Game Theoretic Models of Intangible Learning Data

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ABSTRACT

Learning Analytics is full of situations where features essential to understanding the learning process cannot be measured. The cognitive processes of students, their decisions to cooperate or cheat on an assessment, their interactions with class environments can all be critical contextual features of an educational system that are impossible to measure. This leaves an empty space where essential data is missing from our analysis. This paper proposes the use of Game Theoretic models as a way to explore that empty space and potentially even to generate synthetic data for our models. Cooperating or free-riding on the provisioning of feedback in a class activity is used as a case study. We show how our initially simple model can gradually be built up to help understand potential educator responses as new situations arise, using the emergence of GenAI in the classroom as a case in point.

CCS CONCEPTS

• **Applied computing** → **E-learning**; • **General and reference** → **Estimation**; • **Computing methodologies** → **Modeling and simulation**.

KEYWORDS

game theory, learning analytics, missing data, learning theory

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1 INTRODUCTION

Much of what is important to learning - thinking, social interaction, decisions - is incredibly challenging to measure or even observe. This tension between what learning *is* and what artefacts of learning we can *observe* underpins concerns around whether the analytics we have connects to the learning we care about [9, 12], and questions about the general efficacy of Learning Analytics (LA) in improving learning outcomes [32]. Part of the challenge lies in what aspects of learning produce tangible data artefacts. Naturally the availability of assessment, grades, student demographics, administrative and digital trace data has lead to the development of LA for those data. However some key aspects of the learning experience are less tangible, such as a student's disposition, engagement, or curiosity, and may always be so [18]. However, the difficulty of observing these latent characteristics of our learners does not make them any less important. Indeed, Wise et al. [35] makes a compelling call for us to think carefully and critically about what data might be missing from LA and how this might impact our models. This paper will propose one new method for progressing in our modelling over intangible learning data.

First we must ask what type of data is missing from LA. Existing theories of learning highlight the importance of cognitive processes, interactions between learners and their environment, attention, and the moment-to-moment decision making of learners and educators [15]. Important aspects of learning such as "teacher style, school catchment, parent support, enthusiasm, and so on" are often ignored or controlled for in analysis despite having a potentially greater influence on learning than the variables we are paying attention to [31]. The data that is used is what is available, such as demographics, gender, past academic performance, and digital traces of activity. This has proven enough for successes in prediction and in some cases intervention, particularly for in identifying and supporting students' at risk of failure. However, there is still a lack of knowledge of why some interventions are successful and others are not [17]. In short, learning is a complex process [13] which often involves complex, transient interactions between multiple agents. Importantly, key events often occur in the minds of those agents, which makes them hard, and perhaps even impossible, to observe. From the perspective of learning theory these intangible data are key to the learning experience, which might be an underlying reason why there are concerns of LA being disconnected from critical components of learning [9, 12]. This leaves an empty space, where we know that something important to learning is happening but we do not have clear visibility through data, and may never obtain one [18]. This paper is about how to address the empty space of intangible learning data, and how we might connect this missing data to the data that we do have.

A possible way forward is to examine ways that other fields have modelled multiple interactions between individuals. In particular, where repeated interactions between individuals that we have trouble observing directly, but can still reason about have been studied. *Game Theory* [2, 19] provides one such avenue, as it models various strategies taken by individuals encountering each other in situations of uncertainty.

This paper will explore how Game Theory might be adopted for the purposes of modelling missing data in LA. We will address the following research question:

RQ: How can game theoretic models connect intangible data to the data that we have available in LA?

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We first provide a short primer on Game Theory, a way of modelling interactions between agents. Then we review the work that has been completed to date in using game theoretic models to understand education systems and learning. Next we offer suggestions for how to address intangible data in learning systems through the creation of synthetic data generated using game theoretic models. This is demonstrated by modelling peer-feedback and examining what happens when a tool like Generative AI (GenAI) invades the learning environment. We conclude with a discussion of potential further work in this space.

