

Section 3 Review

Related Rates

1. A particle moves along the curve $x^2y^3 = x + 2$.
When the particle is at the point $(2, 1)$, its x -position changes at the rate 3.
At this point, what is the rate of change of its y -position?
2. The radius of a circle increases at a rate of 2 ft/s.
What is the rate of increase of the area when the area is 9π ft²?

Linear Approximations and Differentials

3. What is the linearization of $f(x) = \sin(x)$ around $a = \pi/3$?
4. Give value with approximate error bounds for the area of a circle if the radius is $r = 10 \pm 2$ ft.

Extreme Values on Closed Interval (Global Max and Min)

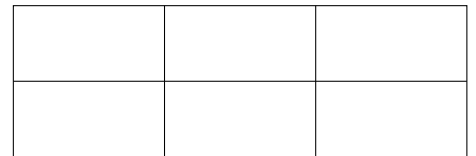
5. Find the critical points of $f(x) = x^{2/3} - 2/3 x$

6. Find the maximum and minimum values on $[-2, 2]$ of $f(x) = \frac{x+3}{x^2-5}$

Applied Optimization (Max and Min Word Problems)

7. What is the minimum value of $f(x, y) = y - 1/x$ if x and y are positive numbers with $x + y = 10$?

8. You plan to fence off a rectangular garden divided into 6 subsections as pictured. What measurements will minimize the amount of fence required so that each subplot has 18 ft^2 area?

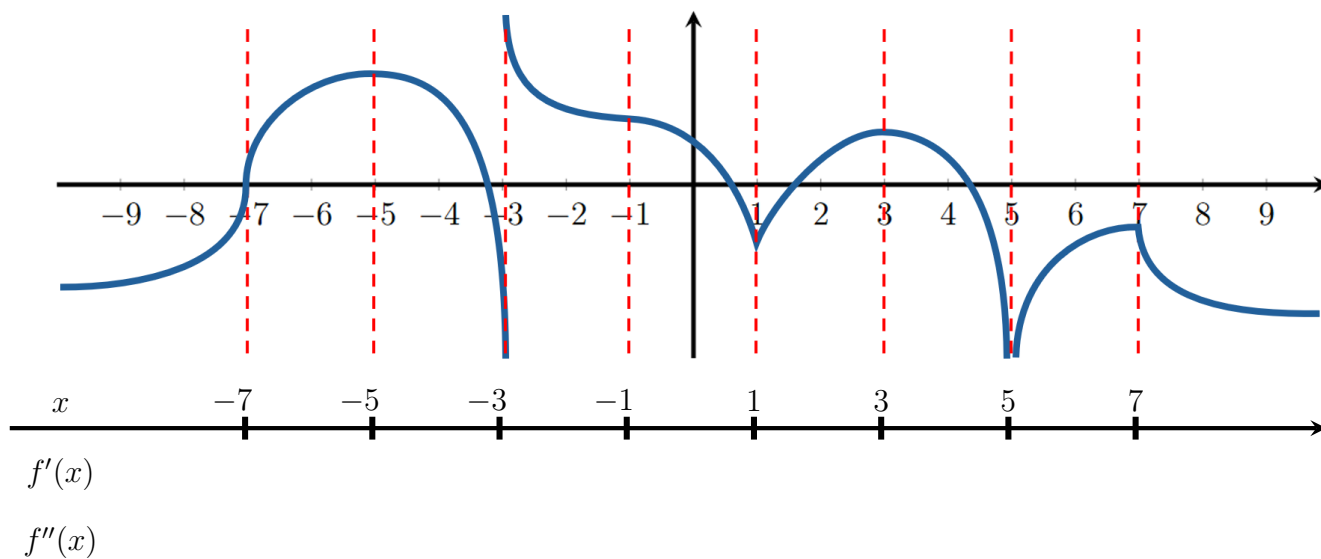


Mean Value Theorem

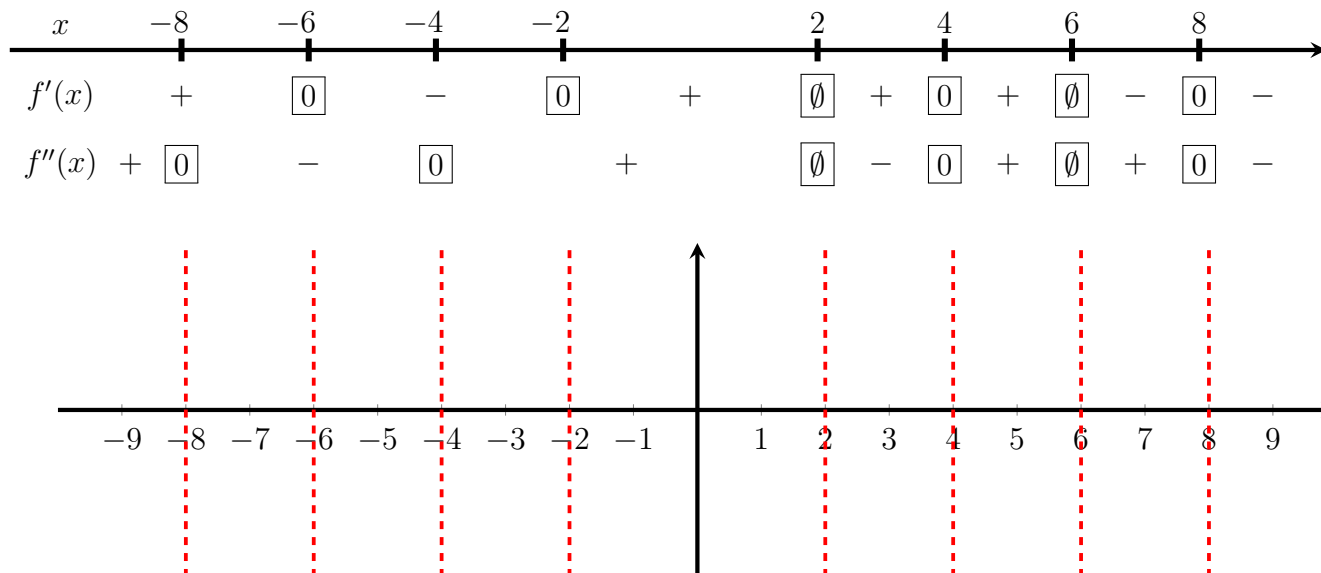
9. If $f(0) = 5$ and $-1 \leq f'(x) \leq 2$ then what values are possible for $f(3)$?

Derivatives and Graphs

10. Convert the graph below to a sign table for f' and f'' .
Indicate values 0 and \emptyset at points; and signs $+$ and $-$ in regions.



11. Convert the sign table for f' and f'' to an approximate graph.



L'Hospital's Rule

12. Compute the limits below.

$$(A) \quad \lim_{x \rightarrow 1} \frac{x^3 + 5x - 6}{x^3 + x - 2}$$

$$(C) \quad \lim_{x \rightarrow 1} \frac{x^4 - x^2 - 2x + 2}{x^4 - x^3 - x + 1}$$

$$(B) \quad \lim_{x \rightarrow 1} \frac{x^3 - 2x + 2}{x^3 + x^2 - 1}$$

$$(D) \quad \lim_{x \rightarrow 0^+} x \ln(x)$$