Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2227388

Risk-Return Balanced Trading Strategies Based on Heuristic and Dynamic Fusion

Summary

Served as one of the most influential decentralized and largely unregulated assets, bitcoin has attracted traders' attention globally with extremely high volatility and potential return. Gold, however, possesses much stabler prices and can offer investors protection against inflation in the long run. In this paper, we develop promising trading strategies to help traders gain a decent return while controlling the downside risk at the same time.

Firstly, based on the renowned sector rotation strategies in the stock market, we propose the **rotation strategy** for bitcoin, gold, and cash, which defines the timing for the trader to switch the position from one type of asset to another. We use the **Momentum** factor to gain insights into the future price movement. The designed strategy withdraws the position from bitcoin rapidly to avoid a potential bear market, achieving a high cumulative return of over 11153% and significantly controlling the maximum drawdown to be -36.33% in the backtesting.

Then, in order to obtain more accurate signals with limited data, we perform traditional **technical analysis** towards the bitcoin and gold separately and build an advanced **adaptive Guangfa Thomas R. DeMark (GFTD) strategy** based on the powerful Thomas R. DeMark indicator system. We tune the model parameters and backtest on bitcoin and gold respectively. The GFTD strategy gives over 15693% and 47.51% cumulative returns while effectively reducing the maximum drawdown to be -48.16% and -12.44% for bitcoin and gold.

Next, after designing several powerful timing signals, we utilize the classic **Markowitz model** in portfolio management science to optimize portfolio weight allocations for traders with different levels of risk preference. We further improve Markowitz's mean-variance utility by creating a **sliding window constraint** and propose the **dynamic strategy fusion method with sliding window** to dynamically combine all the individual strategies.

Moreover, we conduct performance comparisons among all the strategies we have obtained. It turns out that the fused strategy combines the advantages of each individual strategy and gives a decent overall level of performance metrics: a cumulative return of over 11,519.44%, an annualized return of over 158.84%, maximum drawdown within -46.36%, and the highest Sharpe ratio of over 1.7 for a standard risk-averse trader. We further analyze how traders' risk preference level affects the strategy performance. The lower risk the trader can tolerate, the fewer transactions the trader will make to avoid the short-term volatility for bitcoin prices. Lastly, we evaluate the robustness of our strategy by adding uniform random noise to the price series. The fused strategy performance remains stable and robust with different levels of random noise ranging from 0.1% to 1% of the price series.

Finally, we use the reciprocal of daily return to develop a particular indicator that measures the **sensitivity** of our strategy performance against the transaction costs. The higher the transaction costs, the fewer transactions the trader will perform. The empirical study results show that different scales of transaction costs will change the dynamic weight allocation for the portfolio but indeed have little impact on the overall behavior of our fused strategy, such as transaction frequency.

Keywords: Rotation Strategy; Timing Signals; GFTD; Dynamic Strategy Fusion; Sensitivity Analysis

Contents

1 Introduction

1.1 Background

Market traders seek to develop their own timing strategies and asset picking capabilities to generate the excess return, namely, alpha return over the market. As one of the most influential decentralized and largely unregulated assets in the cryptocurrency market, bitcoin attracts traders' attention globally with its significant potential return and also hyperbolic volatility [7]. Gold price, however, remains much stabler over the years due to its tender reaction to market sentiments as a commodity.

Institutional investors actively seek opportunities in bitcoin to perform arbitrage, explore useful information from its trading price and volume, and frequently adjust their positions to maximize their profit. Some aggressive traders are even willing to undergo a huge amount of risk by taking high leverage, while some risk-averse traders prefer to use stable assets such as gold to provide themselves with necessary protections in case the price of bitcoin slumps.

1.2 Problem Statement

In the long run, gold offers investors protection against inflation, but holding this type of asset only may exert a considerable drag on the return of a long-term investor's portfolio [10]. Hence our target is to establish a portfolio consisting of gold, bitcoin, and cash, expecting a promising return and a decent level of the maximum drawdown at the same time.

In this problem, we will design appropriate weight allocation among the assets and explore the best trading strategy based only on the daily price data from September 2016 to September 2021. The following stages will be accomplished throughout the process:

- Design proper models to capture the historical price information of gold and bitcoin;
- Develop informative trading signals and tune the model parameters;
- Combine the useful signals through a dynamic fusion process to address the optimal weight allocation among cash, bitcoin, and gold based on the classic Markowitz model;
- Compare and obtain the best model with decent performance metrics, analyze its sensitivity to the transaction costs and robustness.

2 Assumptions and Notations

2.1 Assumptions

Due to the limited price information, we make several reasonable premises to yield more rigorous and realistic strategies that better fit in real-life trading settings. All assumptions will be re-emphasized once they are used in the model construction.

Assumption 1. Traders will release orders to the market at the beginning of the day at the daily price of the previous trading day. The market has enough liquidity to fulfill all the orders immediately.

Assumption 2. Commission fee and possible slippage will be considered. Limit up, limit down, and stamp duty will not be considered.

Assumption 3. Traders cannot short sell bitcoin or gold. They cannot take leverage to obtain more wealth, and therefore minimal margin rate will not be considered.

2.2 Notations

The primary notations used in this paper are listed in Table 1.

Notations	Definitions
ATR	annualized total return
MDD	maximum drawdown
$RoMaD$	return over maximum drawdown
MOM	momentum factor
SMA	simple moving average
EMA	exponential moving average
BB	bollinger band
$GFTD$	Guangfa Thomas R. DeMark signal
α	commission fee ratio
MPT	modern portfolio theory

Table 1: Notation Descriptions

3 Strategy Construction

3.1 Rotation Strategy

3.1.1 Rotation Strategy in the Stock Market

Strategy Overview A renowned type of strategy institutional investors use on the stock market is called rotation strategy. In particular, sector rotation strategies describe that traders tend to believe that different sectors of stocks will behave alternately as the style of the market rotates cyclically. Scholars also have provided evidence that the rotation strategy earns consistent and economically significant excess returns while requiring only infrequent portfolio rebalancing [2].

Strategy Principles Rotation strategies can be generalized easily according to different levels of the market microstructure of the target assets, including rotation among assets with different liquidity, volume, or volatility. Since bitcoin offers extremely promising returns during the bull market, we expect to hold a large proportion of bitcoin when its price surges. However, when its price tumbles, we need to rapidly rotate our position towards gold, which can provide necessary protections against the drawdown period of bitcoin. When both bitcoin and gold prices fall, we choose to withdraw the asset and keep the cash only.

3.1.2 Detailed Implementation

Several heuristic indicators can be used to facilitate the decision of the rotation timing. We define the momentum factor (MOM) for time t as follows:

$$MOM_t = \frac{Price_t - Price_{t-n}}{Price_{t-n}} \tag{1}$$

The MOM factor has a hyperparameter n, referring to the number of lagged days used in the calculation. It indicates whether the price of a given asset still has the “momentum” to rise. Since bitcoin and gold have different prices, we will use the percent change of the lagged price instead of directly taking the difference.

If the MOM of bitcoin is more significant than that of gold, we will switch our position to hold the bitcoin only, expecting that bitcoin has a more substantial growth potential over gold.

Otherwise, we will rotate the position towards gold. If the momentum of both assets is negative, we withdraw all the positions and hold the cash only to avoid possible future losses. It should be mentioned that MOM can also be substituted by other indicators, such as amplitude factor and volatility, based on individuals' preference.

At the end of each trading day, the momentum factor is calculated using the price on that day and the previous lagged price without involving any future price information. The trader then decides whether to rotate his position at the beginning of the next trading day depending on the calculated MOM value. Since gold is only traded when the market is open, the trader can only withdraw money from bitcoin or switch from cash to bitcoin when the gold does not trade. Commission fees will be deducted for each transaction accordingly.

