

Assignment 1

STAT463

1. Consider the bivariate random variable (S, P). Explicitly write out the joint pmf for (S, P). – 4 points

$$p_{S,P}(s,p) = \begin{cases} \frac{1}{36} & \text{if } (s,p) = (2,1), (4,4), (6,9), (8,16), (10,25), (12,36) \\ \frac{1}{18} & (x,y) \in x_{SP} \\ 0 & \text{else} \end{cases}$$

Where

$$X_{SP} = (3,2), (4,3), (5,4), (6,5), (7,6), \\ (5,6), (6,8), (7,10), (8,12), (7,12), \\ (8,15), (9,18), (9,20), (10,24), (11,30)$$

2. Explicitly write out the marginal pmf of S. **– 2 points

$$p_S(s) = \Pr(S = s) = \Pr(\{o; S(o) = s\})$$

$$p_S(s) = \begin{cases} \frac{1}{36} & \text{if } n = 2, 12 \\ \frac{2}{36} & \text{if } n = 3, 11 \\ \frac{3}{36} & \text{if } n = 4, 10 \\ \frac{4}{36} & \text{if } n = 5, 9 \\ \frac{5}{36} & \text{if } n = 8 \\ 0 & \text{else} \end{cases}$$

3. Explicitly write out the marginal pmf of P. – 2 points

$$p_P(p) = \Pr(P = p)$$

$$p_P(p) = \begin{cases} \frac{1}{36} & \text{if } n = 1, 9, 16, 25, 36 \\ \frac{1}{18} & \text{if } n = 2, 3, 5, 8, 10, 15, 18, 20, 24 \\ \frac{1}{12} & \text{if } n = 4 \\ \frac{1}{9} & \text{if } n = 6, 12 \\ 0 & \text{else} \end{cases}$$

4. Calculate $E[S]$. – ½ point

$$\begin{aligned} E[S] &= 2 \times \frac{1}{36} + 3 \times \frac{1}{18} + 4 \times \frac{1}{12} + 5 \times \frac{1}{9} \\ &\quad + 6 \times \frac{5}{36} + 7 \times \frac{1}{6} + 8 \times \frac{5}{36} + 9 \times \frac{1}{9} \\ &\quad + 10 \times \frac{1}{10} + 11 \times \frac{1}{18} + 12 \times \frac{1}{36} \\ &= 7 \end{aligned}$$

5. Calculate $E[P]$. – ½ point

$$\begin{aligned} E[P] &= \frac{6}{9} + \frac{4}{12} + \frac{1 + 9 + 16 + 25 + 36}{36} \\ &\quad + \frac{2 + 3 + 5 + 8 + 10 + 15 + 18 + 20 + 30}{18} \\ &= 12.25 \end{aligned}$$

6. Calculate $Cov[S, P]$. – 1 point

$\bar{s} = 7$, $\bar{p} = 12.25$, and $n = 36$

$$\begin{aligned} Cov[S, P] &= \frac{\sum (s_i - \bar{s})(p_i - \bar{p})}{n - 1} \\ &= \frac{735}{36 - 1} = \frac{735}{35} \\ &= 21 \end{aligned}$$

7. Calculate $Var[P]$. – ½ point

We have the following equation to be solved.

$$Var[P] = E[P^2] - \mu_P^2$$

We have $E[P^2]$ such that:

$$\begin{aligned} E[P^2] &= \frac{1^2 + 16^2 + 25^2}{36} + \frac{6^2 + 12^2}{9} + \frac{7^2}{6} + \frac{4^2}{12} \\ &\quad + \frac{2^2 + 3^2 + 8^2 + 10^2 + 15^2 + 18^2 + 20^2 + 24^2 + 30^2}{18} \\ &= \frac{3427}{18} = 190.38 \end{aligned}$$

and μ_P^2 such that:

$$\begin{aligned}\mu_P^2 &= E[P]^2 \\ &= (12.25)^2 \\ &= 150.0625\end{aligned}$$

Consequently, we can compute $Var[P]$.

$$Var[P] = \frac{3427}{18} - 150.0625 = 40.3263\bar{8}$$

8. Explicitly write out the conditional pmf of P given S = 7 . – 1.5 points

$$\begin{aligned}P_{P|S}(p_i | y_j) &= \frac{P_{PS}(p_i, s_j)}{P_S(s_j)} \\ P_{P|S}(p | y) &= \begin{cases} \frac{1}{3} & \text{if } p = 6, 10, 12 \\ 0 & \text{else} \end{cases}\end{aligned}$$

9. Calculate $E[P | S = 7]$. – ½ point

$$E[P | S = 7] = \frac{6 + 10 + 12}{3} = \frac{28}{3}$$

10. Calculate $Var[P | S = 7]$. – ½ point

$$Var[P | S = 7] = E[(P | S = 7)^2] - E[P | S = 7]^2$$

$$E[(P | S = 7)^2] = \frac{6^2 + 10^2 + 12^2}{3} = \frac{280}{3}$$

$$Var[P | S = 7] = \frac{280}{3} - \frac{784}{9} = \frac{56}{9} = 6.\overline{222}$$