2 AN INTRODUCTION TO GAME THEORETIC MODELS

Game Theory is a branch of mathematical modelling that analyses the dynamics of a system by creating abstract models called *games* in which *agents* interact with each other by employing different *strategies* [33]. These agents are commonly individuals, such as people in social systems or animals in ecological systems, and each agent can employ a range of strategies in to achieve some desired outcome, such as acquiring resources. The strategies employed by agents describe how they make *decisions* when they interact with each other, with each interaction using the analogy of a game being played between players. Numerous strategies have been studied in different contexts, with common problems such as the Prisoner's Dilemma, Chicken, and even penalty shootouts analysed using this method. By describing these games and the population mathematically inferences can be made using formal logic or simulation. Game Theory models are a deliberate oversimplification of reality, but they can provide valuable insights in situations where data is hard to obtain, and where the cumulative effect of many interactions in a heterogeneous population is important.

A classic example is the *Prisoner's Dilemma* (PD) which imagines a scenario in which the best overall outcome is for the two agents interacting to *cooperate*, but from the individual's perspective the best outcome is to *defect*. In a classroom setting this could manifest where students are asked to work together, such as providing each other with feedback [27], or responding to questions on a forum [1]. For each interaction the student has a choice to *cooperate* by providing meaningful feedback, or answering a forum question, or to *defect* (often referred to as *free-riding* in these settings) by providing limited feedback or not answering a forum question. There are various costs and payoffs depending on which decision (cooperate or defect) is chosen, and an example of these is illustrated in Table 1. The dilemma emerges from the fact that if both students choose their strategy logically then they should both defect (as this will lead to them receiving a better outcome irrespective of the choice of the other student). However, this apparently logical choice leads to the worst outcome overall (a combined payoff of $1 + 1 = 2$). This model may seem trivial and narrow in scope, but it can be generalised into any scenario in which there is a strong payoff for each individual to be selfish, but the overall fitness of the population would be better off if individuals cooperated.

Interestingly, the sub-optimal outcome emerging from this game (termed sub-Pareto optimal in the literature) can be overcome if we allow for repeated games (an *iterative* Prisoners Dilemma, or IPD), agents having memory, or communities of agents [37]. If each of

	Cooperate	*Free-ride*
Cooperate	4	-2
Free-ride	5	1

Table 1: Payoff matrix for a game between Free-riders and Cooperators.

these cases it has been shown that cooperation becomes a more attractive decision option as new strategies become relevant, such as tit-for-tat, where agents adopt the decision that the other agent made in their previous interaction. Understanding which strategies might evolve, under what environments, is known as *Evolutionary Game Theory* (EGT) [10, 19].

EGT moves from trying to understand optimal strategies in a single game, such as the PD, to understanding how a mix of strategies in a population evolves with the iterative version of the game, IPD. This allows for new kinds of questions, such as can a new strategy invade, can a mix of strategies coexists, or should an agent change strategies?

3 GAME THEORETIC MODELS IN EDUCATION

The majority of game theoretic models built in education apply extensions of the Iterative Prisoner's Dilemma (IPD) to understand the dynamics of multiple agents choosing between cooperation or self-interest. Much of this work concerns the dynamics of educational systems, and how various groups or individuals make decisions within them to balance between cooperation and self-interest. For example, a number of papers have analysed interactions between governments and universities under new or future policy decisions [7, 22, 28]. Niklasson [22] explicitly state that they are examining a scenario where data is unavailable, modelling "a new kind of interaction between government and universities". Others have analysed the dynamics that emerge at different scales in a university, between individual faculty staff and the between the faculties themselves [11].

Of more relevance to the field of LA, some models have been used to understand the dynamics of learning environments, specifically the interactions between students and educators under a variety of conditions. Early models used the Prisoner's Dilemma to examine Teacher-Student dynamics around effort [6], with extensions towards grading practices [21], and potential grade inflation [5]. Although these models began as a way to examine the hidden data around effort and grading practices, Montmarquette and Mahseredjian [21] used the results to adjust their models to account for potential grade-inflation by including the resulting data as a latent variable.

Extensions of the IPD have also been implemented to model classroom cooperation, or lack thereof [1, 25, 26], as well as the establishment of trust [30]. Al-Dhanhani et al. [1] use the IPD to model expected behaviour in online forums. They showed that tit-for-tat strategies and their variations become important, with learners making decisions on their degree of cooperation based on their previous interactions. This model led to a series of design suggestions around increasing the visibility of group reputation [1] that were incorporated into an online learning platform with promising results [23].