3.1.3 Strategy Results

Performance Metrics Typical performance metrics of a trading strategy include cumulative return, annualized total return, maximum drawdown, Sharpe ratio, Sortino ratio, etc. Cumulative return is the aggregate amount that the investment has gained or lost over time, while annualized total return (ATR) is the geometric average amount of money earned each year. The maximum drawdown (MDD) is an essential indicator of the downside risk, which is defined by maximum observed loss from a peak to a trough of a portfolio before a new peak is attained:

$$MDD = \frac{Trough\ Net\ Value - Peak\ Net\ Value}{Peak\ Net\ Value} \tag{2}$$

Since bitcoin is exceptionally volatile and free of limit down constraint (the maximum permitted decline in one trading day), we will also focus on the different indicators, including MDD, Sharpe ratio, cumulative return, and return over maximum drawdown (RoMaD) at the same time, where

$$RoMaD = \frac{ATR}{MDD} \tag{3}$$

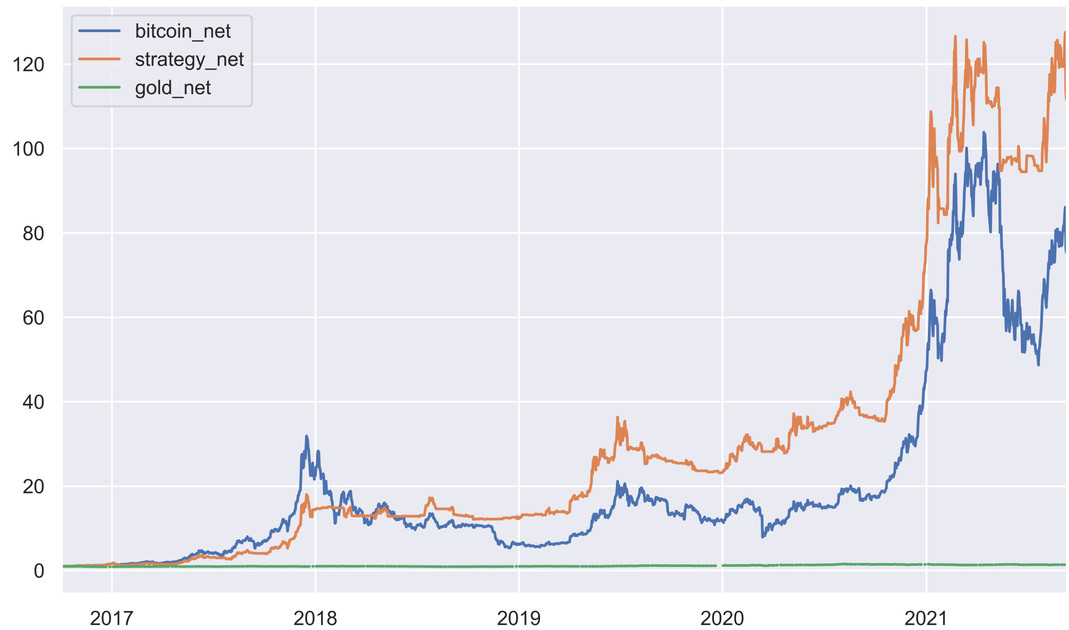


Figure 1: Equity curve of rotation strategy, bitcoin, and gold

Parameter Choice The choice of parameter n involves a trade-off. If the lagged period used is small, traders use short-term information to predict the price movement, which may "misjudge" the actual momentum during the period. On the other hand, a considerable lagged period detects the momentum more accurately, however, at the cost of reacting slowly to the price changes. In this paper we choose the lagged period to be 20 based on a standard rule of thumb.

Performance Analysis and Visualization Figure 1 displays the cumulative return curve if we start with one dollar and invest it in gold, bitcoin, and rotation strategies.

As can be seen from the Figure 1, the designed rotation strategy reaches a higher total return than bitcoin and gold by avoiding the worst plummeted period of bitcoin. Bitcoin price experienced a long bull market in 2017 and 2020 and dropped over 80% in 2018. Rotation strategies effectively control the maximum drawdown by switching the position to pure cash when the bear market occurs.

Table 2 shows the rotation strategy's trading characteristics and performance metrics results. The strategy will take a full position in bitcoin for 1045 days, which occupies most of the period. It reached its maximum drawdown of -36.33% during 2019, which lasted for almost half a year, and it successfully controlled the loss during the plunge period of bitcoin in 2018.

Holding Periods	Bitcoin	Gold	Cash	Total
	1045	456	325	1826
Performance	**Cumulative**	ATR	**Sharpe**	**RoMaD**
	111.53	159.7%	1.64	**4.4**
Drawdown	**MDD**	Drawdown Begin	Drawdown End	Recover Date
	-36.33%	2019/06/27	2019/12/26	2020/05/08

Table 2: Rotation strategy summary sheet

Figure 2 and 3 display the monthly return distribution and the visualization of drawdown using an underwater plot. The strategy earns significant returns during 2017 and is less affected by the bear market in 2018 and late 2021 than the bitcoin. We reach the final RoMaD of 4.4 and a cumulative return of 111.53. This indicates that the initial 1000 dollars will be worth roughly 111530 dollars at the end.

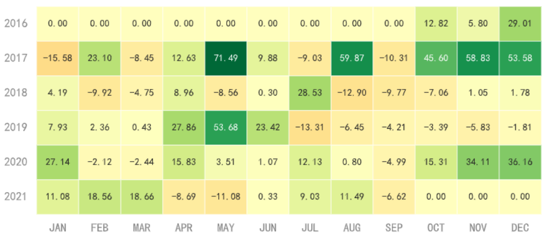


Figure 2: Strategy monthly returns (%)

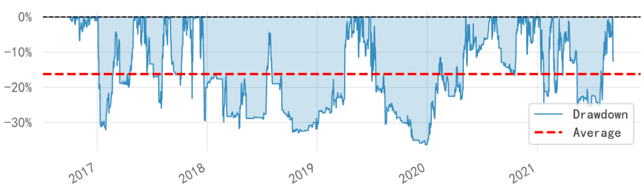


Figure 3: Underwater plot of drawdown

3.2 Signal-based Strategies

3.2.1 Technical Analysis and Timing Signals

A more commonly used type of strategy is performing market timing according to well-designed trading signals. Many trading signals are developed by conducting technical analysis on the historical microstructure information. Typical technical indicators include the momentum factor (MOM), simple moving average (SMA), exponential moving average (EMA), and Bollinger band (BB).

The N-day SMA assigns equal weight to the past m day price information, while the N-day EMA signal assigns a higher weight to recent prices and therefore is more sensitive and reactive to the price changes. It is defined recursively:

$$EMA_t = \frac{N-1}{N+1} \times EMA_{t-1} + \frac{2}{N+1} \times Price_t \tag{4}$$

The 30-day SMA and EMA curve is shown in the Figure 4. The trading signal gives the trader the timing to buy when the short-term moving average curve crosses the long-term curve upward and gives the timing to sell otherwise. Despite the relatively easy calculation formula, trading strategies based on MA generate substantial alpha, utility and Sharpe ratios gains and significantly reduce the severity of drawdowns both in and out of sample [6].

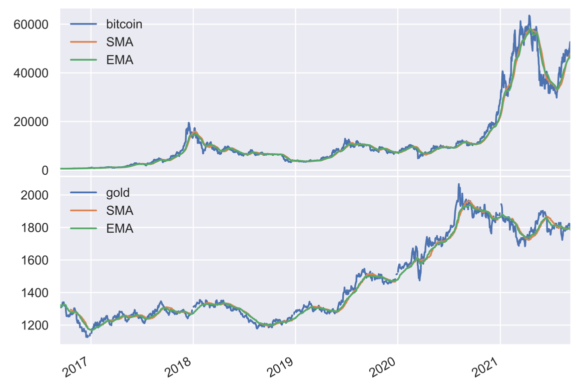


Figure 4: SMA,EMA, and price ($N = 30$)

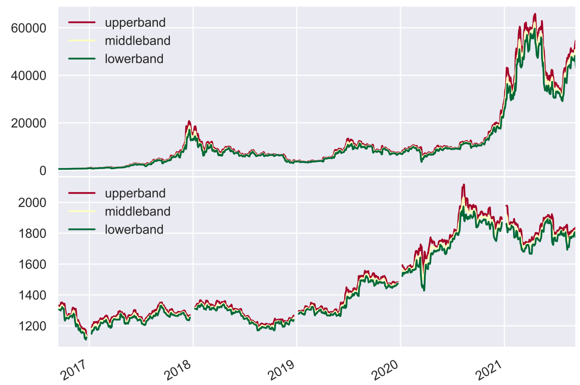


Figure 5: Bollinger band of gold and bitcoin

Bollinger bands are a technical analysis tool developed by John Bollinger for generating oversold or overbought signals, and are proved to be powerful especially in the cryptocurrency market. When the market becomes more volatile the bands widen, while during less volatile periods the bands contract. Figure 5 illustrates the lower, middle, and upper bollinger band with a 20-day rolling window and the width between the two nearest bands equaling to two times of the price standard deviation within the rolling window. A strong trend continuation can be expected when the price moves out of the bands. However, if prices move immediately back inside the band, then the suggested strength is negated [1].

These traditional signals will be considered separately for gold and bitcoin due to their distinctive price characteristics. In addition to the traditional technical signals, we also design an advanced adaptive timing signal called Guangfa Thomas R. DeMark signal (GFTD) based on the powerful technical indicator proposed by Thomas R. DeMark, as illustrated below.

3.2.2 Guangfa Thomas R. DeMark Strategy

TD Signal Introduction Initially developed by the renowned technical analysis master Thomas R. DeMark, his TD indicators have been the premier name in financial market timing over decades. Thomas emphasized that an objective analysis system can also combine subjective judgments [5], indicating that TD indicators can be easily modified according to market characteristics. Many model parameters exist in the indicator and can be adjusted subjectively by traders, which effectively enhance the flexibility and adaptability of the technical indicator.

