

Converting Products and Sums

Sine \iff Mixed Products

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin A + \sin B = 2 \sin\left(\frac{A + B}{2}\right) \cos\left|\frac{A - B}{2}\right|$$

$$\sin A \cos B = \frac{1}{2} \left(\sin(A - B) + \sin(A + B) \right)$$

Cosine \iff Pure Products

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos A + \cos B = 2 \cos\left(\frac{A + B}{2}\right) \cos\left|\frac{A - B}{2}\right|$$

$$\cos A \cos B = \frac{1}{2} \left(\cos|A - B| + \cos(A + B) \right)$$

$$\cos A - \cos B = 2 \sin\left(\frac{A + B}{2}\right) \sin\left(\frac{B - A}{2}\right)$$

$$\sin A \sin B = \frac{1}{2} \left(\cos|A - B| - \cos(A + B) \right)$$

1. Convert sums to products.

(a) $\sin(5x) + \sin(4x)$

(b) $\cos(5x) + \cos(4x)$

(c) $\sin(5x) - \sin(4x)$

(d) $\cos(5x) - \cos(4x)$

2. Convert products to sums.

(a) $\sin(5x) \sin(4x)$

(b) $\cos(5x) \cos(4x)$

(c) $\sin(5x) \cos(4x)$

(d) $\cos(5x) \sin(4x)$

3. Simplify the following formulas to a single function.

(a) $\frac{\cos(4x) - \cos(2x)}{\sin(4x) + \sin(2x)}$

(b) $\frac{\sin(3x) - \sin(5x)}{\cos(3x) + \cos(5x)}$

(c) $\frac{\sin(3x) - \sin x}{\sin x}$

(d) $2 \sin(8x) \cos(6x) - \sin(2x)$