Numerical Analysis I，Homework 1

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Question 1:

Code:

```
diary assign1-output
format compact
% Math 573 Assignment #1 Benwei Jin
% Problem 1a
a=-5; b=5;
n=4;
xin=linspace(a,b,n+1);
yin= 1./(1+xin.^2);
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 1a functions; n=4');
print -deps assign1-a11-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
title('Problem 1a error; n=4');
print -deps assign1-a12-out2.eps

n=8;
xin=linspace(a,b,n+1);
yin= 1./(1+xin.^2);
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 1a functions; n=8');
print -deps assign1-a21-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
title('Problem 1a error; n=8');
print -deps assign1-a22-out2.eps

n=16;
xin=linspace(a,b,n+1);
yin= 1./(1+xin.^2);
```

```matlab
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 1a functions; n=16');
print -deps assign1-a31-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
title('Problem 1a error; n=16');
print -deps assign1-a32-out2.eps

n=32;
xin=linspace(a,b,n+1);
yin= 1./(1+xin.^2);
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 1a functions; n=32');
print -deps assign1-a41-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
title('Problem 1a error; n=32');
print -deps assign1-a42-out2.eps

% Problem 1b
n=4;
xin=linspace(a,b,n+1);
yin= exp(-.4*xin.^2);
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= exp(-.4*xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 1b functions; n=4');
print -deps assign1-b11-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
title('Problem 1b error; n=4');
print -deps assign1-b12-out2.eps

n=8;
xin=linspace(a,b,n+1);
yin= exp(-.4*xin.^2);
xout=linspace(a,b,100);
```

```matlab
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= exp(-.4*xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 1b functions; n=8');
print -deps assign1-b21-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
title('Problem 1b error; n=8');