Other IPD extensions focus on the challenge of student cooperation in providing peer-feedback [27, 36] or knowledge sharing [37]. These models all highlight that cooperative behaviour between students is unlikely to emerge with some incentives. Pandey and Chatterjee [27] explore the importance of learning design and structuring the learning environment such that the peer-feedback system promotes cooperation. In particular they examine the effect of including a threshold of number of reviews a student needs to contribute before they see their own feedback from their peers. Zhang et al. [37] also note that perceived academic value is also critical to promoting cooperation, but extend this model to include social gains as a potential incentive for students to adopt cooperative strategies.

Game theoretic models have been applied to hidden learning processes involving effort, grading, trust, cooperation, social learning and peer-feedback. These models suggest that without incentives rationally acting learners tend to move away from cooperative behaviour, however interventions in the learning design [1] or the social environment [25, 37] can help drive learners to more cooperative behaviour and improve overall outcomes. This aligns with research into cooperative learning as a teaching strategy being both beneficial and challenging to implement [4].

4 CASE STUDY: GENERATIVE AI INVADES THE CLASSROOM

In this section we will illustrate how game theory can be utilized to generate data for features that often remain unaccounted for in LA. Our starting point is an Iterative Prisoner's Dilemma model similar the models of online forum behaviour presented by Al-Dhanhani et al. [1] or peer-feedback systems [27, 37]. (Each of which were discussed above.) Our model will consider the students as a class of agents, which we will refer to as *learners*. Let us assume that the assessment our learners are currently undertaking requires them to provide some level of feedback to one another. Interactions occur between learners as they develop various strategies for providing and receiving feedback from one another. We use P_i to indicate a learner, i, providing feedback, and R_j the learner j receiving feedback. At each interaction the learners must decide upon the amount of effort they will put into collaboration. This choice of strategy is represented using a parameter, Θ which represents how much effort they put into *providing* feedback. At the extremes, a learner can choose to fully cooperate (Strategy $C : \Theta_C = 1$) or to not cooperate, with a 'free-rider' (Strategy $F : \Theta_F = 0$). In this case, let us assume that even if a free-rider provides feedback, they just put so little effort into it that their feedback provides no value. The model allows for choices in between these two extremes, and we designate Strategy: $T : \Theta_T = 0.5$ as the 'token-effort' strategy. A token-effort strategy might look like skimming the other student's work and providing one or two pieces of sound advice — useful, but they could have done more. We assume that the benefit to learners is only dependent on the effort that the provider puts into the feedback, ignoring differences in the students' learning progression. We also define the learning environment using four *environment* parameters, which we will set the same for each learner: cost of providing feedback, c, benefit of receiving feedback, f, benefit to yourself of providing someone else with feedback, b, and a social benefit of working with someone with the same strategy, a. The value attained by the receiver R_j exchanging feedback with a peer P_i is calculated by:

$$V(R_j|P_i) = \Theta_{P_i} f + \Theta_{R_j}(b - c) + (1 - |\Theta_{P_i} - \Theta_{R_j}|)a \qquad (1)$$

Here, $(1 - |\Theta_{P_i} - \Theta_{R_j}|)a$ returns a if R_j and P_i have the same strategy in providing each other feedback ($\Theta_{R_j} = \Theta_{P_i}$), and less the further the paired strategies are from each other. This kind of mechanism is common in evolutionary game theory models where there is a preference for working with like minded individuals, such as in family groups. In this model such an outcome would represent the preference for working with peers putting in a similar amount of effort. Note that the effort of the learner providing the feedback influences the benefit received.

4.1 The peer feedback dilemma

Choosing some specific starting parameters enables us to now compute a payoff matrix between a cooperator C and a free-rider F. A range of different parameter settings for different learning environments are suggested on Table 2.

Scenario	c	f	b	a	Example
Mild provision and social incentives for cooperation	4	5	2	2	[25]
Social incentives help drive cooperation	4	5	1	3	[1, 37]
Learning design incentives drive cooperation	4	5	4	0	[27]
Incentives to cooperate, but low feedback value	2	5	2	2	[38]

Table 2: Some example parameter settings and the type of situation they might represent in an educational scenario.