Adjusted TD Timing Signal The common feature of the TD indicators is that they focus on grasping the timing of the accumulation and exhaustion of market energy. For bitcoin and gold, we expect that the reaction of TD signal towards bitcoin is more sensitive to that towards gold. Hence we adjust the original TD indicator system to be more adaptive and flexible to bitcoin and allow for a delay tolerance for gold. Denote the adjusted TD indicators as Guangfa TD timing signal (GFTD).

Suppose $\{c_t\}$, $\{h_t\}$, $\{l_t\}$ are the series of the daily close, highest, and lowest price of a given asset. GFTD timing signal requires three model parameters:

1) *Lagged period (n_1): Defined as the lagged period used when comparing the historical price with the current price;*

2) *Consecutiveness (n_2): Defined as the one-way consecutive number of price relationships (comparison results between the daily price and the n_1 lagged price);*

3) *Count period (n_3): Defined as the count numbers required to release the final signal.*

Define ud_t to be the comparison result between the close prices:

$$ud_t = \begin{cases} 1 & c_t > c_{t-n_1} \\ -1 & c_t < c_{t-n_1} \\ 0 & otherwise \end{cases} \tag{5}$$

Then the GFTD signal can be generated by the following Algorithm 1. Illustrative graphs of the buying signal, selling signal, buying count period, and selling count period are given in Figure 6 and Figure 7.

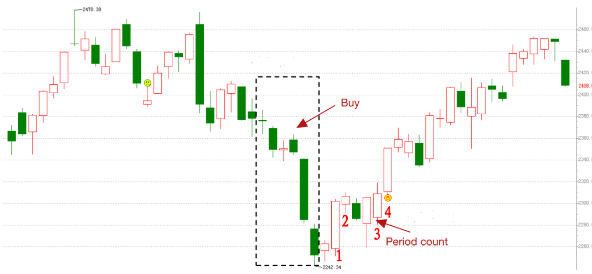


Figure 6: GFTD: Timing to buy illustration

Figure 7: GFTD: Timing to sell illustration

Performance on a Single Asset Since we only have a single price series for each type of asset, the three price series $\{c_t\}$, $\{h_t\}$, $\{l_t\}$ in the GFTD algorithm will be set the same. This may reduce signal accuracy but can guarantee useful insights into the timing of the accumulation and exhaustion of market energy. We fix the count period (n_3) as 2 to let the signal release more quickly to cope with the high short-term volatility in the bitcoin price. The lagged period (n_1) and consecutiveness (n_2) are tuned according to RoMaD using historical price only, as displayed

Algorithm 1: GFTD Signal Generating Algorithm

1 Calculate ud_t sequence using historical price information.

2 Let $udc_t = \sum_{k=0}^{n_2} ud_{t-k}$ be the the rolling summation of ud_t. Generate $pseudo\_signal_t = 1$ if $udc_t = n_2$, and $pseudo\_signal_t = -1$ if $udc_t = -n_2$, where 1 indicates possible selling opportunity and -1 indicates possible buying opportunity.

for *t where* $pseudo\_signal_t = -1$, **do**

  for $k = 1, 2, \ldots$; $count = 0$, **do**

3   Start counting from trading day $t+1$. The *count* variable will increase by 1 if the following three conditions hold for the given value of k:

$$(c_{t+k} \geq h_{t+k-2}) \cap (h_{t+k} > h_{t+k-1}) \cap (c_{t+k} > c_{t+1}) = \text{True}$$

4   **If** $count < n_3$ and $pseudo\_signal_{t+k} = -1$: **break**

5   **Else if** $count = n_3$: set $real\_signal_{t+k} = 1$, **break**

6   **Else**: **continue**

for *t where* $pseudo\_signal_t = 1$, **do**

  for $k = 1, 2, \ldots$; $count = 0$, **do**

7   Start counting from trading day $t+1$. The *count* variable will increase by 1 if the following three conditions hold for the given value of k:

$$(c_{t+k} \leq h_{t+k-2}) \cap (l_{t+k} < l_{t+k-1}) \cap (c_{t+k} < c_{t+1}) = \text{True}$$

8   **If** $count < n_3$ and $pseudo\_signal_{t+k} = 1$: **break**

9   **Else if** $count = n_3$: set $real\_signal_{t+k} = -1$, **break**

10   **Else**: **continue**

11 Release buy signal at day t when $real\_signal_t = 1$ and sell signal otherwise.

in Figure 8 and 9. We then backtest the tuned GFTD strategy on bitcoin and gold respectively and obtain the comparison results in Table 3.

Compared with investing in bitcoin or gold without performing market timing, The well-designed GFTD signal helps boost the cumulative nearly twice, meanwhile reducing the maximum drawdown significantly. Guided by the GFTD signal, 1000 initial dollars fully invested in bitcoin will then be worth over 156930 dollars at the period end. The model parameters are adjusted subjectively according to the specific price behavior of gold and bitcoin and therefore

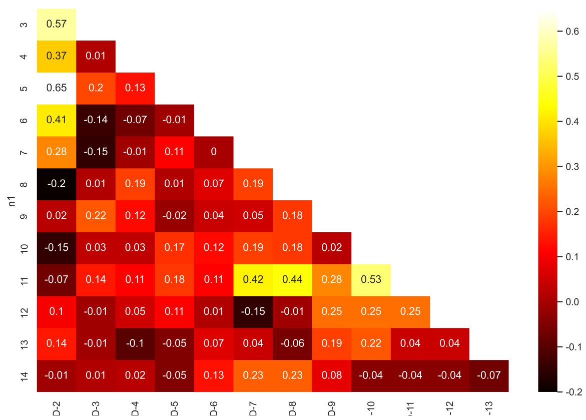


Figure 8: RoMaD for gold with different (n_1, n_2)

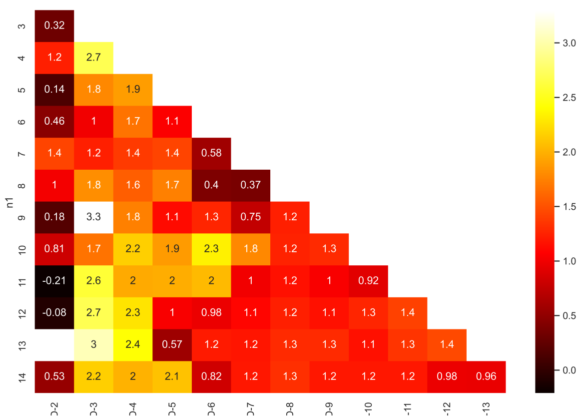


Figure 9: RoMaD for bitcoin with different (n_1, n_2)

	Gold		Bitcoin	
Strategy	**GFTD**	**Original**	**GFTD**	**Original**
Cumulative	47.51%	22.38%	15693.72%	7505.54%
ATR	8.09%	4.12%	175.23%	137.81%
Sharpe	0.81	0.36	1.64	1.24
MDD	-12.44%	-20.84%	-48.46%	-83.37%
RoMaD	0.65	0.197	3.616	1.653

Table 3: GFTD Performance on bitcoin and gold separately

reach a decent backtesting performance on the given period.

In the above process GFTD signals are applied separately to bitcoin and gold. We will propose a fusion method in later sections to combine the separated timing strategies dynamically and address the optimal weight allocation among cash, gold, and bitcoin.

3.3 Prediction-based Strategies

Despite the limited data, we try to use a supervised learning algorithm, “XGBoost" classifier[1], to predict the return of the assets. Since we only have close price of the assets as the feature, we first generate more features for the model input.

Genetic programming (GP) is a technique of evolving programs. It starts from a population of unfit programs and fits for a particular task by applying statistical operations. We define several statistical functions in Table 4.

function	meaning
add(X,Y)	add vector X and vector Y, the ith element is $X_i + Y_i$
sub(X,Y)	subtract vector X and vector Y, the ith element is $X_i - Y_i$
log(X)	the logarithm of vector X, the ith element is $\log(X_i)$
inv(X)	the inverse of vector X, the ith element is $\frac{1}{X_i}$
delay(X,d)	the value of the data d days ago, the ith element is X_{i-d}
rank(X)	the rank of the X_i in vector X

Table 4: Some examples of the functions used for Genetic Programming

In our genetic programming algorithm, we allow the generations to do cross-over and mutation operations. We set our fitness function to be the Mean Absolute Error (MAE). Based on MAE, we train the data in 100 generations and select the best 5 individuals as our results. Here, we call each individual as a "factor", which indicates the real return of the assets in some senses.

Since all features come from the close price of the assets, they tend to have high correlation between each other, which may influence our prediction due to the multicollinearity problems. We therefore apply Gram-Schmidt orthogonalization to reduce the correlation among factors.

