

Trigonometric Integrals

Basic Trigonometric Integrals

$$\int \sin x \, dx = -\cos x + C \quad \int \tan x \, dx = \ln|\sec x| + C \quad \frac{d}{dx} \tan x = \sec^2 x$$

$$\int \cos x \, dx = \sin x + C \quad \int \sec x \, dx = \ln|\sec x + \tan x| + C \quad \frac{d}{dx} \sec x = \sec x \tan x$$

Substitution

$$\begin{aligned}\sin x &= u \\ \cos^2 x &= 1 - u^2 \\ \cos x \, dx &= du\end{aligned}$$

Requirement

cos power
is ODD

$$\begin{aligned}\cos x &= u \\ \sin^2 x &= 1 - u^2 \\ \sin x \, dx &= -du\end{aligned}$$

sin power
is ODD

Substitution

$$\begin{aligned}\tan x &= u \\ \sec^2 x &= u^2 + 1 \\ \sec^2 x \, dx &= du\end{aligned}$$

Requirement

sec power
is EVEN

$$\begin{aligned}\sec x &= u \\ \tan^2 x &= u^2 - 1 \\ \sec x \tan x \, dx &= du\end{aligned}$$

tan power
is ODD

$$1. \int \sin^3 x \cos^4 x \, dx$$

$$2. \int \sin^6 x \cos^3 x \, dx$$

$$3. \int \sec^3 x \tan^3 x \, dx$$

$$4. \int \sec^4 x \tan^4 x \, dx$$

$$5. \int \frac{\sin^3 x}{\cos^4 x} \, dx$$

$$6. \int \frac{\sec^4 x}{\tan^3 x} \, dx$$

Type conversion $\sin x = \frac{\tan x}{\sec x}$ $\cos x = \frac{1}{\sec x}$ $\tan x = \frac{\sin x}{\cos x}$ $\sec x = \frac{1}{\cos x}$

1. $\int \sin x \tan^2 x dx$

2. $\int \frac{\sec x}{\tan^2 x} dx$

Powers to Frequency $\sin x \cos x = \frac{1}{2} \sin(2x)$ $\sin^2 x = \frac{1}{2} (1 - \cos(2x))$

$\cos^2 x = \frac{1}{2} (1 + \cos(2x))$

1. $\int \sin^2 x dx$

2. $\int \sin^2 x \cos^2 x dx$

Reduction Rules $\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$
 $\int \tan^n x dx = \frac{1}{n} \tan^{n-1} x - \int \tan^{n-2} x dx$

1. $\int \cos^4 x dx$

2. $\int \tan^6 x dx$

Mismatched Frequencies

$$\sin(ax) \cos(bx) = \frac{1}{2} \sin((a - b)x) + \frac{1}{2} \sin((a + b)x)$$

$$\cos(ax) \cos(bx) = \frac{1}{2} \cos(|a - b|x) + \frac{1}{2} \cos((a + b)x)$$

$$\sin(ax) \sin(bx) = \frac{1}{2} \cos(|a - b|x) - \frac{1}{2} \cos((a + b)x)$$

1. $\int \sin 4x \cos 3x dx$

2. $\int \cos 4x \cos 3x dx$