Derivatives of Inverses

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$f^{-1}(a) = b$
$\Updownarrow$
$a = f(b)$

At the point (a, b).

$$\left(\tfrac{d}{dx}\, f^{-1}\right)(a) = \frac{1}{f'(b)}$$

Derivative function.

$$\tfrac{d}{dx}\left[\, f^{-1}(x)\right] = \frac{1}{f'\big(f^{-1}(x)\big)}$$

Inverse Trig Derivatives

$$\tfrac{d}{dx}\left[\, \arcsin(x)\,\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\tfrac{d}{dx}\left[\, \arctan(x)\,\right] = \frac{1}{1+x^2}$$

$$\tfrac{d}{dx}\left[\, \operatorname{arcsec}(x)\,\right] = \frac{1}{|x|\sqrt{x^2-1}}$$

$\cos^2\theta = 1 - \sin^2\theta$ $\qquad$ $\sec^2\theta = 1 + \tan^2\theta$ $\qquad$ $\tan^2\theta = \sec^2\theta - 1$

1. If $f(x) = x^3 - 2x - 1$ then what is...

...the slope of $y = f^{-1}(x)$ at $(3, 2)$?

...the tangent line to $y = f^{-1}(x)$ at $(3, 2)$?

2. Use the product rule to compute derivative $f'(x)$ of the following.

A. $f(x) = 3x \arcsin x$

B. $f(x) = 5x^2 \arctan x$

C. $f(x) = 2x^3 \operatorname{arcsec} x$

3. Use the chain rule to compute derivative $f'(x)$ of the following.

A. $f(x) = 2\arcsin(3x)$

B. $f(x) = 5\arctan(x^2)$

C. $f(x) = 3\big(\arctan(x)\big)^2$

D. $f(x) = 4\operatorname{arcsec}(x^3 + x^2)$

4. Compute derivative $f'(x)$ of the following.

A. $f(x) = 2\tan(3x)\ \arcsin(4x)$

B. $f(x) = 5\sin(3x)\ \arctan(2x)$

C. $f(x) = 3\sec(x^2)\ \arctan(x^3)$

D. $f(x) = 4\tan(x^2)\ \text{arcsec}\,(x^3)$

E. $f(x) = 3x\sin(x^2)\ \arcsin(x^3)$

F. $f(x) = 4x^2\tan(x^3)\ \arctan(x^4)$