Arc Length, Surface Area, Centroids

If functions $y = f(x)$ and $y = g(x)$ enclose a region from $x = a$ to $x = b$ then...

Area is $\int h \, dx$

$$\int_{x=a}^{x=b} \left| f(x) - g(x) \right| dx$$

Volume given by rotation around x-axis is $\int \pi \, r^2 \, dx$

$$\pi \int_a^b \left| (f(x))^2 - (g(x))^2 \right| dx$$

... rotating around $y = c$

$$\pi \int_a^b \left| (f(x) - c)^2 - (g(x) - c)^2 \right| dx$$

Volume given by rotation around y-axis is $\int 2\pi \, r \, h \, dx$

$$2\pi \int_a^b \left| x \, (f(x) - g(x)) \right| dx$$

... rotating around $x = c$

$$2\pi \int_a^b \left| (x - c) \, (f(x) - g(x)) \right| dx$$

If function $y = f(x)$ goes from $x = a$ to $x = b$ then

Arc length of curve is

$$\int_{x=a}^{x=b} ds$$

where

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$= \sqrt{1 + \left[f'(x) \right]^2} \, dx$$

Surface Area rotating around x-axis is $\int 2\pi \, r \, ds$

$$2\pi \int_a^b \left| f(x) \right| ds$$

... rotating around $y = c$

$$2\pi \int_a^b \left| f(x) - c \right| ds$$

Surface Area rotating around y-axis is $\int 2\pi \, r \, ds$

$$2\pi \int_a^b \left| x \right| ds$$

... rotating around $x = c$

$$2\pi \int_a^b \left| x - c \right| ds$$

Mass of function

$$m = \int_a^b f(x) \, dx$$

Average Value of function

$$\overline{f} = \frac{m}{b - a} = \frac{\int_a^b f(x) \, dx}{\int_a^b 1 \, dx}$$

Average y-Value of function

$$\overline{y} = \frac{M_x}{m} = \frac{\frac{1}{2} \int_a^b (f(x))^2 \, dx}{\int_a^b f(x) \, dx}$$

... of a region

$$\overline{y} = \frac{\frac{1}{2} \int_a^b (f(x))^2 - (g(x))^2 \, dx}{\int_a^b f(x) - g(x) \, dx}$$

Average x-Value of function

$$\overline{x} = \frac{M_y}{m} = \frac{\int_a^b x f(x) \, dx}{\int_a^b f(x) \, dx}$$

... of a region

$$\overline{x} = \frac{\int_a^b x (f(x) - g(x)) \, dx}{\int_a^b f(x) - g(x) \, dx}$$

1. Consider the region enclosed by $y = \sin x$ and $y = 0$ from $x = 0$ to $x = \pi$.
Write integrals computing the following values. Do not integrate!!

(a) Area inside region.

(b) Volume rotating around line $x = -1$

(c) Volume rotating around line $y = 2$

(d) Arc length of upper function $y = \sin x$

(e) Surface area rotating around line $y = -1$

(f) Surface area rotating around line $x = 4$

(g) Average x-value of region

(h) Average y-value of region