frac{s_{lk}}{\sum_{m=0}^{p} s_{lm}} \quad (4)$$

Then, for any number of people, p, the trust matrix, T, is a right stochastic p x p matrix:

$$T = \begin{bmatrix} t_{00} & \cdots & t_{0p} \\ \vdots & t_{lk} & \vdots \\ t_{p0} & \cdots & t_{pp} \end{bmatrix} \qquad (5)$$

to which the DeGroot Learning algorithm can be applied.

From there, every future choice, f_c, can be calculated as:

$$\lim_{t \to \infty} T^t p^c = f_c \qquad (6)$$

where p^c is the *1* x p array of votes made be persons, p, regarding choice c.

Those votes are then averaged out to find the ultimate ranking of choices:

$$r = \frac{f}{|\ f\ |} \qquad (7)$$

Such that:

$$r = \begin{bmatrix} r_0, & \cdots, & r_c \end{bmatrix} \qquad (8)$$

is the ranking for each choice c.

More info can be got at `https://en.wikipedia.org/wiki/DeGroot_learning`.
Python implementation can be found at `https://github.com/benvolioo/DeGroot`.