In our case, if we choose $c = 4, f = 5, b = 2, a = 1$ then we can achieve the Prisoner's Dilemma outlined earlier in Table 1. We can learn a lot from the values in the payoff matrix alone. The highest value on the diagonal, when cooperators work together, shows us the ideal strategy mix for the whole — we would like everyone to be cooperating, and this results in the best payoff for the system as a whole. But by comparing the expected payoff for each strategy, that is, by looking at the average value for each row, we can see from an individuals perspective that it is best to choose the 'free-rider' strategy. This is where the dilemma arises: the best overall environment involves cooperation, but if individuals choose what is best for them the system will move away from this state. To understand how this might play out in a learning environment, with repeated peer-feedback interactions, we move from analysing the payoff of a single game to the evolutionary dynamics of a mix of strategies.

4.2 Evolving collaborative learning environments

To see what might happen with a mix of strategies in a system we calculate the gradients between three different strategies and

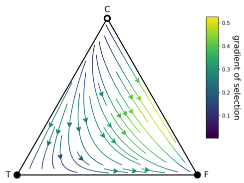


Figure 1: The evolutionary dynamics of cooperation C, token-effort, T, and free-rider (F) strategies in the initial peer-feedback model, Equation 1.

plot them, using the EGTTools library [10]. Each vertex in of a triangle in Figures 1 and 2 represents a pure strategy, and any point within the triangle a mix of strategies taken by learners in that system. The coloured arrows represent a set of computed gradients, each of which shows how the mix of strategies will change from one iteration to the next, given the payoffs outlined by the model. Initially we set $c = 4, f = 5, b = 2, a = 1$ to explore what might happen between cooperator, token-effort, and free-rider strategies in a learning environment. We can see the peer-feedback dilemma play out in Figure 1, where a population of using a mix of cooperator, token-effort, and free-rider strategies will gravitate towards free-riding over time.

Learning designers and teachers know this [4, 24, 25]. To foster collaboration incentives are created so that learners value the providing of feedback for themselves. This could be through setting expectations in the classroom, highlighting the value of providing feedback as a way of learning, or creating a formal academic incentive such as grades for providing feedback. This can then be represented in the model by adjusting the parameter for the individual providing the feedback, b. In Figures 2a, 2b and 2c we illustrate three cases where the value of b in the learning environment is increased. When the cost of providing feedback is equal to the benefit the provider receives for themselves (Figure 2b) the system is evenly balanced: our simulated learners will gravitate towards the initially most common strategy, away from the central, even mix of strategies. In this scenario it is desirable to guide student behaviour towards more cooperation, leveraging the accumulation of positive learning experience. Papers such as Jennings and Greenberg [14] demonstrate this very role for educators in building a pro-social learning environment. When the benefit of providing feedback is greater than the cost of providing the feedback ($b > c$) then the system dynamics changes and learner strategy is drawn towards a choice to cooperate (Figure 2c).

4.3 The invasion of GenAI

We now extend the previous model to allow for the invasion of new strategies, based on the use of Generative AI (GenAI). This is achieved by extending the original model with a new strategy parameter:

- $\Pi \in \{0, 1\}$: The use of GenAI to provide feedback (1) or not (0), for the receiver (Π_R) and the provider (Π_P).

Additionally, there are two new environment parameters:

- q: The quality of the GenAI for feedback.
- k: The reduction in cost of providing feedback using AI.

We set the quality variable $q = 0.8$ to indicate a situation where the GenAI provides slightly worse feedback than what a peer would provide if they were to use full effort. We also use $k = 0.1$ to indicate a low cost for using GenAI when compared to the cost of providing the feedback yourself. We assume that all users are providing the same level of GenAI feedback, and that receiving multiple feedback from a GenAI source does not provide additional value. We are also ignoring the use of GenAI for providing feedback for yourself without peer-interaction. The assumption here is that this is relatively uniform across the population and is of less importance to the behavioural dynamics, however the option to include this is or explore this modelling in more detail is available in the supplementary code repository.

