Week 3. Probability and Events

Probability of an *event* is the **proportion** of times the event *occurs* in identical conditions

→ **Number** (fraction or percentage)

- between **0** and **1** (=100%)
- Low value – *"usually does not* occur"
- High value – *"frequently does* occur"

Proportion $\dfrac{\text{\# times of event}}{\text{\# times possible}}$

- **Experiment** – generates an observable *outcome*
- **Outcome** – result of an *experiment*
- **Sample Space** – set of possible *outcomes*
- **Event** – collection of *outcomes* from *sample space* (something to check probability of)
- $\mathrm{P}(E)$ – probability of event E
- **Equiprobable** sample space has all outcomes **equally likely**.
  - In this case, $P(\text{Event}) = \dfrac{\text{\# Event Outcomes}}{\text{\# Total Outcomes}}$

Example.

Flip a coin and check: Heads or Tails?

- **Experiment** – flip coin
- **Outcome** – top side of result
- **Sample Space** – {H, T}

Example.

Flip a coin until Tails appears.
Count # flips.
What is probability of 3 or less flips?

- **Experiment** – flip coin until T
- **Outcome** – number of flips
- **Sample Space** – {1,2,3,4,5,...}
- **Event** – {1,2,3}
- $\mathrm{P}(E) = \dfrac{7}{8}$

Roll a die.

$$P(\text{even}) = \frac{\#\{2,4,6\}}{\#\{1,2,3,4,5,6\}}$$

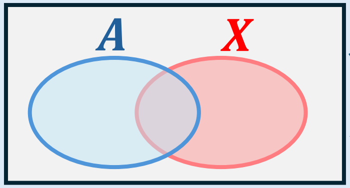
Flip 2 coins.

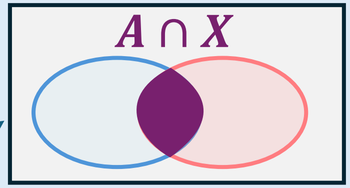
$$P(\text{at least 1 H}) = \frac{\#\{HT, TH, HH\}}{\#\{TT, HT, TH, HH\}}$$

Roll 2 dice.

$$P(\text{sum is 5}) = \frac{4}{36}$$

Week 3. Mixing Events and Counting Probabilities

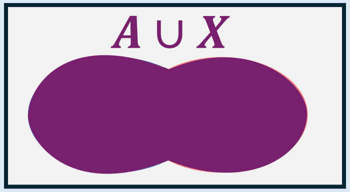




Intersection

$A \cap X$

"in **both** A **and** X"



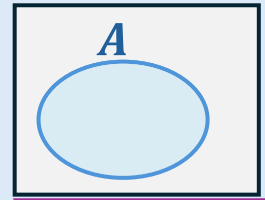
Union

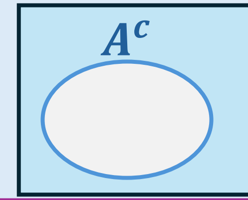
$A \cup X$

"in **either** A **or** X"

Complement A^c **or** A'

"outcomes **not** in A"



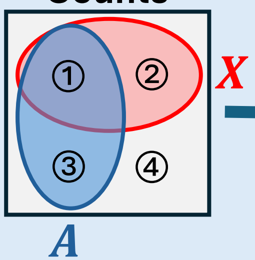


$$P(A^c) = 1 - P(A)$$

For interactions of two events, use **table**

("contingency" / "marginal" / "joint" table)

Counts



into table

	A		*Total*
X	①	②	① + ②
	③	④	③ + ④
Total	① + ③	② + ④	① + ② + ③ + ④

total in X

total *not* in X

total in A

total *not* in A

total outcomes

$$P(A) = \frac{\text{total in } A}{\text{total outcomes}} \qquad P(X) = \frac{\text{total in } X}{\text{total outcomes}}$$

$$P(A \cap X) = \frac{A, X \text{ box}}{\text{total outcomes}} \qquad P(A \cup X) = \frac{A \text{ and } X \text{ boxes}}{\text{total outcomes}}$$

Conditional Probability

$P(A \mid X)$ *"Probability of A given X"*

Example.

$$P\left(\begin{matrix}\text{two dice}\\ \text{sum to 6}\end{matrix}\right) = \frac{5}{36}$$

$$P\left(\begin{matrix}\text{two dice}\\ \text{sum to 6}\end{matrix} \,\middle|\, \begin{matrix}\text{one is}\\ \text{even}\end{matrix}\right) = \frac{2}{27}$$

Probability of A, ***taking into account*** that X happened.

	A		*Total*
X	①	②	① + ②
	③	④	③ + ④
Total	① + ③	② + ④	① ③

$P(A \mid X)$ restricts sample space to X

$$P(A \mid X) = \frac{A, X \text{ box}}{\text{total in } X}$$

If $P(A \mid X) = P(A)$ then A and X are ***independent***.