Let the number of factors be F, and the number of dates be N. Define our data matrix as $X_{N\times F}$. We want our final result as $\hat{X}_{N\times F} = X_{N\times F}S_{F\times F}$. Then, we apply the Gram-Schimdt orthogonalization method to obtain the expression for $S_{F\times F}$.

[1] `https://github.com/dmlc/xgboost`

factor	expressions
1	add(sin(inv(close)), sin(inv(close)), argmax(return(close)))
2	add(minmaxcps(rank(close)), delta(close), minmaxcps(rank(close)), skewness(close))
3	add(inv(tsmin(close)), inv(close))
4	add(delta(zscoresquare(sign(delta(close)))), skewness(close), delta(close))
5	add(min(close, zscore(sigmoid(mindiff(sigmoid(zscore(close)))))), return(close))

Table 5: Some examples of the expression of the factor for gold or bitcoin

$$\begin{aligned} \hat{f}^1 &= f^1 \\ \hat{f}^k &= f^k - \frac{\langle f^k, \hat{f}^1 \rangle}{\langle \hat{f}^1, \hat{f}^1 \rangle} \hat{f}^1 - \frac{\langle f^k, \hat{f}^2 \rangle}{\langle \hat{f}^2, \hat{f}^2 \rangle} \hat{f}^2 - \cdots - \frac{\langle f^k, \hat{f}^{k-1} \rangle}{\langle \hat{f}^{k-1}, \hat{f}^{k-1} \rangle} \hat{f}^{k-1} \end{aligned} \tag{6}$$

$$S_{K \times K} = \begin{bmatrix} \beta_1^1 & \beta_1^2 & \dots & \beta_1^F \\ 0 & \beta_2^2 & \dots & \beta_2^F \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & \beta_F^F \end{bmatrix} \tag{7}$$

where $\beta_k^k = \frac{1}{\|\hat{f}^k\|}$, $e^k = \frac{\hat{f}^k}{\|\hat{f}^k\|} = \beta_1^k f^1 + \cdots + \beta_k^k f^k$.

After applying orthogonalization, we use these factors as the features for prediction task. First, we set labels for the assets on different dates. If the asset has a higher close price in the next day, we set the label to be 1. Otherwise, we set the label to be 0. Then, we apply the factors to the XGBoost classifier. Start from the 10th day, each day we use all the history data to train the XGBoost model and predict its label. Additionally, we do hyperparamter tune on the training data and decide the optimal hyperparameters. After getting all the predicted labels, we define our signal for decision making. We check the rolling labels for the past 10 dyas, if there are more than 4 days with predicted label as 1, we buy the asset. Otherwise, we sell the asset if we have some. Finally, we get a strategy with higher cumulative return.

	Gold		**Bitcoin**	
Strategy	**XGBoost**	**Original**	**XGBoost**	**Original**
Cumulative	28.5%	22.38%	9910.26%	7505.54%
ATR	5.15%	4.12%	151.24%	137.81%
Sharpe	0.45	0.36	1.32	1.24
MDD	-20.74%	-20.84%	-76.07%	-83.37%
RoMaD	0.248	0.197	1.988	1.653

Table 6: XGBoost performance on bitcoin and gold separately

As shown in Table 6, by applying XGBoost classifier, we can get slightly higher cumulative return, Sharpe ratio, and lower MDD over the individual assets. However, due to insufficient data, the trained machine learning model may be too flexible and has high variance, so that the performance will worsen when encountering different price series.

3.4 Strategy Fusion

Given a set of timing strategies, each having its own risk and return traits, we now propose a fusion method to form a portfolio that possesses the advantages of these individual strategies. To effectively fuse the strategies, we first introduce a set of notions and theories in asset allocation.

3.4.1 Theory of Asset Allocation

Asset allocation is the process of allocating different weights to different assets to obtain a portfolio with satisfying returns and risks. To measure the degree of satisfaction, or more formally preference, one approach is to use utility functions, where options with higher utility are preferred over those with lower utility. Fortunately, for most cases, a continuous real-valued utility function is proved to exist [4]. In the literature on asset allocation, the objective can be simply expressed as expected utility maximization, which takes both the risk and return into account when the utility function u is increasing and concave [9]. Consider a portfolio whose random return $r(\theta)$ is related to decision parameter θ, the optimal parameter under expected utility theory is $\theta^* = \text{argmax}_\theta \mathbb{E}[u(r(\theta))]$.

Mean-Variance Objective Under certain assumptions[2], the expected utility maximization objective can be reduced to a mean-variance objective [3]. That is

$$\theta^* = \text{argmax}_\theta v(\mu(\theta), \sigma(\theta)) \tag{8}$$

where $\mu(\theta) = \mathbb{E}[r(\theta)]$, $\sigma(\theta) = \sqrt{\mathbb{E}[(r(\theta) - \mu(\theta))^2]}$, and v is a real-valued function of mean μ and standard deviation σ. Following modern portfolio theory (MPT), the optimal portfolio should be selected from the efficient frontier [8]. For each portfolio on the efficient frontier, it has the smallest variance among other portfolios that have the same expected return, namely

$$\Theta = \{\theta | \min_\theta \sigma^2(\theta) \text{ such that } \mu(\theta) = z, z \in R\} \tag{9}$$

where R is the range of possible expected return of all portfolios. The optimal portfolio is then $\theta^* = \text{argmax}_{\theta \in \Theta} v(\mu(\theta), \sigma(\theta))$.

3.4.2 Solving Mean-Variance Objective

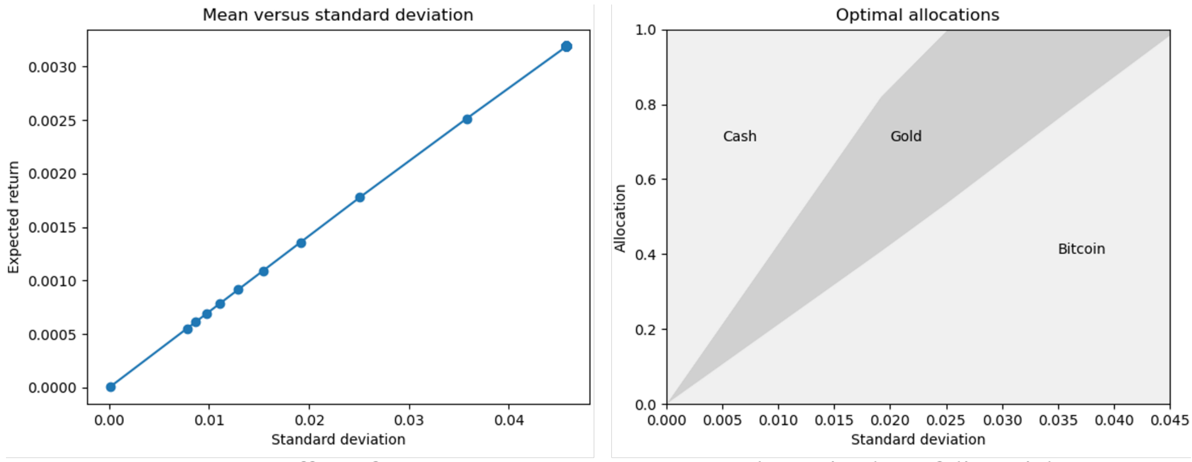
Consider the case with a total number of n assets and denote r_i to be the random variable representing the return of asset i and let $r = [r_1, r_2, ..., r_n]^\top$. Denote $\mu = \mathbb{E}[r]$ and Σ to be the covariance matrix of r. The efficient frontier can then be simplified as

$$\begin{aligned} \Theta = \{w | \min_w w^\top \Sigma w \text{ such that } \mu^\top w = z, 1^\top w = 1, w \geqslant 0, \\ z \in [\min(\mu_1, ..., \mu_n), \max(\mu_1, ..., \mu_n)]\} \end{aligned} \tag{10}$$

where w is the weight of each asset in the portfolio. From the KKT condition, the optimal portfolio (which is on the efficient frontier) satisfies the following criteria

$$\begin{aligned} 2\Sigma w + \lambda_1 \mu + \lambda_2 - \mathbf{h} &= 0 \\ \mu^\top w &= z \\ 1^\top w &= 1 \\ w, \mathbf{h} &\geqslant 0 \\ \mathbf{h} \otimes w &= 0 \end{aligned} \tag{11}$$

[2]Such as the return $r(\theta)$ is normally distributed.



(a) Effect of γ (b) Optimal portfolio weights

Figure 10: A demonstration of optimal portfolio under mean-variance objective with three asset, i.e., bitcoin, gold, and cash. μ and Σ are estimated from data. (a): the mean and standard deviation of the optimal portfolio from Equation (13) with different γ. The larger γ is, the smaller the standard deviation is. (b): the optimal portfolio weight given the standard deviation of the portfolio. The section of the shaded areas on the y-axis represent the weight of the corresponding asset.