This extended scenario doubles the number of strategies due to the choice to use GenAI or not:

- N: No GenAI use, $\Pi = 0$.
- AI: Using GenAI to provide feedback for others, $\Pi = 1$. This reduces the cost of providing feedback to $c \times k$ (as $0 \leq k < 1$), reduces the benefit of providing feedback to $b \times k$, and also changes the value of the feedback received to $q \times \Theta_P \times f$ instead of $\Theta_P \times f$.

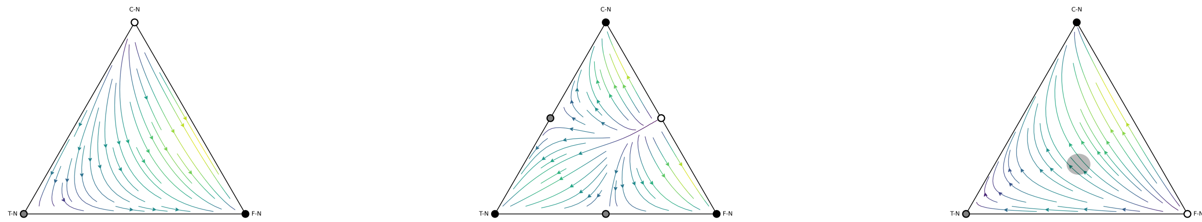
Using Game Theory enables us to combine the cooperation strategies and GenAI strategies in our model. So, for example, *C-N* would represent a learner that is fully cooperating, and not using GenAI either to generate feedback for other learners. *T-AI* would indicate that a token-effort strategy (half effort towards cooperation) that is using the GenAI for providing feedback to others. The new, extended, value calculation is more complicated:

$$\begin{aligned} V(R_i|P_j) =& (1 - \Pi_{P_j} + q\Pi_{P_j})\Theta_{P_j} f \\ &+ (1 - \Pi_{R_i} + k\Pi_{R_i})\Theta_{R_i}(b - c) \\ &+ (1 - |\Theta_{R_i} - \Theta_{P_j}|)a \end{aligned} \quad (2)$$

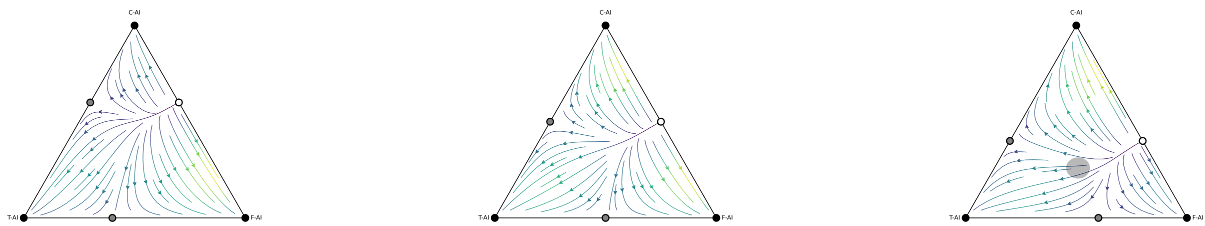
The model outlined in Equation 2 reduces to the non-GenAI scenario when $\Pi_R = \Pi_P = 0$. To see how it changes with the introduction of GenAI observe what happens to the benefit from feedback received as Π_P changes from 0 to 1. When $\Pi_P = 0$ the expression $(1 - \Pi_P + q\Pi_P)\Theta_P f = \Theta_P f$, but changes to $q\Theta_P f$ when $\Pi_P = 1$.

We now explore the same scenario of attempting to promote cooperative behaviour (by changing the value of b) but with a mix of strategies where GenAI is being used to help provide feedback to peers. The results are shown in Figure 2.

As is the case in the no AI scenario, when the value of of b increases in the GenAI scenario this results in a greater pull towards the cooperative strategy — a greater area of the triangle flows towards the top vertex *C-AI*. What is interesting is the difference in how each system changes when the perceived benefit changes from this balanced position, when $b = c$ (Figures 2b and 2e). In the non AI scenario the system changes much more dramatically when we change the perceived benefit of providing feedback than a similar change in the presence of GenAI, both in positive and negative directions. The GenAI system appears less sensitive to both attempts to promote cooperation and circumstances that are unfavourable to cooperation.



