Volumes of Rotation

If functions $y = f(x)$ and $y = g(x)$ enclose a region from $x = a$ to $x = b$ then...

Area of region is $\int \Delta y$

$$\int_{x=a}^{x=b} \left| f(x) - g(x) \right| \, dx$$

Volume given by rotation around x-axis is $\int \pi r^2$

$$\pi \int_a^b \left| \left(f(x)\right)^2 - \left(g(x)\right)^2 \right| \, dx$$

... rotating around $y = c$

$$\pi \int_a^b \left| \left(f(x) - c\right)^2 - \left(g(x) - c\right)^2 \right| \, dx$$

Volume given by rotation around y-axis is $\int 2\pi r h$

$$2\pi \int_a^b \left| x \left(f(x) - g(x)\right) \right| \, dx$$

... rotating around $x = c$

$$2\pi \int_a^b \left| (x - c) \left(f(x) - g(x)\right) \right| \, dx$$

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1. Consider the region enclosed by $y = x^2 - x + 1$ and $y = 3x - 2$.

   (a) Write an integral computing the **area** in this region.
   Do not integrate!

   (b) Write an integral computing the **volume** if the region is rotated around the x-axis.
   Do not integrate!

   (c) Write an integral computing the **volume** if the region is rotated around the y-axis.
   Do not integrate!

(d) Write an integral computing the **volume** if the region is rotated around $y = -2$.
Do not integrate!

(e) Write an integral computing the **volume** if the region is rotated around $x = 6$.
Do not integrate!