

# Trigonometric Substitution (Part II)

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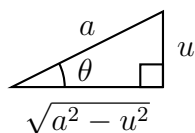
**Simplify**  $(a^2 - u^2)$

**Apply:**  $1 - \sin^2 \theta = \cos^2 \theta$

**Substitute:**  $u = a \sin \theta$

$$a^2 - u^2 = a^2 \cos^2 \theta$$

$$du = a \cos \theta \, d\theta$$



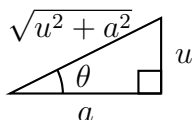
**Simplify**  $(u^2 + a^2)$

**Apply:**  $\tan^2 \theta + 1 = \sec^2 \theta$

**Substitute:**  $u = a \tan \theta$

$$u^2 + a^2 = a^2 \sec^2 \theta$$

$$du = a \sec^2 \theta \, d\theta$$



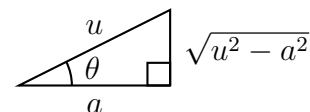
**Simplify**  $(u^2 - a^2)$

**Apply:**  $\sec^2 \theta - 1 = \tan^2 \theta$

**Substitute:**  $u = a \sec \theta$

$$u^2 - a^2 = a^2 \tan^2 \theta$$

$$du = a \sec \theta \tan \theta \, d\theta$$



1. Undo the indicated substitution to write answers in terms of  $x$ .

(a) **Substitution:**  $x = \sin \theta$

(b) **Substitution:**  $x = \tan \theta$

$$2 \sec^2 \theta \tan \theta + 3 \sec \theta + C$$

$$2 \sin \theta \cos \theta + 3\theta + C$$

(c) **Substitution:**  $2x = \sin \theta$

(d) **Substitution:**  $3x = 2 \sin \theta$

$$5 \sec \theta \tan \theta + 3\theta + C$$

$$7 \sec \theta \tan \theta + 5\theta + C$$

(e) **Substitution:**  $2x = 3 \sec \theta$

(f) **Substitution:**  $3x = 2 \tan \theta$

$$5 \sin^2 \theta \cos \theta + 7 \sin \theta + C$$

$$7 \sin \theta \cos \theta + 5\theta + C$$

2. Solve completely!

(a)  $\int \frac{5}{x^2 \sqrt{4 - 9x^2}} dx$

Note:  $\int \csc^2 \theta d\theta = -\cot \theta + C$

(b)  $\int \frac{5}{x \sqrt{9x^2 + 4}} dx$

Note:  $\int \csc \theta d\theta = -\ln |\csc \theta + \cot \theta| + C$

(c)  $\int \frac{5}{x^2 \sqrt{9x^2 - 4}} dx$