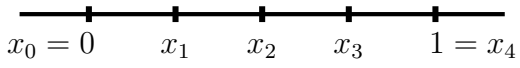
Riemann Sums

We will investigate the area under $f(x) = x^3$ on the interval from $x = a$ to $x = b$.

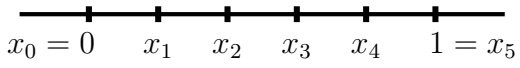
1. Begin with right endpoint Riemann Sums for area under $f(x) = x^3$ from $x = 0$ to $x = 1$.

   - Divide $[0, 1]$ into 4 segments as indicated.
     What are x_1, x_2, x_3, and Δx?

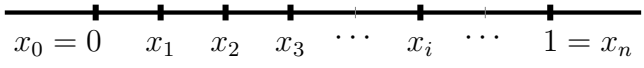
     Write the right endpoint Riemann sum for $f(x) = x^3$ using this division of $[0, 1]$.

   - Divide $[0, 1]$ into 5 segments as indicated.
     What are x_1, x_2, x_3, x_4, and Δx?

     Write the right endpoint Riemann sum for $f(x) = x^3$ using this division of $[0, 1]$.

   - Now divide $[0, 1]$ into n segments.
     What are x_1, x_2, x_3, and Δx?
     What is the ith value x_i?

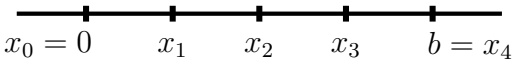
     Use summation notation to write the right endpoint Riemann sum for $f(x) = x^3$.

     Use the formula $\displaystyle\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$ to express this without a summation.

     Area is $\lim_{n\to\infty}$ of the expression above.
     Use your knowledge from last semester to compute this limit!

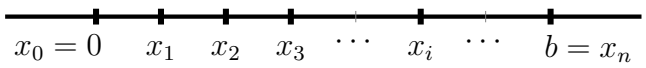
2. Now look at right endpoint Riemann Sums for area under $f(x) = x^3$ from $x = 0$ to $x = b$.

- Divide $[0, b]$ into 4 segments as indicated.
  What are x_1, x_2, x_3, and Δx?



  Write the right endpoint Riemann sum for $f(x) = x^3$ using this division of $[0, b]$.

- Now divide $[0, b]$ into n segments.
  What are x_1, x_2, x_3, and Δx?
  What is the ith value x_i?



  Use summation notation to write the right endpoint Riemann sum for $f(x) = x^3$.

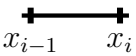
  Use the formula $\displaystyle\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$ to express this without a summation.

  Area is $\lim_{n\to\infty}$ of the expression above.
  Use your knowledge from last semester to compute this limit!

Summary. Dividing interval $[a, b]$ in to n segments yields:

$$\Delta x = \frac{b-a}{n}$$

$$\begin{aligned} x_0 &= a \\ x_1 &= a + \Delta x \\ x_2 &= a + 2\,\Delta x \\ &\vdots \\ x_i &= a + i\,\Delta x \\ &\vdots \\ x_n &= a + n\,\Delta x = b \end{aligned}$$

Segment i of the division is: 

Right endpoint Riemann sum: $\displaystyle\sum_{i=1}^{n} f(x_i)\,\Delta x$

Left endpoint Riemann sum: $\displaystyle\sum_{i=1}^{n} f(x_{i-1})\,\Delta x$