Homework 07 - Cosets and Lagrange

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1. Let H be a subgroup of G and $g \in G$.
Prove. If gH is also a subgroup, then $gH = H$.

2. Let H be a subgroup of G.
Prove. If $gHg^{-1} \subset H$ for all $g \in G$, then $gH = Hg$.

3. Let H be a subgroup of G with $[G : H] = 2$ and $g \in G$.
Prove. $gH = Hg$.

4. Let H be a subgroup of G with $[G : H] = 2$.
Prove. If $a, b \notin H$, then their product $ab \in H$.

5. Let H and K be subgroups of G and $g \in G$.
Prove. $g(H \cap K) = gH \cap gK$.

6. Let H and K be two subgroups of G and $a, b \in G$.
Prove. If $aH = bK$ then $H = K$.