Homework 02 - Set Maps and Equivalence

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1. Consider the composition $A \xrightarrow{f} B \xrightarrow{g} C$.

In class a student showed that if the composition is surjective, then so is the last map, g.

The student also gave an example showing that $g \circ f$ could be surjective without f being surjective.

- **Prove:** If $g \circ f$ is surjective **and** g is injective, then f is surjective.

2. The **identity** map on a set A is $\mathrm{Id}_{\mathrm{A}} : A \to A$ defined by $\mathrm{Id}(a) = a$ for all $a \in A$. Id is a bijection.

The **inverse** of a map $f : A \to B$ is $f^{-1} : B \to A$ such that $f^{-1} \circ f = \mathrm{Id}_A$ and $f \circ f^{-1} = \mathrm{Id}_B$.

- **Prove:** If $f : A \to B$ has an inverse, then f is a bijection.

3. A map $f : A \to B$ defines an equivalence relation on A by $a_1 \sim a_2$ if $f(a_1) = f(a_2)$.

- Show that $\sim$ is an equivalence relation and describe the equivalence classes.

4. Mathematicians write $\mathbb{R}/\mathbb{Z}$ for the set $\mathbb{R}/\sim$ where $x \sim y$ if $x - y \in \mathbb{Z}$.

- Show that $\sim$ is an equivalence relation and describe the equivalence classes.

5. The **projective line** is $(\mathbb{R}^2 \setminus \{(0,0\})/\sim$ where $(x, y) \sim (\lambda x, \lambda y)$ for all $\lambda \neq 0$.

The **projective plane** is $(\mathbb{R}^3 \setminus \{(0,0,0\})/\sim$ where $(x, y, z) \sim (\lambda x, \lambda y, \lambda z)$ for all $\lambda \neq 0$.

- Describe the equivalence classes for the projective line and projective plane.