print -deps assign1-b22-out2.eps

n=16;
xin=linspace(a,b,n+1);
yin= exp(-.4*xin.^2);
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= exp(-.4*xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 1b functions; n=16');
print -deps assign1-b31-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
title('Problem 1b error; n=16');
print -deps assign1-b32-out2.eps

n=32;
xin=linspace(a,b,n+1);
yin= exp(-.4*xin.^2);
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= exp(-.4*xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 1b functions; n=32');
print -deps assign1-b41-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
title('Problem 1b error; n=32');
print -deps assign1-b42-out2.eps

dairy off
```
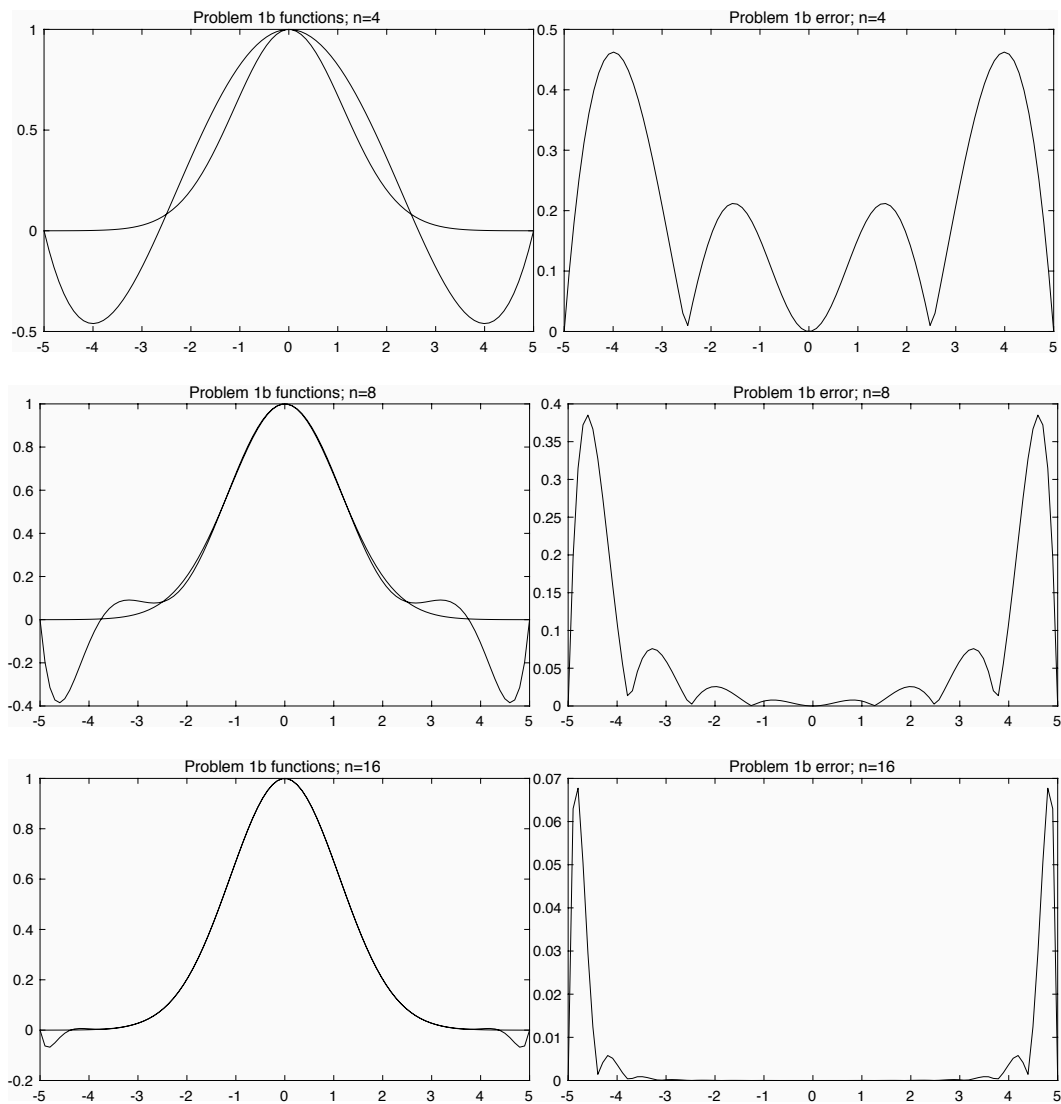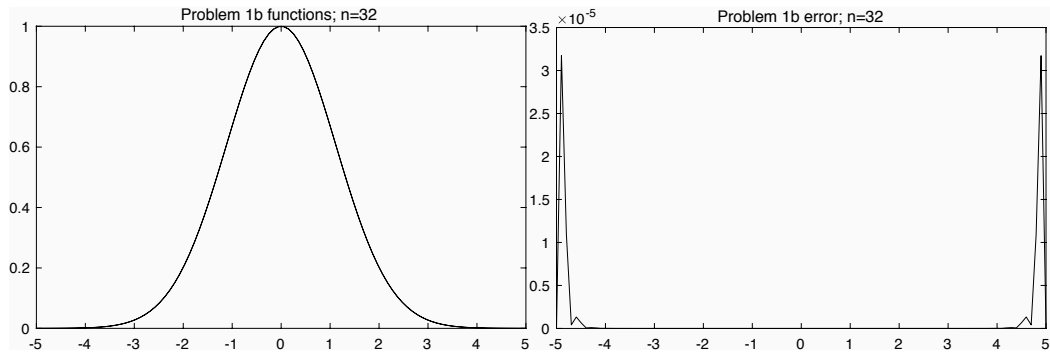
1(a) graphs:
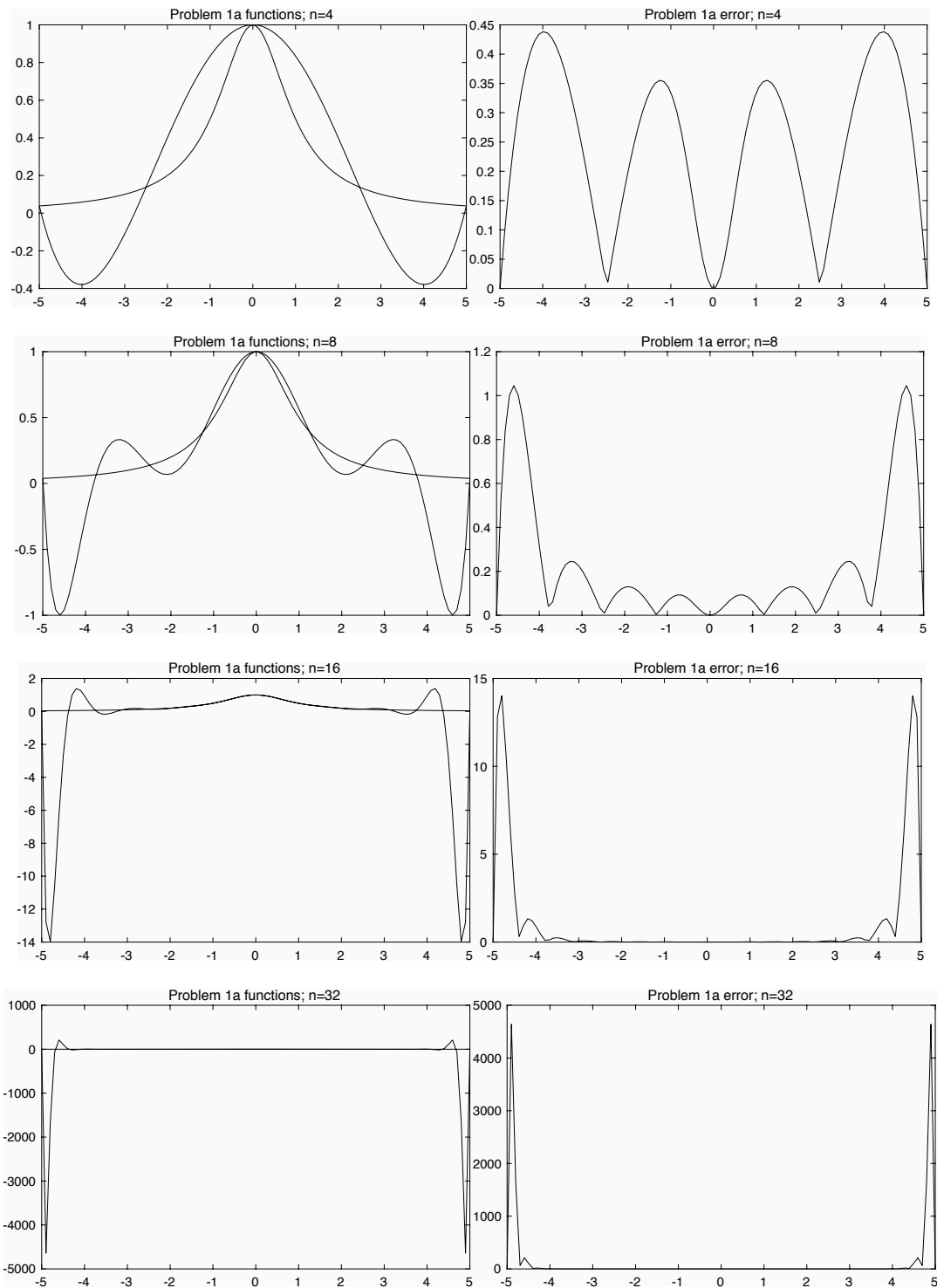
Problem 1a		
n	Max|e(x)|	Approx. location
4	0.4383	±3.9899
8	1.0451	±4.5960
16	14.0135	±4.7980
32	4641.2	±4.8990

