

Optimization

1. If two positive numbers sum to 10, then what are the minimum and maximum values possible for their sum of squares?
2. What are the maximum and minimum values of $f(x, y) = x^2 - y$ if x and y are positive numbers with $x + y = 10$?
3. Find the point on $2x + y = 3$ which is closest to $(2, 4)$.
4. What is the maximum area for a rectangle with open top with perimeter (not counting top) of 12?

5. What is the minimum perimeter (not counting top) for a rectangle with open top whose area is 18?
6. What is the maximum volume for an open-top box with square base whose surface area (not counting top) is 12?
7. What is the minimum surface area (not counting top) for an open-top box with square base whose volume is 4?
8. An open box is made out of a 3 ft by 5 ft piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. Find the dimensions of the resulting box that has the largest volume.