By choosing appropriate γ, the solution in Equation (11) is also the solution in

$$\begin{aligned} \gamma\Sigma w - \mu + \lambda_2 - \mathbf{h} &= 0 \\ 1^\top w &= 1 \\ w, \mathbf{h} &\geqslant 0 \\ \mathbf{h} \otimes w &= 0 \end{aligned} \tag{12}$$

which is the KKT condition for

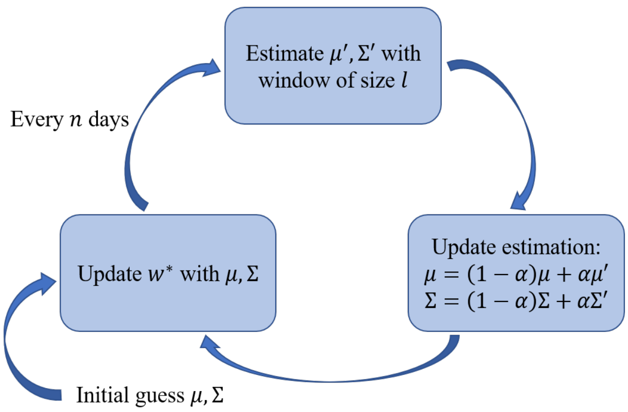
$$\max_w \mu^\top w - \frac{1}{2}\gamma w^\top \Sigma w \text{ such that } 1^\top w = 1, w \geqslant 0 \tag{13}$$

Therefore, the efficient frontier and the optimal portfolio on it can be obtained by solving Equation (13) with appropriate γ, which is usually positive. Equation (13) can be understood intuitively: the objective of a trader is a balance between high expected return and high variance (or risk) and γ can be seen as the tolerance of risk for the trader. For simplicity, we denote the solution to Equation (13) as $w^*(\mu,\Sigma)$ against μ,Σ, or just w^*, which can be easily solved using quadratic programming solvers. An illustrative example of the optimal portfolio under mean-variance objective is shown in Figure 10.

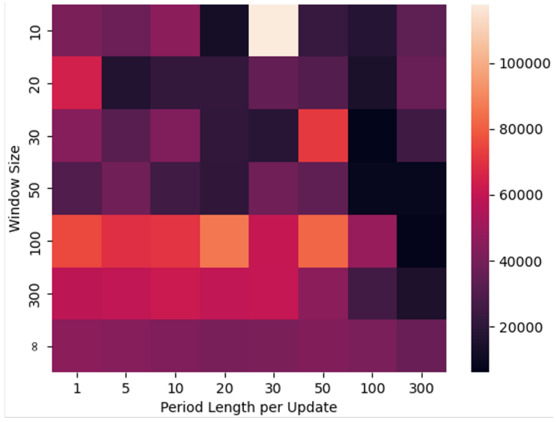
3.4.3 Dynamic Strategy Fusion with Sliding Windows

From previous analysis, given the expected returns and the covariances of all assets, the optimal portfolio is fixed, static and can be obtained by solving Equation (13). This result depends on two assumptions: 1) The true expectation μ and covariance Σ can be obtained or accurately estimated; 2) μ and Σ remain constant over time. Unfortunately, both assumptions fail in the real-life settings. The behavior of assets is likely to change over time and the estimation is imprecise if the period of observed data is too short or too long (the true μ,Σ may change in a long period). To adjust the theoretical framework and better fit the real-life settings, we periodically re-estimate μ and Σ and re-calculate the optimal portfolio weights w^*.

The overall process is described in Figure 11(a). The returns of the dynamic fusion with different period lengths for portfolio update and different window sizes for parameter estimation are shown in Figure 11(b), where both large period lengths and small period lengths have



(a) Dynamic fusion process



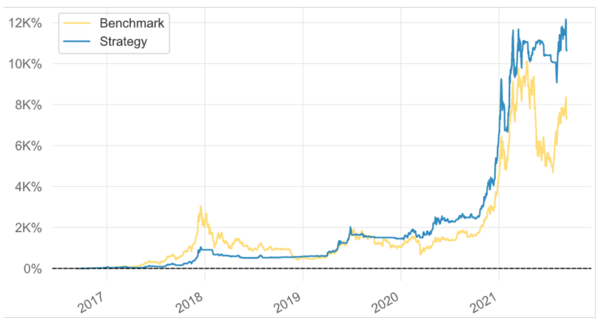
(b) Effects of n and l

Figure 11: (a): the graphical overview of the process of dynamic fusion with sliding window. (b): the net value of the portfolio under dynamic fusion at 2021/09/10 against period length per update n and window size l. (b) is plotted with only three assets, i.e., bitcoin, gold, and cash.

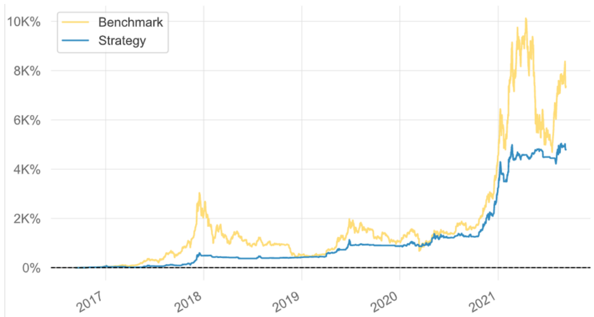
inferior cumulative return as expected. Moreover, shorter update period performs slightly better than longer update period, indicating the benefit of capturing the change trend may outweigh the commission fee.

3.5 Overall Strategy Performance

Using dynamic strategy fusion described in section 3.4.3, we are able to construct various portfolios optimized to different risk preferences with 7 "assets", i.e., bitcoin, gold, cash, and 4 artificial assets, i.e., EMA bitcoin, EMA gold, MA bitcoin (short for SMA bitcoin), MA gold (short for SMA gold), GFTD bitcoin, GFTD gold, and the rotation strategy. The parameters in dynamic strategy fusion, i.e., period length per update n and window size l, are chosen to be 5



(a) $\gamma = 3$



(b) $\gamma = 10$

Figure 12: The performance of the portfolio formed by dynamic strategy fusing with different risk preference γ. "Benchmark" represents the performance of bitcoin. Higher γ indicates less risk tolerance.

γ	terminal value	daily mean return	daily standard deviation
3	\$116316.26	0.2995%	2.803%
5	\$104321.46	0.2904%	2.687%
10	\$49170.63	0.2396%	2.301%

Table 7: The terminal value, average daily return, and standard deviation of the daily return of the portfolios constructed using dynamic strategy fusion.

and 50 as the result of the rule of thumb. Example strategy performances under different risk preference γ are shown in Figure 12 and the terminal values of portfolios on 2021/09/10 with \$1,000 cash on 2016/09/11 are in Table 7.

4 Performance Comparison and Strategy Analysis

4.1 Performance Comparison

We now compare the performance of all the strategies we have obtained so far. From the Table 8, both rotation and fused strategy have much higher cumulative return, ATR, and Sharpe ratio than the asset itself (bitcoin or gold), indicating the profitability power of the designed strategy. Besides, both strategies have much lower MDD and higher RoMaD, indicating that our strategies effectively control the downside risk during the given period.

Indicator	Asset		Strategy	
	Gold	Bitcoin	Rotation	fused
Cumulative	35.48%	7358.97%	11053.48%	11519.44%
Sharpe	0.42	1.24	1.63	1.7
MDD	-18.54%	-83.37%	-36.33%	-46.36%
ATR	6.26%	136.88%	156.73%	158.84%
RoMaD	0.34	1.64	4.31	3.42

Table 8: Ratio comparison summary sheet

Indeed, the rotation and fused strategy share similar performance metrics. Rotation strategy possesses lower MDD while fused strategy obtains higher cumulative return and Sharpe ratio.

We further display the monthly return heatmap for the our fused strategy and underwater plot for the drawdown periods in the backtesting.

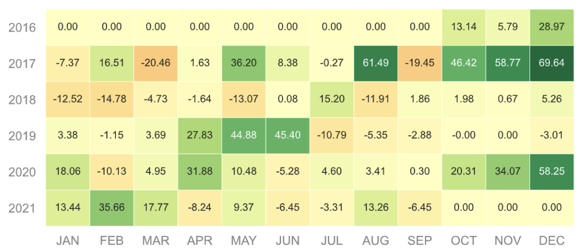


Figure 13: The monthly return for fused strategy

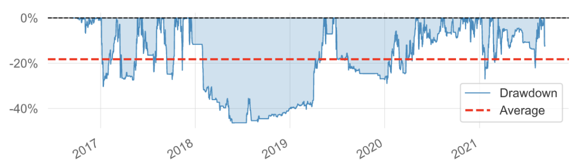


Figure 14: Underwater plot for fused strategy

From Figure 13 and 14, our strategy hits successive drawdowns between 2018 and 2019 due to the severe bear market of bitcoin. The highest monthly return reaches around 70%, while the max monthly drawdown is around 20%.

