

Trigonometric Substitution (Part II)

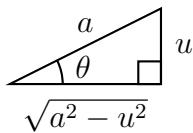
Simplify $(a^2 - u^2)$

$$\text{Apply: } 1 - \sin^2 \theta = \cos^2 \theta$$

Substitute: $u = a \sin \theta$

$$a^2 - u^2 = a^2 \cos^2 \theta$$

$$du = a \cos \theta d\theta$$



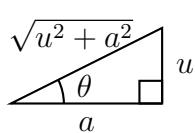
Simplify $(u^2 + a^2)$

$$\text{Apply: } \tan^2 \theta + 1 = \sec^2 \theta$$

Substitute: $u = a \tan \theta$

$$u^2 + a^2 = a^2 \sec^2 \theta$$

$$du = a \sec^2 \theta d\theta$$



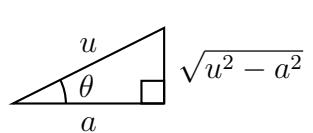
Simplify $(u^2 - a^2)$

$$\text{Apply: } \sec^2 \theta - 1 = \tan^2 \theta$$

Substitute: $u = a \sec \theta$

$$u^2 - a^2 = a^2 \tan^2 \theta$$

$$du = a \sec \theta \tan \theta d\theta$$



1. Undo the indicated substitution to write answers in terms of x .

(a) **Substitution:** $x = \sin \theta$

$$2 \sec^2 \theta \tan \theta + 3 \sec \theta + C$$

(b) **Substitution:** $x = \tan \theta$

$$2 \sin \theta \cos \theta + 3\theta + C$$

(c) **Substitution:** $2x = \sin \theta$

$$5 \sec \theta \tan \theta + 3\theta + C$$

(d) **Substitution:** $3x = 2 \sin \theta$

$$7 \sec \theta \tan \theta + 5\theta + C$$

(e) **Substitution:** $2x = 3 \sec \theta$

$$5 \sin^2 \theta \cos \theta + 7 \sin \theta + C$$

(f) **Substitution:** $3x = 2 \tan \theta$

$$7 \sin \theta \cos \theta + 5\theta + C$$

2. Solve completely!

$$(a) \int \frac{5}{x^2 \sqrt{4 - 9x^2}} dx$$

$$\text{Note: } \int \csc^2 \theta d\theta = -\cot \theta + C$$

$$(b) \int \frac{5}{x \sqrt{9x^2 + 4}} dx$$

$$\text{Note: } \int \csc \theta d\theta = -\ln |\csc \theta + \cot \theta| + C$$

$$(c) \int \frac{5}{x^2 \sqrt{9x^2 - 4}} dx$$