(a) $b < c$**, no AI** **(b)** $b = c$**, no AI** **(c)** $b > c$**, no AI**



(d) $b < c$**, GenAI** **(e)** $b = c$**, GenAI** **(f)** $b > c$**, GenAI**

Figure 2: Comparison of a peer-feedback system without AI (top row) and a peer-feedback system with GenAI support (bottom row). The benefit of providing feedback increases as we move from left to right. Each triangle has the cooperative strategy at the top ($C-$**), the token-effort strategy on the bottom-left (**$T-$**) and the free-rider strategy on the bottom-right (**$F-$**). The hollow circles indicate unstable equilibrium points, full (black) circles indicate stable equilibrium points, and grey circles indicate a saddle point.**

To see what this model has to say about the introduction of GenAI to classroom dynamics we should examine the differences in each vertical pair. The systems are also nearly identical when the perceived benefit of providing feedback, b, is equal to the cost of providing feedback (Figures 2b and 2e). In situations where cooperation is not flourishing ($b < c$, Figure 2a — imagine a large online course without targeted interventions), the introduction of GenAI has the potential to promote cooperative behaviour for populations where the initial mix of strategies favours some cooperation (Figure 2d). Conversely, in situations where cooperation is already established ($b > c$, Figure 2c — imagine a face-to-face class with a high value on providing feedback as a learning experience) the introduction of GenAI has the potential to result in sub-optimal learning environments. To see this, imagine a population with an almost evenly mixed set of the three strategies, but with slightly fewer cooperators (inside the grey circle). In the no AI scenario (Figure 2c) the educator would expect, in this established learning environment, that the strategies move towards cooperation with the number of free-riders to reduces quickly. In the GenAI scenario (Figure 2f) we would also see the free-riders also reduce initially, however as the strategy mix approaches the *T-AI* to *C-AI* line it may either turn towards cooperation or token-effort. The attraction of GenAI as a tool to supplement peer-feedback, according to this model, is making a token-effort approach more attractive in the same learning environment.

4.4 Connecting to tangible data

Up to this point the model has provided qualitative proposals of how the system might behave, through stories told from examining the change in dynamics under different parameter settings. Without data on levels of cooperation, say, validation of the model and trust in what it says relies on a qualitative comparison with those who understand the peer-feedback learning environment. To proceed in using this approach to connect to existing data we need to be precise about the strengths and weaknesses of game theoretic models.

To be clear, this kind of model tells us little about individual learners or precise measurements at a given point in time. Although the models are built from describing behaviour between individual agents, the knowledge provided is around what kind of effects we might see emerging at a larger scale from these numerous interactions. What we can do is generate data through simulation and build a picture of possible ways the system might change and evolve. This simulated data can then inform other modelling, where little is known about this aspect of the system. A natural mechanism for this to work through is by informing prior distributions of parameters — encoding what we know about the system before training the statistical model on new data [20].

To see how this might work in our example we generate a small data set from the model described in Equation 2. We can simulate the interaction of a set number of learners with randomly sampled initial strategies, for a given set number of peer-interactions.

f	b	q	$\Delta\hat{C}$	$\Delta\hat{\Pi}$	$\Delta\hat{V}$
5.04	4.01	0.71	0.09	0.15	0.59
5.34	2.41	0.93	0.01	0.02	-0.36
5.21	5.24	1.01	-0.06	-0.02	-0.31

Table 3: Change in proportion of cooperative strategies ($\Delta\hat{C}$), average use of AI ($\Delta\hat{\Pi}$), and average value of the peer-feedback interactions ($\Delta\hat{V}$) for different environment parameters (only f, b and q shown here). Simulation was with 100 learners over five peer-feedback interactions. Learners could switch strategy after each interaction.

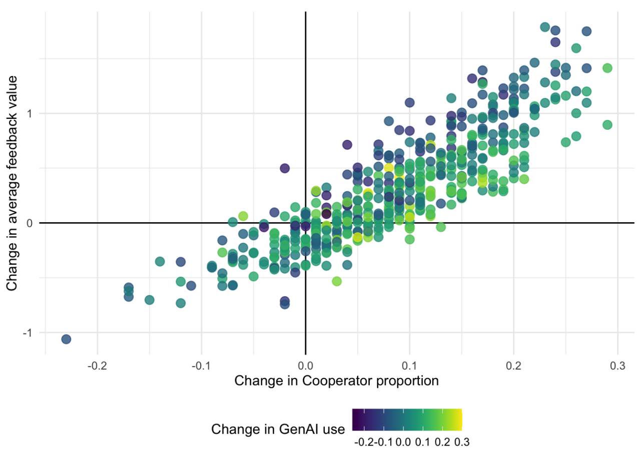


Figure 3: Simulated data of 500 different environment parameter scenarios, looking at the joint distribution of the changes in cooperation, GenAI use, and average value of feedback.

