

Type I and II Partial Fractions

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$$\frac{P(x)}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

$$A = \frac{P(a)}{\cancel{(a-a)}(a-b)(a-c)}$$

$$\frac{P(x)}{(x-a)^2 Q(x)} = \left[\frac{A_2}{(x-a)^2} + \frac{A_1}{x-a} \right] + \dots$$

$$A_2 = \frac{P(a)}{Q(a)}$$

Basic Forms.

$$\int \frac{1}{x-a} dx = \ln|x-a| + C$$

$$\int \frac{1}{(x-a)^2} dx = -\frac{1}{x-a} + C$$

$$\int \frac{1}{(x-a)^3} dx = -\frac{1}{2} \frac{1}{(x-a)^2} + C$$

1. Split the following Type I rational functions into sums of fractions.

(a) $\frac{5x+1}{(x-1)(x+2)}$

(b) $\frac{7x+2}{x(x-1)(x+2)}$

2. Split the following Type II rational functions into sums of fractions.

(a) $\frac{9x}{(x-1)^2(x+2)}$

(b) $\frac{6x-4}{x^3(x+2)}$

3. Solve completely!

(a) $\int \frac{4x - 6}{(x - 1)(x - 2)(x - 3)} dx$

(b) $\int \frac{3x - 4}{(x - 1)(x - 2)^2} dx$

(c) $\int \frac{x}{(x - 1)(x - 2)^3} dx$