Homework 12 - Group Actions

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1. A group action on X is **transitive** if for every $x, y \in X$ there is $g \in G$ with $y = g \cdot x$.
This means all of X is in a single orbit, $X = Gx$.

A group action on X is **free** if for each $x \in X$ the only $g \in G$ with $g \cdot x = x$ is $g = e$.
This means all stabilizers are trivial, $G_x = \{e\}$.

Question. For each of the group actions below, state whether or not they are free and whether or not they are transitive, and explain.
(Note: there are four possibilities and four problems.)

(a) $G = \mathbb{Z}_3$ acting on $X = \{0, 1, 2, 3, 4, 5\}$ by $g \cdot x = 2g + x \pmod 6$.

(b) $G = \mathbb{Z}_4$ acting on $X = \{0, 1\}$ by $g \cdot x = g + x \pmod 2$.

(c) $G = \mathbb{Z}_5$ acting on $X = \{0, 1, 2, 3, 4\}$ by $g \cdot x = g + x \pmod 5$.

(d) $G = \mathbb{Z}_6$ acting on $X = \{0, 1, 2, 3\}$ by $g \cdot x = 2g + x \pmod 4$

2. For each of the group actions (a)-(d) above, write the following

(i) $G1 =$ the orbit of $1 \in X$.

(ii) $G_1 =$ the stabilizer subgroup of $1 \in X$.

(Your answers here may help you verify 3.)

3. What does the Orbit-Stabilizer Theorem say in each of the three situations below?
Explain your answers.

(a) If the action of G on X is transitive?

(b) If the action of G on X is free?
(Hint: how many cosets does $\{e\}$ have?)

(c) If the action of G on X is free and transitive?