L'Hospital's Rule

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If $\begin{cases} \lim\limits_{x\to a} f(x) = 0 \\ \lim\limits_{x\to a} g(x) = 0 \end{cases}$ or $\begin{cases} \lim\limits_{x\to a} f(x) = \pm\infty \\ \lim\limits_{x\to a} g(x) = \pm\infty \end{cases}$ then $\displaystyle\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

Indeterminate Forms

Quotients

$\frac{0}{0}$ $\quad \frac{\infty}{\infty}$

Use L'H on these

$\leftarrow$ algebra

Products	**Differences**	**Powers**
$0 \cdot (\pm\infty)$	$\infty - \infty$	$\mathbf{0^0} \to$ "$0 \cdot \ln 0$" $\infty^{\mathbf{0}} \to$ "$0 \cdot \ln \infty$" $\mathbf{1}^{\infty} \to$ "$\infty \cdot \ln 1$"

Use algebra to convert these to quotients, **then** use L'H

1. (Basic L'H) Calculate the limits below.

A. $\displaystyle\lim_{x\to 1} \frac{x^4 - 3x^2 + x + 1}{x^3 - 2x + 1}$

D. $\displaystyle\lim_{x\to 1} \frac{x^3 - 3x + 2}{x^3 - x^2 - x + 1}$

B. $\displaystyle\lim_{x\to 1} \frac{x^2 + 3x - 4}{\ln(x)}$

E. $\displaystyle\lim_{x\to \infty} \frac{x^2 + 3x - 4}{\ln(x)}$

C. $\displaystyle\lim_{x\to 0} \frac{\sin(2x)}{x + \tan(x)}$

F. $\displaystyle\lim_{x\to 0} \frac{x^3}{\sin(x) - x}$

2. (Not L'H) Calculate the limits below.

A. $\displaystyle\lim_{x\to 1} \frac{x^3 + 3x^2 - 3x + 1}{x^3 + 2x - 1}$

B. $\displaystyle\lim_{x\to \infty} \frac{\sqrt{9x^4 + 3x + 1}}{x^2 + 1}$

3. (Indeterminate Products) Calculate the limits below.

A. $\lim_{x \to 0^+} \sqrt{x}\, \ln(x)$

C. $\lim_{x \to \infty} x^3\, e^{-x}$

B. $\lim_{x \to 0^+} x^2\, e^{(1/x)}$

D. $\lim_{x \to \pi/2} \tan(x)\, \sin(2x)$

4. (Indeterminate Powers) Calculate the limits below.

A. $\lim_{x \to 0} \big(\sin x\big)^x$

C. $\lim_{x \to \infty} \big(\ln(x)\big)^{1/x}$

B. $\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{-x}$

D. $\lim_{x \to 0^+} \big(1 - x\big)^{\ln x}$

5. (Indeterminate Differences) Calculate the limits below.

A. $\lim_{x \to 0} \left(\frac{2}{x} - \frac{1}{\sin x}\right)$

B. $\lim_{x \to 1^+} \Big(\ln(x^3 - 1) - \ln(x^3 + x - 2)\Big)$