Learners can change their strategy after each interaction based on population information (they see classmates with different strategies do better) or local information (the person they shared feedback with did better). From the resulting simulation we calculate quantities that are more readily observable and less scale dependent, such as: overall change in cooperative strategies (proportion of strategies with $\Theta > 0$: $\hat{C}$), use of GenAI (average of Π: $\hat{\Pi}$), and the average value in feedback interaction (average of V: $\hat{V}$). This can be done for a range of parameter values in any combination, an example with the first five of 500 simulations shown in Table 3.

This builds a simulated joint distribution of changes in cooperative behaviour, GenAI use, and perceived feedback value. An example of this simulated data is shown in Figure 3, showing that we might expect a positive relationship between change in cooperative behaviour and change in the value of feedback, but a more complicated one in relation to the change in GenAI use.

This model simulated data can be used as a weakly informative *implicit* prior in a Bayesian workflow, where there is insufficient data or transferable expert knowledge to achieve stability of the model. These priors can be iteratively strengthened (for instance by gradually restricting the sampled distribution of parameters) until the Bayesian model has informative enough priors to become stable [8]. There is also the potential to include a human in this loop, iteratively using the generated data to narrow in on a suitable prior with support from an expert [3].

Outside of the potential of using simulated data as an implicit prior there has also been work on using simulated data from a game theoretic models to augment training data in machine learning models. Shen et al. [29] augmented existing training data using a 2 player pursuit-evasion game to improve the performance of a 3D convolution neural network classifying satellite behaviour. The additional simulated data helped improve the model performance where there was a lack of well labelled training data available.

5 DISCUSSION

Much of the data we can obtain in LA is a shallow proxy representing unmeasurable and latent features which are critical to the learning process. In order to improve our models we will need methods that enable us to account for this missing data. Modeling and simulation provide us with an avenue for generating synthetic data, and this paper proposes that models from game theory can be used in this process. This possibility would enable us to create simulations of possible student responses to varying educational strategies. It would also potentially support the creation of statistical tests of the likely occurrence of unknown behaviour in a classroom (e.g. what percentage of students are using GenAI to free ride on an assignment).

One limitation of the model results presented here is the gradient calculations based on only three strategies at a time, not on the entire mix of possible strategies once GenAI was introduced. Future work will explore in more depth agent-based simulation of the games to encapsulated interactions between the full suite of strategies. The simulation approach would also enable varying of the environment parameters, so it would be possible to explore scenarios where the learning environment is evolving, such as where the quality of GenAI is improving over time. A cautious reader might challenge the use of the models themselves: how do we know that they are a good representation of our intangible data? A model is always a necessary simplification, and so there will inevitably be aspects of learning that it does not capture. Although game theoretic models are clearly toy-like representations of the real world, this paper has demonstrated that they offer a window into intangible data that can help us to hypothesise and reason about important aspects of learning where we otherwise might be blind. The resulting models can then be tested against the reality that we can observe with the data we do have access to.

As the learning environment changes there is the potential to use this method as a relatively low cost way to to imagine future learning environments. The effect of new technologies such as GenAI are difficult to anticipate from existing data. They are clearly ‘out of sample’ but the last 5 years have demonstrated the importance of being able to anticipate “unknown unknowns” in education. Future work will also seek to investigate the tuning of these models against emerging data, where available. We will also explore other games and strategies that might be adopted for various educational settings.

LA has often advocated that we must measure what is important to learning, rather than simply relying on what data is available [16, 34, 35]. This paper has demonstrated a way in which we can use modelling and simulation to account for some intangible learning data. As such it is a promising way forward that could help us to open up new avenues for understanding learners and the environments in which they learn.

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