We further compare the fusion strategy with bitcoin itself. The Figure 15 shows the fused strategy starts outperforming the bitcoin after its bear market in 2018. The strategy succeeds in avoiding the huge drawback in 2021 and thus achieves a much higher cumulative return. Figure 16 displays that bitcoin has the rolling volatility plot with a period of 6 months much higher than the fused strategy, indicating the dynamic fusion process indeed minimizes the variance.

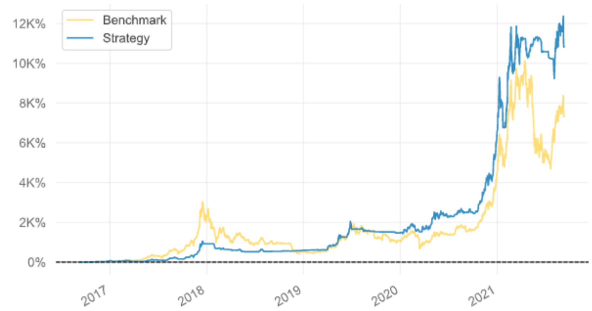


Figure 15: Cumulative return comparison

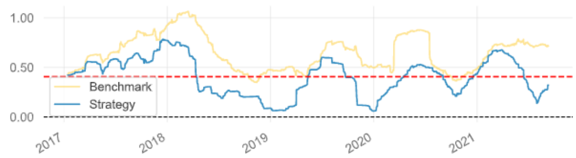


Figure 16: Rolling volatility comparison

4.2 Risk Preference Analysis

Since our fused strategy is constructed based on Markowitz model, we are able to give distinctive solutions to traders with different levels of risk preference. We adjust the model parameter γ to obtain the backtesting results for traders with different risk preferences.

Parameter γ defines the risk tolerance of an individual. Traders with higher γ tend to avoid possible large fluctuations in the market. When facing large short-term volatility in bitcoin prices, these traders tend to withdraw their positions in bitcoin and wait for the price to stabilize. Traders with low γ, on the other hand, are aggressive to take more risks in order to maximize their return. When facing the plummet period in bitcoin prices, however, this type of traders may undergo substantial financial loss as well.

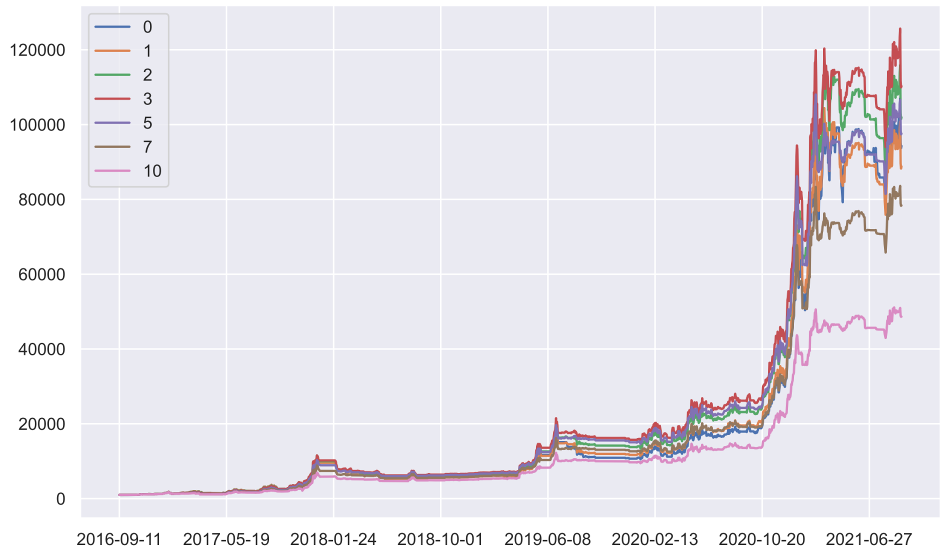


Figure 17: Cumulative return of fused strategies based on different γ

The cumulative return plot in Figure 17 shows that from 2016 to 2021, traders with a standard risk preference ($\gamma = 3$) obtains the highest cumulative return. The realized total return hurts whenever a trader is too aggressive (small γ) or too conservative (large γ). Besides, the more risk averse the trader is, the smoother the equity curve becomes.

Table 9 indicates when changing γ from 0 to 10, both cumulative return and Sharpe ratio first increase and then decrease, and reach the peak when γ equals to 3. Moreover, the MDD decreases as γ increases, since the trader tries to reduce the risk. In summary, the fused strategy is adaptive to traders with varying levels of risk tolerance.

Indicator	γ					
	1	**2**	**3**	**5**	**7**	**10**
Cumulative	8779.22%	9922.57%	11519.44%	9522.70%	7631.38%	4760.90%
Sharpe	1.55	1.62	1.7	1.69	1.68	1.65
MDD	-50.67%	-49.9%	-46.36%	-41.83%	-38.61%	-36.94%
ATR	145.29%	151.3%	158.84%	149.26%	138.59%	117.44%
RoMaD	2.87	3.03	3.42	3.57	3.59	3.18

Table 9: Risk preference summary sheet

4.3 Robustness Analysis

After comparing the performance indicators and the effect of different levels of risk tolerance on the fused strategy, we now evaluate the robustness of our strategy by adding some noise to the original bitcoin and gold price.

Denote ε as the uniform random noise in the range [-d, d], where the bound value d can be set as $0.1\%, 0.2\%, \ldots 1\%$. We backtest our strategies above again on the price series after disturbance. We list the comparison results for the previous GFTD strategy in Table 10 as a reference. As display in the table, when the random noise increases, some performance metrics are slightly affected. The overall performance remains stable, i.e. the returns, MDD, RoMaD, and Sharpe are all fluctuating in a reasonable level. Although only GFTD strategy is tested on robustness due to time limit, it is still reasonable to expect that other strategies and the fused strategy perform similarly and have high robustness.

	Gold			**Bitcoin**		
bound (d)	**0.1%**	**0.5%**	**1%**	**0.1%**	**0.5%**	**1%**
Cumulative	47.1%	46.8%	47.74%	15665.3%	15655.12%	15696.64%
ATR	8.39%	7.97%	7.66%	173.28%	178.82%	180.04%
Sharpe	0.83	0.85	0.79	1.60	1.61	1.68
MDD	-13.53%	-13.36%	-13.11%	-47.21%	-50.04%	-52.15%
RoMaD	0.62	0.60	0.58	3.67	3.57	3.45

Table 10: Comparison of GFTD strategy performance with different levels of disturbance

4.4 Theoretical Analysis

Theoretically, our individual strategy construction and dynamic strategy fusion with sliding window approach has some desirable attributes, including theoretical optimality, specificity, and generalizability.

Theoretical Optimality From analysis in section 3.4.1 about asset allocation, under certain conditions, the solution to Equation (13) is the optimal portfolio weight of a set of n assets. The conditions include:

• The utility function measuring the preference satisfies certain conditions or the distribution of the returns of assets is normal;

- The returns of assets are stationary, indicating time-invariant mean and covariance.

Although the utility function is of various forms and the return distribution is not normal in the real world, the mean-variance objective is still a close-to-optimal objective. Assuming the trend of the returns is slowly changing (but not stationary), with appropriate window size, the local mean and covariance[3] can be accurately estimated. Therefore, dynamic strategy fusion gives an approximate optimal portfolio.

Specificity Unlike strategies in section 3.1 and 3.2, dynamic strategy fusion is capable of tuning the portfolio to any specific level of risk tolerance. This indicates that our approach provide tailor-made portfolio to specific investors.

Generalizability Besides the specific scenario in this problem, our approach can be applied to real world cases. In a market of n total assets, assume N "assets" (including strategies built upon the n assets) are available, the dynamic strategy fusion is still applicable, without significant decrease in performance or increase of computational difficulties.

4.5 Sensitivity Analysis

In this section, we explore the sensitivity of the model against the transaction cost. The relation between transaction cost and the risk preference γ is shown in Figure 18. Surprisingly, the commission fee ratio[4] seems not much related to the risk preference γ. From the decreasing standard deviation and increasing transaction frequency, the lower the risk tolerance, the more frequent the transactions, however, each transaction has lower transaction amount. Therefore, in the following analysis, we assume risk preference γ does not affect the transaction fee and ignore the role of γ.

