Assignment 1

STAT463

1. Consider the bivariate random variable (S, P). Explicitly write out the joint pmf for (S, P). **– 4 points**

$$p_{S,P}(s,p) = \begin{cases} \frac{1}{36} & \text{if } (s,p) = (2,1),(4,4),(6,9),(8,16),(10,25),(12,36) \\ \frac{1}{18} & (x,y) \in x_{SP} \\ 0 & else \end{cases}$$

Where

$$\begin{aligned} X_{SP} = \{&(3,2),(4,3),(5,4),(6,5),(7,6), \\ &(5,6),(6,8),(7,10),(8,12),(7,12), \\ &(8,15),(9,18),(9,20),(10,24),(11,30)\} \end{aligned}$$

2. Explicitly write out the marginal pmf of S. **– 2 points

$$p_S(s) = \Pr(S = s) = \Pr(\{o; S(o) = s\})$$

$$p_S(s) = \begin{cases} \frac{1}{36} & \text{if } s = 2,12 \\ \frac{1}{18} & \text{if } s = 3,11 \\ \frac{1}{12} & \text{if } s = 4,10 \\ \frac{1}{9} & \text{if } s = 5,9 \\ \frac{5}{36} & \text{if } s = 6,8 \\ \frac{1}{6} & \text{if } s = 7 \\ 0 & else \end{cases}$$

3. Explicitly write out the marginal pmf of P. **– 2 points**

$$p_P(p) = \Pr(P = p)$$

$$p_P(p) = \begin{cases} \frac{1}{36} & \text{if } n = 1,9,16,25,36 \\ \frac{1}{18} & \text{if } n = 2,3,5,8,10,15,18,20,24,30 \\ \frac{1}{12} & \text{if } n = 4 \\ \frac{1}{9} & \text{if } n = 6,12 \\ 0 & \text{else} \end{cases}$$

4. Calculate $E[S]$. **– ½ point**

$$\begin{aligned} E[S] &= 2\times\frac{1}{36}+3\times\frac{1}{18}+4\times\frac{1}{12}+5\times\frac{1}{9} \\ &+6\times\frac{5}{36}+7\times\frac{1}{6}+8\times\frac{5}{36}+9\times\frac{1}{9} \\ &+10\times\frac{1}{10}+11\times\frac{1}{18}+12\times\frac{1}{36} \\ &=7 \end{aligned}$$

5. Calculate E[P]. **– ½ point**

$$\begin{aligned} E[P] &= \frac{4}{12}+\frac{6+12}{9}+\frac{1+9+16+25+36}{36} \\ &+\frac{2+3+5+8+10+15+18+20+24+30}{18} \\ &=12.25 \end{aligned}$$

6. Calculate $Cov[S,P]$. **– 1 point**

$\overline{s}=7$, $\overline{p}=12.25$, and $n=36$

$$\begin{aligned} Cov[S,P] &= \frac{\sum(s_i-\overline{s})(p_i-\overline{p})}{n-1} \\ &= \frac{735}{36-1}=\frac{735}{35} \\ &=21 \end{aligned}$$

7. Calculate Var[P]. **– ½ point**

We have the following equation to be solved.

$$Var[P]=E[P^2]-\mu_P^2$$

We have $E[P^2]$ such that:

$$\begin{aligned} E[P^2] &= \frac{1^2+9^2+16^2+25^2+36^2}{36}+\frac{6^2+12^2}{9}+\frac{4^2}{12} \\ &+\frac{2^2+3^2+5^2+8^2+10^2+15^2+18^2+20^2+24^2+30^2}{18} \\ &=\frac{8281}{36}=230.02\overline{77} \end{aligned}$$

and μ_P^2 such that:

$$\begin{aligned}\mu_P^2 &= E[P]^2 \\ &= (12.25)^2 \\ &= 150.0625\end{aligned}$$

Consequently, we can compute $Var[P]$.

$$Var[P] = \frac{8281}{36} - 150.0625 = 79.9652\overline{77}$$

8. Explicitly write out the conditional pmf of P given S = 7 . **– 1.5 points**

$$P_{P|S}(p_i \mid y_j) = \frac{P_{PS}(p_i, s_j)}{P_S(s_j)}$$

$$P_{P|S}(p \mid y) = \begin{cases} \frac{1}{3} & \text{if } p = 6, 10, 12 \\ 0 & \text{else} \end{cases}$$

9. Calculate $E[P \mid S = 7]$. **– ½ point**

$$E[P \mid S = 7] = \frac{6 + 10 + 12}{3} = \frac{28}{3}$$

10. Calculate $Var[P \mid S = 7]$. **– ½ point**

$$Var[P \mid S = 7] = E[(P \mid S = 7)^2] - E[P \mid S = 7]^2$$

$$E[(P \mid S = 7)^2] = \frac{6^2 + 10^2 + 12^2}{3} = \frac{280}{3}$$

$$Var[P \mid S = 7] = \frac{280}{3} - \frac{784}{9} = \frac{56}{9} = 6.\overline{222}$$