

# Computing Limits Algebraically

## Polynomial

For polynomial functions,

$$\lim_{x \rightarrow a} p(x) = p(a)$$

## Rational

For rational functions,

$$\lim_{x \rightarrow a} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)} \text{ if } q(a) \neq 0$$

## Comparison

If  $f(x) = g(x)$  for  $x \neq a$  then

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x)$$

1. If plugging in does not give division by 0 then do it!
  2. Otherwise do algebra (factor, multiply by conjugate, combine) to get  $(x - a)$  in the top and bottom.
  3. Cancel  $(x - a)$  and return to step 1.
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1. Compute the limits below.

(A)  $\lim_{x \rightarrow 1} 5$

(B)  $\lim_{x \rightarrow 1} x$

(C)  $\lim_{x \rightarrow 2} x^2 + 3x - 2$

(D)  $\lim_{x \rightarrow 2} \sqrt{2x - 1}$

2. Compute the limits below.

(A)  $\lim_{x \rightarrow 1} \frac{x + 1}{x^2 + 1}$

(B)  $\lim_{x \rightarrow 1} \frac{x - 1}{x^2 - 1}$

(C)  $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x^2 - 3x + 2}$

(D)  $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 2x}$

3. Compute the limits below.

(A)  $\lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{x - 4}$

(B)  $\lim_{x \rightarrow 1} \frac{\sqrt{3x + 1} - 2}{1 - x^2}$

4. Compute the limits below.

$$(A) \lim_{x \rightarrow 2} \frac{1 - \frac{2}{x}}{x - 2}$$

$$(B) \lim_{x \rightarrow 2} \frac{\frac{2}{x+2} - \frac{1}{2}}{x^2 - x - 2}$$

5. Compute the limits below.

$$(A) \lim_{x \rightarrow 0} x \left( \frac{2}{x} + x^2 \right)$$

$$(B) \lim_{x \rightarrow 0} \left( \frac{1}{x} + \frac{5}{x^2 - 5x} \right)$$

$$(C) \lim_{x \rightarrow 2} (x - 2) \left( 1 + \frac{3x}{x^2 + x - 6} \right)$$

$$(D) \lim_{x \rightarrow 3} \left( \frac{1}{x - 3} - \frac{4}{x^2 - 2x - 3} \right)$$