4.5.1 How Sensitive the Strategy Is to Transaction Cost

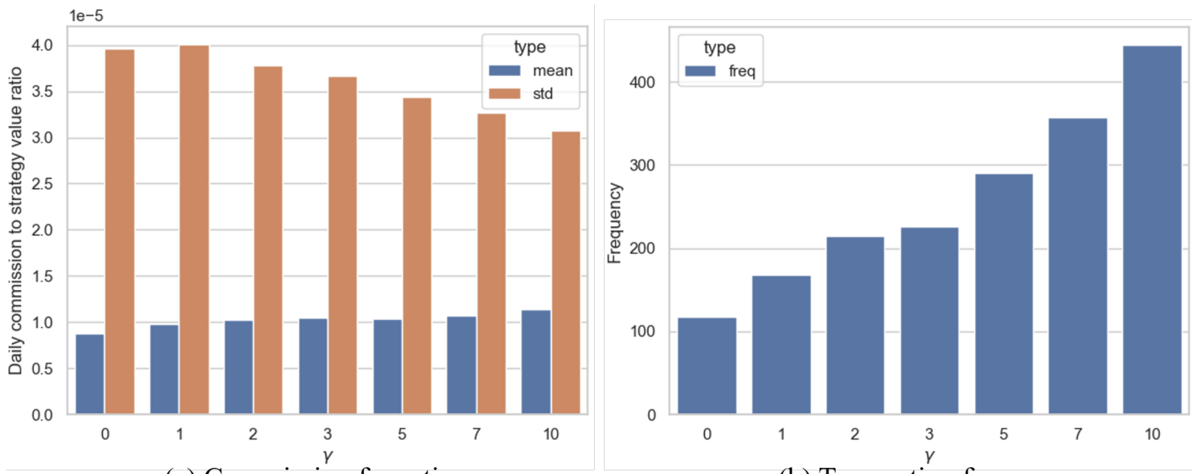
Assume the commission fee ratio α of a portfolio constructed using dynamic strategy fusion is constant for different γ. Further denote the daily return of the portfolio is r_0, the return after transaction cost is $(1+r_0)(1-\alpha)-1 \approx r_0 - \alpha$, the decrease in return due to transaction cost is approximately $\frac{\alpha}{1+r_0}$. Therefore, the sensitivity of the performance of the portfolio to commission fee can be measured using $\frac{1}{1+r}$. For higher γ, the risk tolerance is lower and the daily return is lower, hence more sensitive to transaction cost. The relation between the decrease in average daily return and α is shown in Figure 19, which matches the linear relation. Due to randomness, the slope in Figure 19(a) is not exactly $\frac{1}{1+r_0}$, but the correlation coefficient between slope k and $\frac{1}{1+r_0}$ is 0.69, which implies that $\frac{1}{1+r_0}$ is a good indicator for sensitivity to commission fee. The sensitivity of portfolios with different γ measured using $\frac{1}{1+r_0}$ is shown in Table 11. Larger value implies more sensitive to transaction cost.

4.5.2 How Transaction Cost Affect the Strategies

In this section, we analyze the change of optimal portfolio along with the change of transaction cost.

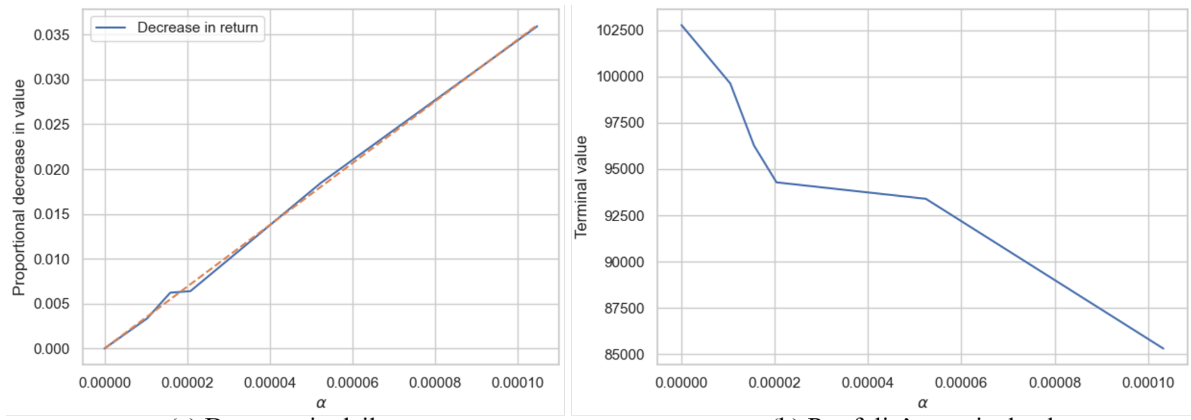
[3]"local" means a small interval containing current point.

[4]Commission fee ratio is short for daily commission fee to portfolio value ratio



(a) Commission fee ratio (b) Transaction frequency

Figure 18: Average commission fee ratio and transaction frequency with different γ.



(a) Decrease in daily return (b) Portfolio's terminal value

Figure 19: Sensitivity of portfolio to transaction cost with $\gamma = 5$. (a): decrease in daily return versus commission fee rate α. The dotted line is fitted. (b): terminal value of portfolios versus commission fee rate α. Different αs are obtained by scaling the original commission cost $\alpha_{bitcoin}, \alpha_{gold}$.

γ	1	5	7	10
$\frac{1}{1+r_0}$	0.997085	0.997113	0.997286	0.997596

Table 11: Sensitivity of the portfolios with different risk preference γ.

Transaction Cost and Optimal Individual Strategies For the purpose of investigating the change of optimal strategies when the transaction cost changes, we tune the parameters in each individual strategy. Surprisingly, the optimal parameters do not vary along with the change in transaction cost. A summary of the performance of different strategies under different scale of transaction costs is in Table 12. As expected, for strategies with low number of transaction, the effect of transaction costs is limited. Note that, although different transaction cost levels do not affect the behaviors of individual strategies, they do influence the return of strategies and play a role in construction of the overall portfolio.

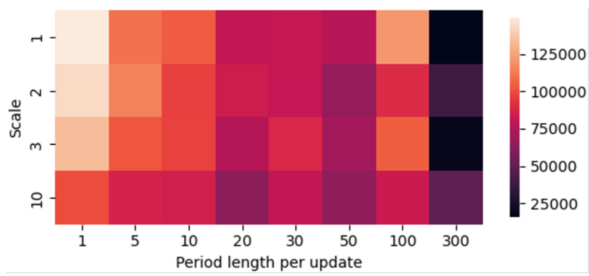
Transaction Cost and Optimal Portfolio Since the period length per update n in the dynamic strategy fusion process is likely to influence the transaction cost by influencing transaction times and the window size l aims to accurately estimate the mean and covariance of the returns and

strategy	$n_{transaction}$	terminal value ($\times 10^3$)			
		1×	2×	3×	10×
EMA bitcoin	13	159.39	158.73	158.28	155.21
MA bitcoin	57	103.25	101.47	100.30	92.48
GFTD bitcoin	88	157.93	155.18	152.47	134.79
EMA gold	16	1.23	1.23	1.23	1.21
MA gold	40	1.55	1.55	1.54	1.50
GFTD gold	3	1.47	1.47	1.47	1.46
rotation	186	111.53	107.56	103.74	80.51

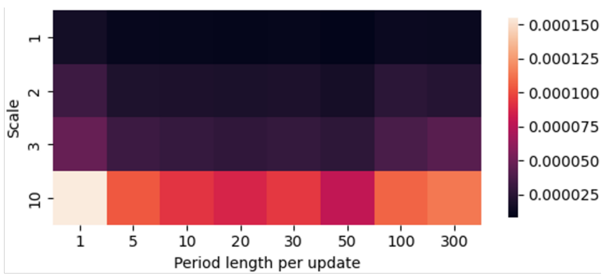
Table 12: The performance of individual strategies with optimal parameters under different scale of transaction cost. "$n\times$" means the transaction cost $\alpha_{bitcoin}, \alpha_{gold}$ are $n\times$ the original transaction costs. "$n_{transaction}$" means the number of transactions in the period.

is not expected to be influenced by the transaction cost, we keep l to be 50 as a rule of thumb and explore the optimal period length per update n against the transaction cost $\alpha_{bitcoin}, \alpha_{gold}$. To obtain different scale of transaction cost, we multiply original transaction cost $\alpha_{bitcoin}$ and α_{gold} with 1, 2, 3, and 10.

The performance of the portfolio with different transaction cost scale and period length per update is shown in Figure 20. Surprisingly, even with much higher transaction cost, e.g. 10 times the original cost, frequent portfolio update is still preferred, which counters the intuition that frequent portfolio update increases transaction cost and decreases the return. However, this is still reasonable because for small update period length, two consecutive updates are close to each other in time and the estimated mean and covariance should not have much change, hence similar portfolio weight for two updates and small transaction cost for each update. Therefore, it is possible for portfolios with frequent updates to have similar or smaller transaction costs than portfolios with larger update period length. From Figure 20(b), for scale from 1 to 3, the transaction cost does not increase significantly when increasing update frequency. For scale equal 10, the commission fee ratio is slightly higher for update period length of 1, but frequent update captures the trend of assets more accurately and compensate for the increasing transaction costs. Moreover, the relative lower volatility in the daily commission fee ratio (indicated by Figure 21) also suggests that the increase in the transaction cost is not as significant as the increasing average commission fee ratio. In summary, the extra transaction cost brought by increasing update frequency is not significant and can be compensated by extra return.



