

# Linear Approximation and Differentials

## Linearization and Linear Approx.

Approximate  $f(x)$  using  $f(a)$  and  $f'(a)$ .

**Linearize**  $L(x) = f(a) + f'(a)(x - a)$

**Approx.**  $f(a + \Delta x) \approx f(a) + f'(a) \Delta x$

## Differentials

Derivatives

$$y = f(x)$$

$$\Downarrow \frac{d}{dx}$$

$$y' = f'(x)$$

Rel. Rates

$$y = f(x)$$

$$\Downarrow \frac{d}{dt}$$

$$y' = f'(x) x'$$

Differentials

$$y = f(x)$$

$$\Downarrow d$$

$$dy = f'(x) dx$$

1. **(Linearization)** Linearize the functions below around the given value.

A.  $f(x) = \sqrt{x}$  around  $a = 25$

B.  $f(x) = x^3$  around  $a = 2$

C.  $f(x) = \sqrt[3]{x}$  around  $a = 125$

D.  $f(x) = x^{3/4}$  around  $a = 16$

2. **(Linear Approximation)** Use linear approximation to approximate the values below.

A.  $\sqrt{27}$

B.  $\sqrt{23}$

C.  $(2.12)^3$

D.  $(1.78)^3$

**3. (Differentials)** Compute differentials for the functions below.

**A.**  $y = x^2 + 4x + 5$

**B.**  $y = \ln(x^2 + 4x + 5)$

**C.**  $y = x^2 \ln x$

**D.**  $y = (x + \ln x)^2$

**4. (Error)** Approximate error for the situations below.

**A.** Error in area if radius of circle is  $30 \pm 2$

**B.** Error in cube volume if side length is  $10 \pm 0.2$