

Trigonometric Integrals

Basic Trigonometric Integrals

$\int \sin x \, dx = -\cos x + C$

$\int \tan x \, dx = \ln|\sec x| + C$

$\frac{d}{dx} \tan x = \sec^2 x$

$\int \cos x \, dx = \sin x + C$

$\int \sec x \, dx = \ln|\sec x + \tan x| + C$

$\frac{d}{dx} \sec x = \sec x \tan x$

Substitution

$\sin x = u$

$\cos^2 x = 1 - u^2$

$\cos x \, dx = du$

$\cos x = u$

$\sin^2 x = 1 - u^2$

$\sin x \, dx = -du$

Requirement

\cos power
is ODD

\sin power
is ODD

Substitution

$\tan x = u$

$\sec^2 x = u^2 + 1$

$\sec^2 x \, dx = du$

$\sec x = u$

$\tan^2 x = u^2 - 1$

$\sec x \tan x \, dx = du$

Requirement

\sec power
is EVEN

\tan power
is ODD

1. $\int \sin^3 x \cos^4 x \, dx$

2. $\int \sin^6 x \cos^3 x \, dx$

3. $\int \sec^3 x \tan^3 x \, dx$

4. $\int \sec^4 x \tan^4 x \, dx$

5. $\int \frac{\sin^3 x}{\cos^4 x} \, dx$

6. $\int \frac{\sec^4 x}{\tan^3 x} \, dx$

Type conversion	$\sin x = \frac{\tan x}{\sec x}$	$\cos x = \frac{1}{\sec x}$	$\tan x = \frac{\sin x}{\cos x}$	$\sec x = \frac{1}{\cos x}$
------------------------	----------------------------------	-----------------------------	----------------------------------	-----------------------------

1. $\int \sin x \tan^2 x \, dx$

2. $\int \frac{\sec x}{\tan^2 x} \, dx$

Powers to Frequency

$\sin x \cos x = \frac{1}{2} \sin(2x)$

$\sin^2 x = \frac{1}{2} (1 - \cos(2x))$

$\cos^2 x = \frac{1}{2} (1 + \cos(2x))$

1. $\int \sin^2 x \, dx$

2. $\int \sin^2 x \cos^2 x \, dx$

Reduction Rules

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

$$\int \tan^n x \, dx = \frac{1}{n} \tan^{n-1} x - \int \tan^{n-2} x \, dx$$

1. $\int \cos^4 x \, dx$

2. $\int \tan^6 x \, dx$

Mismatched Frequencies

$$\sin(ax) \cos(bx) = \frac{1}{2} \sin((a-b)x) + \frac{1}{2} \sin((a+b)x)$$

$$\cos(ax) \cos(bx) = \frac{1}{2} \cos(|a-b|x) + \frac{1}{2} \cos((a+b)x)$$

$$\sin(ax) \sin(bx) = \frac{1}{2} \cos(|a-b|x) - \frac{1}{2} \cos((a+b)x)$$

1. $\int \sin 4x \cos 3x \, dx$

2. $\int \cos 4x \cos 3x \, dx$