1(b) graphs:

Problem 1b		
n	Max|e(x)|	Approx. location
4	0.4624	±3.9899
8	0.3852	±4.5960
16	0.0677	±4.7980
32	$3.1730*10^{-5}$	±4.8990

Question 2:

Code：

```
diary assign1-output
format compact
% Math 573 Assignment #1 Benwei Jin
% Problem 2
a=-5; b=5;
n=4;
x=linspace(1,2*n+1,n+1);
xin = (a+b)/2 + (b-a)/2 * cos(x*pi/(2*n+2));
yin= 1./(1+xin.^2);
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 2 functions; n=4');
print -deps assign1-211-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
```

```
title('Problem 2 error; n=4');
print -deps assign1-212-out2.eps


n=8;
x=linspace(1,2*n+1,n+1);
xin = (a+b)/2 + (b-a)/2 * cos(x*pi/(2*n+2));
yin= 1./(1+xin.^2);
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 2 functions; n=8');
print -deps assign1-221-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
title('Problem 2 error; n=8');
print -deps assign1-222-out2.eps


n=16;
x=linspace(1,2*n+1,n+1);
xin = (a+b)/2 + (b-a)/2 * cos(x*pi/(2*n+2));
yin= 1./(1+xin.^2);
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 2 functions; n=16');
print -deps assign1-231-out1.eps
pause(5);
plot(xout,abs(ytrue-yout))
title('Problem 2 error; n=16');
print -deps assign1-232-out2.eps


n=32;
x=linspace(1,2*n+1,n+1);
xin = (a+b)/2 + (b-a)/2 * cos(x*pi/(2*n+2));
yin= 1./(1+xin.^2);
xout=linspace(a,b,100);
[yout, cof] = Interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,yout,xout,ytrue)
title('Problem 2 functions; n=32');
print -deps assign1-241-out1.eps
pause(5);
```
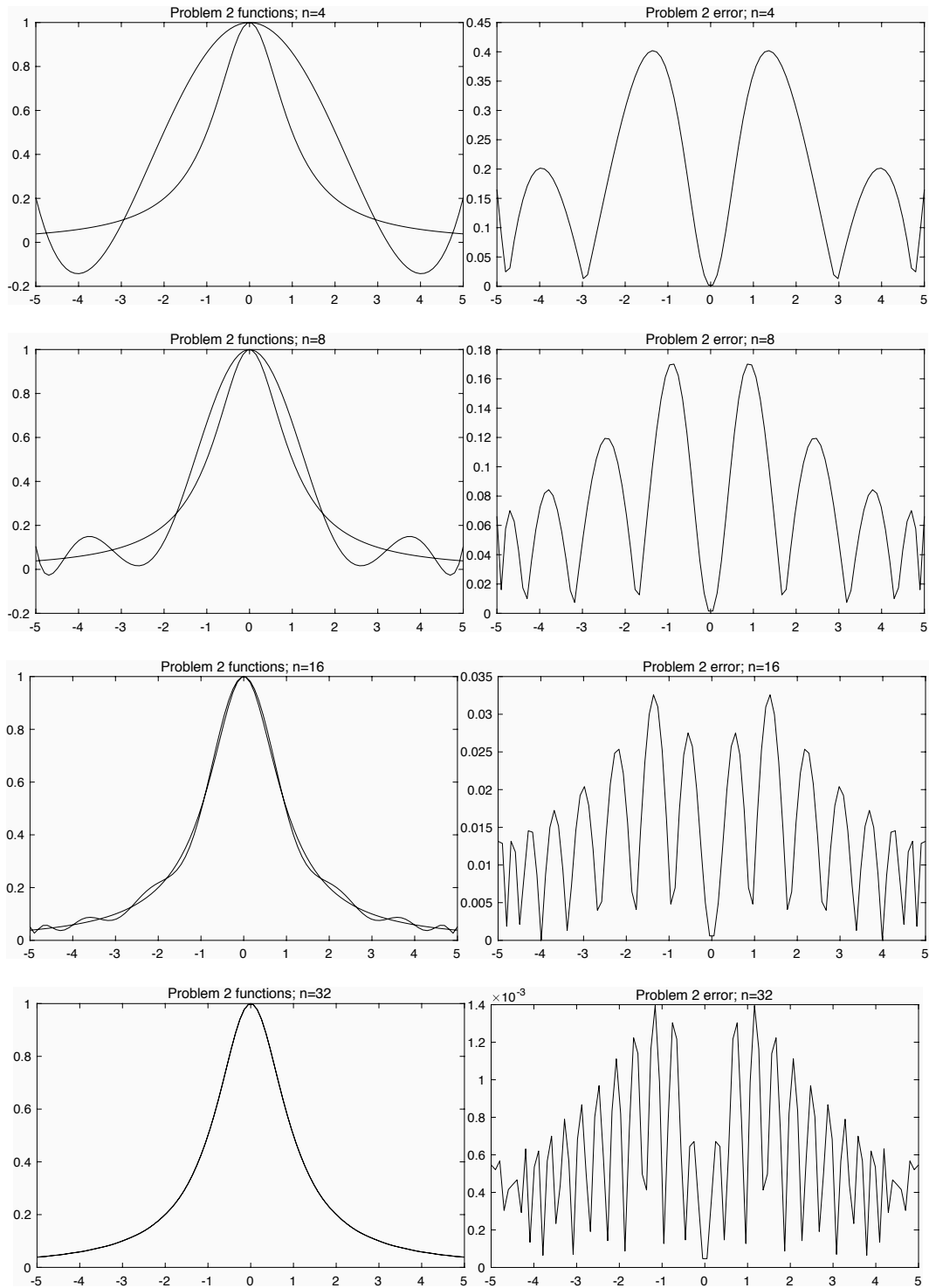
```
plot(xout,abs(ytrue-yout))
title('Problem 2 error; n=32');
print -deps assign1-242-out2.eps

diary off
```

