Double- and Half-Angle Laws

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \qquad \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\begin{aligned} \cos(2x) &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)) \qquad \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x))$$

$$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1}{2} - \frac{1}{2}\cos(x)} \qquad \cos\left(\frac{x}{2}\right) = \sqrt{\frac{1}{2} + \frac{1}{2}\cos(x)}$$

1. If x is an angle in Quadrant I and $\sin(x) = \frac{1}{3}$ then compute the following.

   $\sin(2x)$

   $\cos(2x)$

   $\sin\left(\frac{x}{2}\right)$

   $\cos\left(\frac{x}{2}\right)$

2. If x is an angle in Quadrant II and $\tan(x) = -3$ then compute the following.

   $\sin(2x)$

   $\cos(2x)$

   $\sin\left(\frac{x}{2}\right)$

   $\cos\left(\frac{x}{2}\right)$

3. Write $\cos(4x)$ using only powers of $\cos(x)$.

   $\cos(4x) \;=$

4. Write $\sin^4(x)$ using no powers – only use $\cos(2x)$ and $\cos(4x)$.

   $\sin^4(x) \;=$

5. Simplify.

   $\sin(x+y) \;+\; \sin(x-y) \;=$