Homework 06 - Permutation Groups

We discussed two methods to describe permutations $\sigma \in S_n$.

- **Function form** for a permutation explicitly states all values. For example,

$$\sigma : \begin{cases} 1 \mapsto 5 \\ 2 \mapsto 1 \\ 3 \mapsto 4 \\ 4 \mapsto 3 \\ 5 \mapsto 2 \end{cases}$$

- **Cyclic form** for a permutation expresses it as a product of disjoint cycles. For example,

$$(1\ 5\ 2)\,(3\ 4)$$

Recall that a cycle $(x\ y\ z)$ corresponds to the permutation

$$(x\ y\ z) : \begin{cases} x \mapsto y \\ y \mapsto z \\ z \mapsto x \end{cases}$$

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1. Convert from cyclic form to function form.

$$\sigma = (1\ 6\ 3)\,(2\ 5)$$

2. Convert from function form to cyclic form.

$$f : \begin{cases} 1 \mapsto 4 \\ 2 \mapsto 6 \\ 3 \mapsto 1 \\ 4 \mapsto 3 \\ 5 \mapsto 5 \\ 6 \mapsto 2 \end{cases}$$

3. Simplify the product below to an expression using disjoint cycles.

$$(1\ 2\ 3)\,(2\ 4)\,(1\ 4\ 3)$$

4. Write all elements in the subgroup $\big\langle (1\ 2\ 3\ 4) \big\rangle$.

5. What is the order of the subgroup $\big\langle (1\ 2\ \cdots\ n) \big\rangle$?

Explain.

6. Write all elements in the subgroup $\big\langle (1\ 2\ 3)\,(4\ 5) \big\rangle$.

(Hint: this is easier if you remember that disjoint cycles commute!)