Homework 10 - Group Homomorphisms

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1. Define the **opposite** of a group G to be G^{op}, using the same set, but with multiplication $*$ given by multiplying in the opposite order $a * b = (ba)$.

Prove. The map $\phi : G \to G^{\text{op}}$ given by $\phi(g) = g^{-1}$ is a group homomorphism.

Hint: Need to show that $\phi(ab) = \phi(a) * \phi(b)$.

2. Multiplication defines a map $\mu : G \times G \to G$ by $\mu(a, b) = ab$.

(a) **Prove.** Multiplication is a group homomorphism if and only if G is abelian.

Hint: $(a, e)(e, b) = (a, b) = (e, b)(a, e) \in G \times G$

(b) **Question.** What is the kernel of the multiplication map $\mu : G \times G \to G$?

3. The projection map $\pi_1 : G_1 \times G_2 \to G_1$ is $\pi_1(g_1, g_2) = g_1$.

(a) **Prove.** The projection map is a group homomorphism.

(b) **Question.** What is the kernel of the projection map $\pi_1 : G_1 \times G_2 \to G_1$?

4. Let $\mathbb{Z}[x]$ be the additive group of polynomials with integer coefficients.
The "evaluation at 2" map $\text{ev}_2 : \mathbb{Z}[x] \to \mathbb{Z}$ is given by $\text{ev}_2(f) = f(2)$.

(a) **Prove.** The map ev_2 is a homomorphism.

(b) **Question.** Describe the kernel of ev_2 geometrically.

5. Suppose that $\phi : \mathbb{Z}_{30} \to \mathbb{Z}_{30}$ is a homomorphism with $\operatorname{Ker} \phi = \{0, 10, 20\}$.
If $\phi(23) = 9$ then what is $\phi^{-1}(9)$?