

Trigonometric Substitution (Part I)

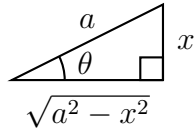
Simplify $(a^2 - x^2)$

Apply: $1 - \sin^2 \theta = \cos^2 \theta$

Substitute: $x = a \sin \theta$

$$a^2 - x^2 = a^2 \cos^2 \theta$$

$$dx = a \cos \theta \, d\theta$$



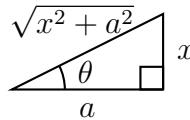
Simplify $(x^2 + a^2)$

Apply: $\tan^2 \theta + 1 = \sec^2 \theta$

Substitute: $x = a \tan \theta$

$$x^2 + a^2 = a^2 \sec^2 \theta$$

$$dx = a \sec^2 \theta \, d\theta$$



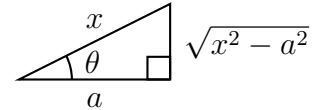
Simplify $(x^2 - a^2)$

Apply: $\sec^2 \theta - 1 = \tan^2 \theta$

Substitute: $x = a \sec \theta$

$$x^2 - a^2 = a^2 \tan^2 \theta$$

$$dx = a \sec \theta \tan \theta \, d\theta$$



1. Identify and perform the correct trigonometric substitution to simplify the integrals below.
(Substitute only! Do not integrate!)

(a) $\int x^2 \sqrt{16 - x^2} \, dx$

(b) $\int \frac{\sqrt{x^2 - 25}}{x^2} \, dx$

(c) $\int x^2 \sqrt{16 + x^2} \, dx$

(d) $\int \frac{(x^2 + 9)^{3/2}}{x^4} \, dx$

(e) $\int x^2 \sqrt{1 - 16x^2} \, dx$

(f) $\int \frac{\sqrt{25x^2 - 1}}{x^2} \, dx$

(g) $\int x^2 \sqrt{1 + 16x^2} \, dx$

(h) $\int \frac{(9x^2 + 1)^{3/2}}{x^4} \, dx$

(i) $\int x^2 \sqrt{16 - 25x^2} \, dx$

(j) $\int \frac{\sqrt{9x^2 - 25}}{x^2} \, dx$