2 graphs:

Problem 2		
n	Max|e(x)|	Approx. location
4	0.4020	±1.3636
8	0.1701	±0.8586
16	0.0326	±1.3636
32	0.0014	±1.1616

Question 3:

3(a) Without solving any equations, find a cubic polynomial P3(x)

satisfying P3(0) = 0,P3(1/3) = 0, P3(2/3) = 0, and P3(1) = 1

Solution: We write:

$$P_3(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$
$$+ f[x_0, x_1, x_2 x_3](x - x_0)(x - x_1)(x - x_2)$$

The dividend difference table:

x_i		$f[x_i]$	f[,]	f[, ,]	f[, , ,]
x_0	0	0	0	0	9/2
x_1	1/3	0	0	9/2	
x_2	2/3	0	3		
x_3	1	1			

$$P_3(x) = \frac{9}{2}x(x - \frac{1}{3})(x - \frac{2}{3})$$

3b. Suppose P2(x) is a given quadratic polynomial and xi, i = 0, 1, 2, 3 are distinct points. Construct in as simple a manner as possible a cubic polynomial P3(x) satisfying P3(xi) = P2(xi), i = 0, 1, 2, and P3(x3) = 1. Your answer will involve P2(x).

Solution: To write $P_3(x)$ in Newton Form:

$$P_3(x) = P_2(x) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$

When $x = x_3$,

$$P_3(x_3) = P_2(x_3) + f[x_0, x_1, x_2, x_3](x_3 - x_0)(x_3 - x_1)(x_3 - x_2) = 1$$

∴

$$f[x_0, x_1, x_2, x_3] = \frac{1 - P_2(x_3)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}$$

∴

$$P_3(x) = P_2(x) + \frac{1 - P_2(x_3)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}(x - x_0)(x - x_1)(x - x_2)$$

Question 4:

4. Consider the problem of determining a polynomial P2(x) of degree $\leqslant$ 2 satisfying

P2(x0) = f(x0), P'2(x1) = f'(x1), P2(x2) = f(x2),

where f, x0, x1, x2 are given and x0 < x1 < x2. Does this problem have a solution for all given f in C1[x0, x2] and all x0, x1, x2 ? If so, find it; if not,

give an example for which there is no solution.

Solution: suppose：

$$P_2(x) = P_1(x) + A(x - x_0)(x - x_2)$$

$$= P_1(x) + A(x^2 - (x_0 + x_2)x + x_0 x_2)$$

And

$$P_1(x) = f(x_0) + \frac{f(x_2) - f(x_0)}{x_2 - x_0}(x - x_0)$$

The derivative of $P_2(x)$ is:

$$P_2{}'(x) = P_1{}'(x) + A(2x - (x_0 + x_2))$$

$$= \frac{f(x_2) - f(x_0)}{x_2 - x_0} + A(2x - (x_0 + x_2))$$

When $x = x_1$:

$$P_2'(x_1) = \frac{f(x_2) - f(x_0)}{x_2 - x_0} + A(2x_1 - (x_0 + x_2)) = f'(x_1)$$

$\therefore$

$$A = \frac{f'(x_1) - \dfrac{f(x_2) - f(x_0)}{x_2 - x_0}}{2x_1 - (x_0 + x_2)}$$

When $x_2 - x_0 = 0$ OR $2x_1 - (x_0 + x_2) = 0$, the value of A is

undefined and $P_2(x)$ cannot be determined.

So the problem might **NOT** have a solution.

Example: There is no solution for $P_2(x)$ satisfying $f(x) = ln(x)$

$$x_0 = 1, x_1 = 2, x_2 = 3,$$

$$P_2(x_0) = 0, P_2{}'^{(x_1)} = \frac{1}{2}, P_2(x_2) = ln(3)$$