(a) Portfolio's terminal value



(b) Average commission fee ratio

Figure 20: The relation between portfolio performance and different transaction cost scale. (a): the terminal value of portfolios versus the scale of transaction cost and the period length per update in dynamic strategy fusion. (b): the average commission fee ratio versus the scale of transaction cost and the period length per update.

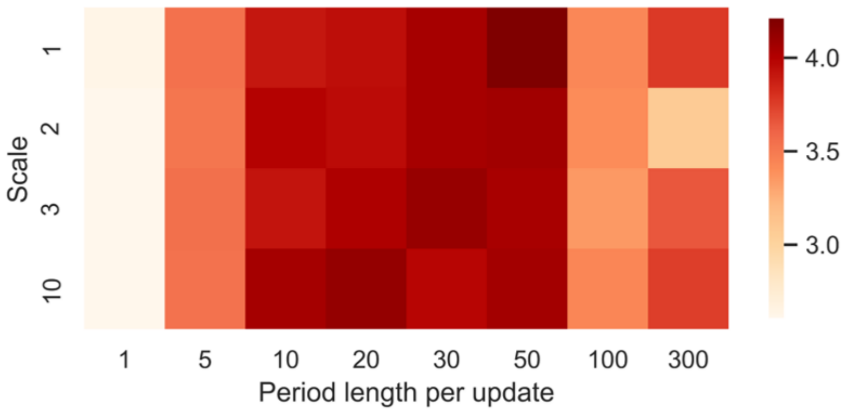


Figure 21: The coefficient of variation of the commission fee ratio versus transaction cost scale and period length per update. Coefficient of variation of a random variable is defined as σ/μ, where σ, μ denotes the mean and standard deviation respectively.

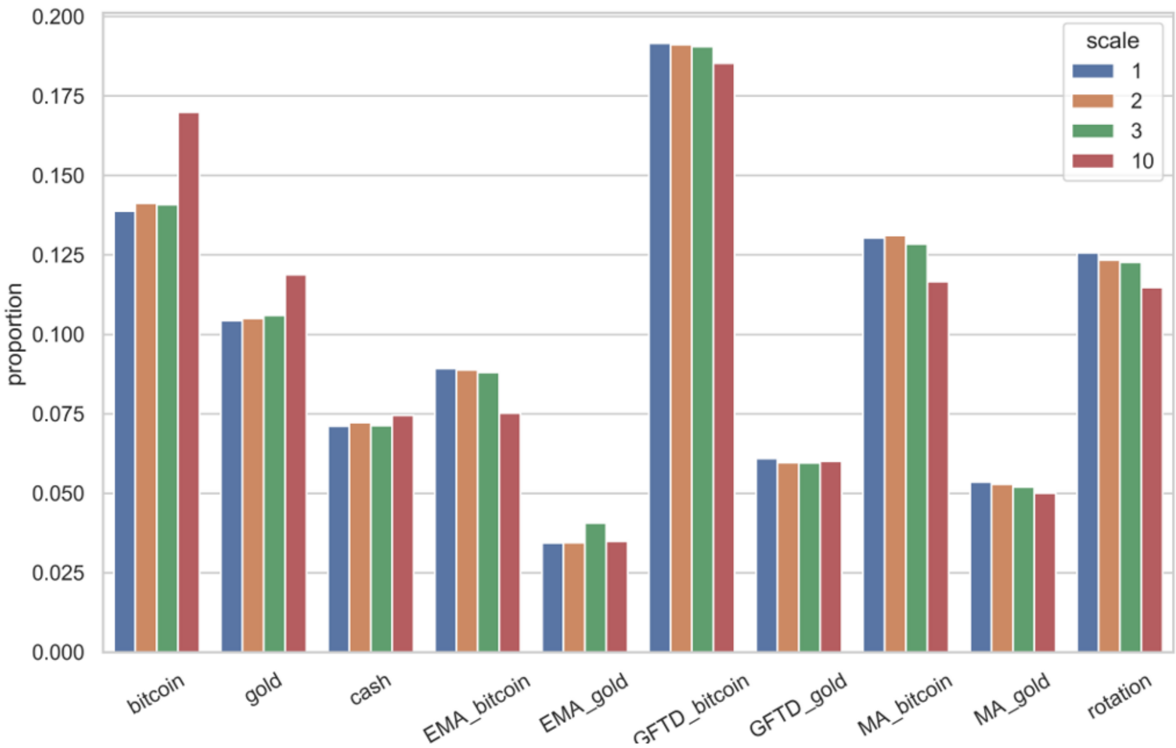


Figure 22: The average proportion of different "assets" in the optimal portfolio under different transaction cost scale.

We explore the the relation between the behavior of the optimal portfolio and the transaction cost scale. The number of transactions over the period for transaction cost scale 1, 2, 3, 10 is 479, 476, 477, and 477 respectively. The average proportion of the optimal portfolio under different transaction cost scales is shown in Figure 22. For "assets" with low transaction cost, such as bitcoin, gold, and cash which does not have transaction cost during the holding period, the average proportion increases when the transaction cost increases. For "assets" with high transaction cost, such as EMA bitcoin, MA bitcoin, and rotation which have relatively large transaction cost during the holding period, the average proportion decreases when the transaction cost increases.

5 Strengths and Weaknesses

5.1 Strengths

• **Great flexibility and adaptability:** The dynamic strategy fusion process can be applied to any developed trading strategies based on timing signals. Parameters in each individual strategy (such as rotation and GFTD strategy) can be flexibly adjusted based on experience.

• **Satisfying strategy performance:** The fused final strategy possesses the advantages of both rotation and GFTD strategy and has a decent level of annualized return and drawdown;

• **High maneuverability:** Our strategy framework can be used for traders with different risk preference and can be easily generalized to other assets such as stocks;

• **High robustness:** Our strategy can tolerate slight random noise added in the price curve.

5.2 Weaknesses

• **Limitations of our acquired data:** Many useful timing signals require the information of highest, lowest, open, and close price data or volume information. Hence many potentially useful signals cannot be used in the strategy fusion process;

• **Limitations in real-life settings:** When used in real transactions, the strategy framework still needs to be adjusted according to the face value of each contract (such as 0.01 bitcoin), possible leverage and the minimal margin rate, and different regulation rules in different cryptocurrency exchanges (such as Binance).

Memorandum

To: The Traders in The MCM Society
From: Team 2227388
Date: February 21st, 2022
Subject: Trading Strategies Report, Suggestions for Futher Improvement

Dear traders in the MCM Society,

Thank you for giving us the opportunities for developing the trading strategy for gold and bitcoin. We have evaluated our designed strategies comprehensively and here we present to you with our detailed solutions.

Our team designed a dynamic fusion strategy by combining distinctive timing strategies and then optimize the weight allocation for our portfolio. The distinctive timing strategies were developed using rotation strategy, traditional technical analysis, and Guangfa Thomas R. DeMark (GFTD) strategy. The dynamic fusion strategy was constructed by introducing a sliding window constraint to the classic Markowitz model and using optimizer to solve the optimal weight allocation for the portfolio, namely, cash, gold, and bitcoin. With the model we developed, we reached the following valuable conclusions:

• Though rotation strategy is primarily used in the stock market, it can be flexibly adjusted to determine the timing for switching the position. The performance is quite satisfying, since the strategy withdraws the position of bitcoin in time to avoid a potential bear market.

• GFTD strategy involves the combination of objective strategy execution and subjective parameter choices, which indicates that the GFTD indicator can be generalized easily to different types of assets with different price series.

• The fused strategy combines the advantages of each individual strategy, possessing a significant cumulative return while controlling the maximum drawdown at the same time. It gives a decent overall level of performance metric: a cumulative return of over 11,519.44%, an annualized return of over 158.84%, maximum drawdown within -46.36%, and the highest Sharpe ratio of over 1.7 for a standard risk-averse trader.

• If we can acquire more information such as the highest, lowest, and open price series or even orderbook data, we are able to develop many more informative timing signals that can be fused together to give potential better performance metrics.

• In real-life settings, traders can short sell the bitcoin and also take leverage to magnify their returns and losses. Many more individual strategies can be added in the dynamic fusion process, such as the renowned "Turtle Trading strategy", "Dual Thrust strategy", and many more quantitative strategies that are used in the derivative market.

• Our fused strategy is robust and is not sensitive to small changes in the commission fees. However, in real-life bitcoin transactions, possible issues such as the minimal face value of a contact need to be considered. Hence the strategy may be more sensitive to commission fees under the regulations of different cryptocurrency exchanges.

We expect to build a more rigorous and realistic trading strategy in the future, taking all the aspects including leverage, margin rate, slippage, and short selling issues into account. We also expect to utilize more data to develop more informative timing signals. The above are the details of our designed strategies and we sincerely hope they can provide you with useful insights into the market.

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