Assignment 1

STAT463

1. Consider the bivariate random variable (S, P). Explicitly write out the joint pmf for (S, P). **– 4 points**

$$p_{S,P}(s,p) = \begin{cases} \frac{1}{36} & \text{if } (s,p) = (2,1),(4,4),(6,9),(8,16),(10,25),(12,36) \\ \frac{1}{18} & (x,y) \in x_{SP} \\ 0 & else \end{cases}$$

Where

$$\begin{aligned} X_{SP} = &\{(3,2),(4,3),(5,4),(6,5),(7,6), \\ &(5,6),(6,8),(7,10),(8,12),(7,12), \\ &(8,15),(9,18),(9,20),(10,24),(11,30)\} \end{aligned}$$

2. Explicitly write out the marginal pmf of S. **– 2 points

$$p_S(s) = \Pr(S = s) = \Pr(\{o; S(o) = s\})$$

$$p_S(s) = \begin{cases} \frac{1}{36} & \text{if } n = 2, 12 \\ \frac{2}{36} & \text{if } n = 3, 11 \\ \frac{3}{36} & \text{if } n = 4, 10 \\ \frac{4}{36} & \text{if } n = 5, 9 \\ \frac{5}{36} & \text{if } n = 8 \\ 0 & else \end{cases}$$

3. Explicitly write out the marginal pmf of P. **– 2 points**

$$p_P(p) = \Pr(P = p)$$

$$p_P(p) = \begin{cases} \frac{1}{36} & \text{if } n = 1, 9, 16, 25, 36 \\ \frac{1}{18} & \text{if } n = 2, 3, 5, 8, 10, 15, 18, 20, 24 \\ \frac{1}{12} & \text{if } n = 4 \\ \frac{1}{9} & \text{if } n = 6, 12 \\ 0 & \text{else} \end{cases}$$

4. Calculate $E[S]$. **– ½ point**

$$
\begin{aligned}
E[S] &= 2 \times \frac{1}{36} + 3 \times \frac{1}{18} + 4 \times \frac{1}{12} + 5 \times \frac{1}{9} \\
&+ 6 \times \frac{5}{36} + 7 \times \frac{1}{6} + 8 \times \frac{5}{36} + 9 \times \frac{1}{9} \\
&+ 10 \times \frac{1}{10} + 11 \times \frac{1}{18} + 12 \times \frac{1}{36} \\
&= 7
\end{aligned}
$$

5. Calculate E[P]. **– ½ point**

$$
\begin{aligned}
E[P] &= \frac{6}{9} + \frac{4}{12} + \frac{1+9+16+25+36}{36} \\
&+ \frac{2+3+5+8+10+15+18+20+30}{18} \\
&= 12.25
\end{aligned}
$$

6. Calculate $Cov[S, P]$. **– 1 point**

$\overline{s} = 7$, $\overline{p} = 12.25$, and $n = 36$

$$
\begin{aligned}
Cov[S, P] &= \frac{\sum(s_i - \overline{s})(p_i - \overline{p})}{n-1} \\
&= \frac{735}{36-1} = \frac{735}{35} \\
&= 21
\end{aligned}
$$

7. Calculate Var[P]. **– ½ point**

redo #7 using $E[P]$
We have the following equation to be solved.

$$
Var[P] = E[P^2] - \mu_P^2
$$

We have $E[P^2]$ such that:

$$
\begin{aligned}
E[P^2] &= \frac{1^2 + 16^2 + 25^2}{36} + \frac{6^2 + 12^2}{9} + \frac{7^2}{6} + \frac{4^2}{12} \\
&+ \frac{2^2 + 3^2 + 8^2 + 10^2 + 15^2 + 18^2 + 20^2 + 24^2 + 30^2}{18} \\
&= \frac{3427}{18} = 190.3\overline{8}
\end{aligned}
$$

and μ_P^2 such that:

$$\begin{aligned}\mu_P^2 &= E[P]^2 \\ &= (12.25)^2 \\ &= 150.0625\end{aligned}$$

Consequently, we can compute $Var[P]$.

$$Var[P] = \frac{3427}{18} - 150.0625 = 40.3263\overline{8}$$

8. Explicitly write out the conditional pmf of P given S = 7 . **– 1.5 points**

9. Calculate $E[P \mid S = 7]$. **– ½ point**

10. Calculate $Var[P \mid S = 7]$. **– ½ point**