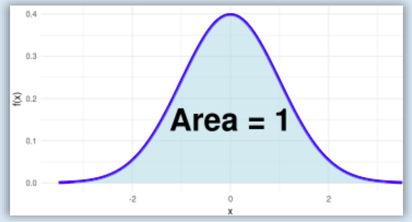
Week 5. Continuous Distributions: *pdf* and Probabilities

- A **continuous random variable** X is an **interval** of values, with probabilities for **ranges**.
- Usually, continuous random variables **MEASURE** things.

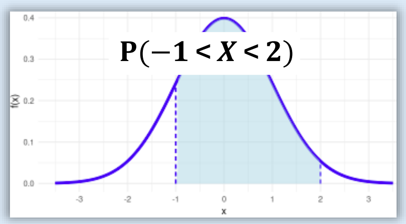
pdf Properties.

- $0 \leq f(x)$ for all x
- **Area** under $f(x)$ is **1**

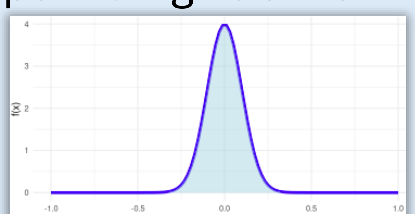


Notes.

- $\mathrm{P}(X{=}x) = 0$ **always!**
- Probability is Area.

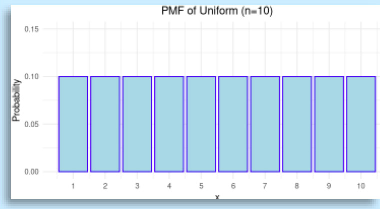


- pdf **can** go above 1



Change from COUNTING to MEASURING

Uniform n



Pick randomly from
n possible choices,
each of them equally likely

$f(x_i) = \frac{1}{n}$ for each x_i

$E[X]$ = average value of x_i

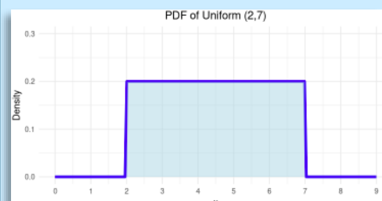
Example:
Uniform on set $\{1,2,3,4,5,6,7,8\}$

$$\mathrm{P}(X \geq 3) = \frac{\#\{3,4,5,6,7,8\}}{\#\{1,2,3,4,5,6,7,8\}} = \frac{6}{8}$$

$$\mathrm{P}(3 \leq X \leq 5) = \frac{\#\{3,4,5\}}{\#\{1,2,3,4,5,6,7,8\}} = \frac{3}{8}$$

$\Downarrow$

Uniform (a, b)



Pick randomly from
interval $a \leq x \leq b$
each value equally likely

$f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$

$E[X]$ = middle of interval $= \frac{b+a}{2}$

Example:
Uniform on interval $(1, 8)$

$$\mathrm{P}(X \geq 3) = \frac{\text{length}}{\text{length}} = \frac{5}{7}$$

$$\mathrm{P}(3 \leq X \leq 5) = \frac{\text{length}}{\text{length}} = \frac{2}{7}$$

Conditional Probabilities

Uniform on set $\{3,4,5,6,7,8,9,10\}$

$$P(X \leq 6 \mid X \geq 5) = \frac{\#\{\cancel{3,4,}5,6\}}{\#\{\cancel{3,4,}5,6,7,8,9,10\}} = \frac{2}{6}$$

$$P(X \leq 6 \mid X \leq 7) = \frac{\#\{3,4,5,6\}}{\#\{3,4,5,6,7,\cancel{8,9,10}\}} = \frac{4}{5}$$

Uniform on interval $(3, 10)$

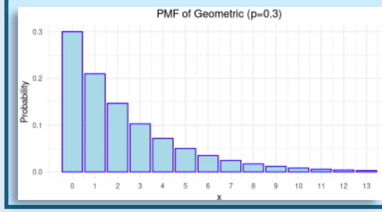
Conditional probability restricts to $x \geq 5$

$$P(X \leq 6 \mid X \geq 5) = \frac{\text{length}}{\text{length}} = \frac{1}{5}$$

$$P(X \leq 6 \mid X \leq 7) = \frac{\text{length}}{\text{length}} = \frac{3}{4}$$

Week 5. Geometric and Exponential *(extra)*

Geometric p



Waiting time with Bernoulli trials

Count **#trials** until success

$p =$ Probability of "success"

Expected value: $\mathrm{E}[X] = {}^{1}/_{p}$

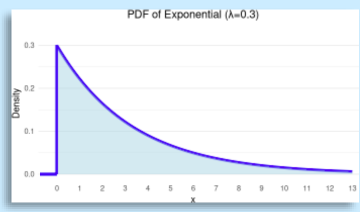
$f(x) = p(1-p)^x$ for integer x

$P(X = t + s \mid X \geq s) = P(X = t)$

"Waiting a long time doesn't make it more likely for something to happen."

"Memoryless" distributions

Exponential λ



Continuous waiting time

Measure length of time until something happens.

$\lambda =$ expected rate of occurrence

Expected value: $\mathrm{E}[X] = {}^{1}/_{\lambda}$

$f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$

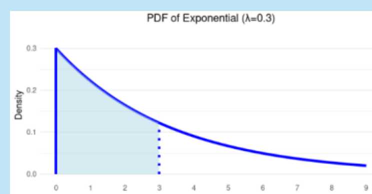
$P(X \geq t + s \mid X \geq s) = P(X \geq t)$

"Waiting a long time doesn't make it more likely for something to happen."

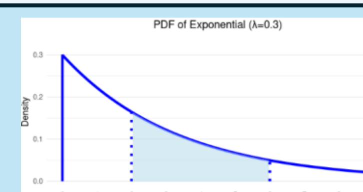
Examples of Exponential Data:

- Waiting time until earthquake
- Waiting time for bus
- Length of phone call
- Time between phone calls
- Waiting time until equipment failure
- Amount of change in pocket
- Distance between roadkills on highway
- Length of surgery time
- Emergency room waiting time
- Length of time that ER is empty

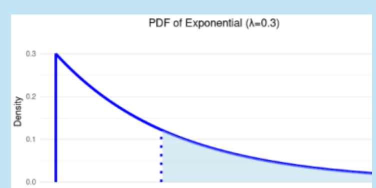
Probability is given by area.



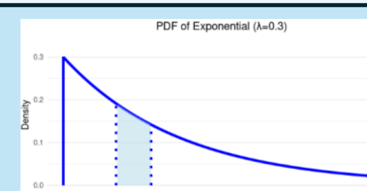
$\boldsymbol{P(X \leq 3)}$



$\boldsymbol{P(2 \leq X \leq 6)}$



$\boldsymbol{P(X \geq 3)}$



$\boldsymbol{P(X = 2 \pm {}^{1}/_{2})}$