Implicit Differentiation

Implicit Differentials of Formulas

$$\frac{d}{dx}\left[\,x\,\right] = 1 \qquad \frac{d}{dx}\left[\,y\,\right] = \boldsymbol{y'}$$
$$\frac{d}{dx}\left[\,x^{\boldsymbol{2}}\,\right] = \boldsymbol{2}x \qquad \frac{d}{dx}\left[\,y^{\boldsymbol{2}}\,\right] = \boldsymbol{2}y\,\boldsymbol{y'}$$
$$\frac{d}{dx}\left[\,x^{\boldsymbol{n}}\,\right] = \boldsymbol{n}x^{\boldsymbol{n-1}} \qquad \frac{d}{dx}\left[\,y^{\boldsymbol{n}}\,\right] = \boldsymbol{n}y^{\boldsymbol{n-1}}\,\boldsymbol{y'}$$
$$\frac{d}{dx}\left[\sin(x)\right] = \cos(x) \qquad \frac{d}{dx}\left[\sin(y)\right] = \cos(y)\,\boldsymbol{y'}$$

Computing with Implicit Diff

1. Compute $\frac{d}{dx}$ of **equation**
   (use **product rule** on terms with x and y)
2. Plug in x and y values (if given)
3. Solve for $\boldsymbol{y'}$

1. (Implicit Derivatives of Expressions) Compute the implicit derivative $\frac{d}{dx}$ of the expressions.

A. $\frac{d}{dx}\left[\,1 + 2x + 3y + 4xy\,\right]$

B. $\frac{d}{dx}\left[\,x^2y^3 + x\,\right]$

C. $\frac{d}{dx}\left[\,\left(x^2y^3 + x\right)^4\,\right]$

D. $\frac{d}{dx}\left[\,\sin\left(x^2y^3 + x\right)\,\right]$

2. (Implicit Derivatives of Equations) Use implicit differentiation to find $y' = \frac{dy}{dx}$.

A. $x^2y^3 + x = 1$

B. $x^2y^3 + y = 1$

3. (Implicit Derivatives at Points) Use implicit differentiation to find $y' = \frac{dy}{dx}$ at the given point.

A. $2y^2 = x^3y - 6$ at $(2,\,3)$

B. $2y^2 - x^2y + 3x = 1$ at $(3,\,2)$