

Linear Approximation and Differentials

Linearization and Linear Approx.

Approximate $f(x)$ using $\mathbf{f(a)}$ and $\mathbf{f'(a)}$.

Linearize $L(x) = \mathbf{f(a)} + \mathbf{f'(a)}(x - a)$
Approx. $f(a + \Delta x) \approx \mathbf{f(a)} + \mathbf{f'(a)}\Delta x$

Differentials

Derivatives	Rel. Rates	Differentials
$y = f(x)$	$y = f(x)$	$y = f(x)$
$\Downarrow \frac{dy}{dx}$	$\Downarrow \frac{dy}{dt}$	$\Downarrow d$
$y' = f'(x)$	$y' = f'(x)x'$	$dy = f'(x)dx$

1. (Linearization) Linearize the functions below around the given value.

A. $f(x) = \sqrt{x}$ around $a = 25$

B. $f(x) = x^3$ around $a = 2$

C. $f(x) = \sqrt[3]{x}$ around $a = 125$

D. $f(x) = x^{3/4}$ around $a = 16$

2. (Linear Approximation) Use linear approximation to approximate the values below.

A. $\sqrt{27}$

B. $\sqrt{23}$

C. $(2.12)^3$

D. $(1.78)^3$

3. (Differentials) Compute differentials for the functions below.

A. $y = x^2 + 4x + 5$

B. $y = \ln(x^2 + 4x + 5)$

C. $y = x^2 \ln x$

D. $y = (x + \ln x)^2$

4. (Error) Approximate error for the situations below.

A. Error in area if radius of circle is 30 ± 2

B. Error in cube volume if side length is 10 ± 0.2