**Numerical Analysis I，Homework 1**

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**Question 1:**

Code:

diary assign1-output

format compact

% Math 573 Assignment #1 Benwei Jin

% Problem 1a

a=-5; b=5;

n=4;

xin=linspace(a,b,n+1);

yin= 1./(1+xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= 1./(1+xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 1a functions; n=4');

print -deps assign1-a11-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 1a error; n=4');

print -deps assign1-a12-out2.eps

n=8;

xin=linspace(a,b,n+1);

yin= 1./(1+xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= 1./(1+xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 1a functions; n=8');

print -deps assign1-a21-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 1a error; n=8');

print -deps assign1-a22-out2.eps

n=16;

xin=linspace(a,b,n+1);

yin= 1./(1+xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= 1./(1+xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 1a functions; n=16');

print -deps assign1-a31-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 1a error; n=16');

print -deps assign1-a32-out2.eps

n=32;

xin=linspace(a,b,n+1);

yin= 1./(1+xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= 1./(1+xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 1a functions; n=32');

print -deps assign1-a41-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 1a error; n=32');

print -deps assign1-a42-out2.eps

% Problem 1b

n=4;

xin=linspace(a,b,n+1);

yin= exp(-.4\*xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= exp(-.4\*xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 1b functions; n=4');

print -deps assign1-b11-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 1b error; n=4');

print -deps assign1-b12-out2.eps

n=8;

xin=linspace(a,b,n+1);

yin= exp(-.4\*xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= exp(-.4\*xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 1b functions; n=8');

print -deps assign1-b21-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 1b error; n=8');

print -deps assign1-b22-out2.eps

n=16;

xin=linspace(a,b,n+1);

yin= exp(-.4\*xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= exp(-.4\*xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 1b functions; n=16');

print -deps assign1-b31-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 1b error; n=16');

print -deps assign1-b32-out2.eps

n=32;

xin=linspace(a,b,n+1);

yin= exp(-.4\*xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= exp(-.4\*xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 1b functions; n=32');

print -deps assign1-b41-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 1b error; n=32');

print -deps assign1-b42-out2.eps

dairy off

1(a) graphs:









|  |  |  |
| --- | --- | --- |
| Problem 1a | | |
| n | Max|e(x)| | Approx. location |
| 4 | 0.4383 | ±3.9899 |
| 8 | 1.0451 | ±4.5960 |
| 16 | 14.0135 | ±4.7980 |
| 32 | 4641.2 | ±4.8990 |

1(b) graphs:









|  |  |  |
| --- | --- | --- |
| Problem 1b | | |
| n | Max|e(x)| | Approx. location |
| 4 | 0.4624 | ±3.9899 |
| 8 | 0.3852 | ±4.5960 |
| 16 | 0.0677 | ±4.7980 |
| 32 | 3.1730\* | ±4.8990 |

**Question 2:**

Code：

diary assign1-output

format compact

% Math 573 Assignment #1 Benwei Jin

% Problem 2

a=-5; b=5;

n=4;

x=linspace(1,2\*n+1,n+1);

xin = (a+b)/2 + (b-a)/2 \* cos(x\*pi/(2\*n+2));

yin= 1./(1+xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= 1./(1+xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 2 functions; n=4');

print -deps assign1-211-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 2 error; n=4');

print -deps assign1-212-out2.eps

n=8;

x=linspace(1,2\*n+1,n+1);

xin = (a+b)/2 + (b-a)/2 \* cos(x\*pi/(2\*n+2));

yin= 1./(1+xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= 1./(1+xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 2 functions; n=8');

print -deps assign1-221-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 2 error; n=8');

print -deps assign1-222-out2.eps

n=16;

x=linspace(1,2\*n+1,n+1);

xin = (a+b)/2 + (b-a)/2 \* cos(x\*pi/(2\*n+2));

yin= 1./(1+xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= 1./(1+xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 2 functions; n=16');

print -deps assign1-231-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 2 error; n=16');

print -deps assign1-232-out2.eps

n=32;

x=linspace(1,2\*n+1,n+1);

xin = (a+b)/2 + (b-a)/2 \* cos(x\*pi/(2\*n+2));

yin= 1./(1+xin.^2);

xout=linspace(a,b,100);

[yout, cof] = Interppoly(xin,yin,xout);

ytrue= 1./(1+xout.^2);

plot(xout,yout,xout,ytrue)

title('Problem 2 functions; n=32');

print -deps assign1-241-out1.eps

pause(5);

plot(xout,abs(ytrue-yout))

title('Problem 2 error; n=32');

print -deps assign1-242-out2.eps

diary off

2 graphs:









|  |  |  |
| --- | --- | --- |
| Problem 2 | | |
| n | Max|e(x)| | Approx. location |
| 4 | 0.4020 | ±1.3636 |
| 8 | 0.1701 | ±0.8586 |
| 16 | 0.0326 | ±1.3636 |
| 32 | 0.0014 | ±1.1616 |

**Question 3:**

3(a) Without solving any equations, find a cubic polynomial P3(x) satisfying P3(0) = 0,P3(1/3) = 0, P3(2/3) = 0, and P3(1) = 1

**Solution:** We write:

The dividend difference table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | | f[] | f[,] | f[, ,] | f[, , ,] |
|  | 0 | 0 | 0 | 0 | 9/2 |
|  | 1/3 | 0 | 0 | 9/2 |  |
|  | 2/3 | 0 | 3 |  |  |
|  | 1 | 1 |  |  |  |

3b. Suppose P2(x) is a given quadratic polynomial and xi, i = 0, 1, 2, 3 are distinct points. Construct in as simple a manner as possible a cubic polynomial P3(x) satisfying P3(xi) = P2(xi), i = 0, 1, 2, and P3(x3) = 1. Your answer will involve P2(x).

**Solution: T**o write in Newton Form:

When ,

∴

∴

**Question 4:**

4. Consider the problem of determining a polynomial P2(x) of degree ≤ 2 satisfying

P2(x0) = f(x0), P‘2(x1) = f’(x1), P2(x2) = f(x2),

where f, x0, x1, x2 are given and x0 < x1 < x2. Does this problem have a solution for all given f in C1[x0, x2] and all x0, x1, x2 ? If so, find it; if not, give an example for which there is no solution.

**Solution:** suppose：

And

The derivative of is:

When :

∴

When OR , the value of A is undefined and cannot be determined.

So the problem might **NOT** have a solution.

Example: There is no solution for satisfying