Computing Improper Integrals

1. Evaluate the following Type I improper intergrals.

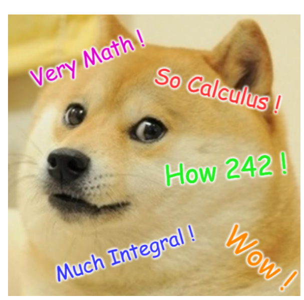
(a) $\displaystyle\int_1^{\infty} \frac{1}{\sqrt[4]{x}}\,dx$

(b) $\displaystyle\int_4^{\infty} \frac{1}{x\,\ln x}\,dx$

(c) $\displaystyle\int_{-\infty}^{-2} \frac{1}{x^3}\,dx$

(d) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx$

Scan the QR code!
Write an evaluation
for math 242!



2. Evaluate the following Type II improper intergrals.

(a) $\displaystyle\int_0^1 \frac{1}{\sqrt[4]{x}}\,dx$

(c) $\displaystyle\int_{-2}^0 \frac{1}{x^3}\,dx$

(b) $\displaystyle\int_0^1 \frac{\ln x}{x}\,dx$

(d) $\displaystyle\int_0^{1/2} \frac{1}{x\,\ln x}\,dx$

3. Torricelli's trumpet (also called Gabriel's Horn) is the infinite 3D shape formed by rotating the region between $y = \frac{1}{x}$ and $y = 0$ for $x \geq 1$ around the x-axis.

Compute the volume of Toricelli's trumpet.

$$\begin{aligned} \text{Volume} &= \pi \int_1^\infty \big(r(x)\big)^2\,dx \\ &= \pi \int_1^\infty \left(\tfrac{1}{x}\right)^2\,dx \end{aligned}$$

For surface area, use the comparison theorem.

$$\begin{aligned} \text{Surface Area} &= \int_1^\infty 2\pi\, r(x)\,ds \\ &= 2\pi \int_1^\infty \frac{1}{x}\sqrt{1+\left(-\tfrac{1}{x^2}\right)^2}\,dx \end{aligned}$$

This was considered a major paradox during the 1600s. The strange nature of infinity and its properties caused much dispute and struggle among mathematicians and philosophers (including Galileo).