```
import numpy as np
import pandas as pd
import scipy as sp
import matplotlib.pyplot as plt
```

Problem 2 Code

2 Linear Regression with Regularization [10pts]

In class we derived and discussed linear regression in detail. Find the result of minimize the loss of sum of the squared errors; however, add in a penalty for an L_2 penalty on the weights. More formally,

$$\arg\min_{\mathbf{w}} \left\{ \sum_i (\mathbf{w}^\mathsf{T}\mathbf{x}_i - y_i)^2 + \lambda \|\mathbf{w}\|_2^2 \right\}$$

How does this change the solution to the original linear regression solution? What is the impact of adding in this penalty?

Write your own implementation of logistic regression and implement your model on either real-world (see Github data sets: https://github.com/gditzler/UA-ECE-523-Sp2018/tree/master/data), or synthetic data. If you simply use Scikit-learn's implementation of the logistic regression classifier, then you'll receive zero points. A full 10/10 will be awarded to those that implement logistic regression using the optimization of cross-entropy using stochastic gradient descent.

Problem 2 functions

```
def split_dataset(dataset,train_percent):
    """
    Input: Dataset : dataframe  train_percent: percentage (0.0 - 1.0)
    Output: 4x dataframes: x-testing,x-training,y-testing,y-training
    """
    #dataset = shuffle_dataset(dataset)
    y = dataset.iloc[:,-1]
    x = dataset.iloc[:, 0:-1]
    test_size = int(train_percent * y.size)
    #create testing
    yts = y.iloc[:test_size,].reset_index(drop=True)
    xts = x.iloc[:test_size,].reset_index(drop=True)
    #create training
    ytr = y.iloc[test_size:,].reset_index(drop=True)
    xtr = x.iloc[test_size:,].reset_index(drop=True)
    return xts,xtr,yts,ytr

def shuffle_dataset(xt,yt):
    """
    Shuffles the data collectively
    Input:    xt : feature data    yt : class data
    Output:   Separate feature and class vector in appropriate order
    """
    xy = pd.concat([xt, yt], axis=1)
```

```
    xy2 = xy.sample(frac=1).reset_index(drop=True)
    y = xy2.iloc[:,-1]
    x = xy2.iloc[:, 0:-1]
    return x,y

def logistic_function(x,w):
    """
    Logistic Function 1/(1+exp(-wTx))
    """
    wTx = np.dot(x, w)
    temp = 1 / (1 + np.exp(-wTx))
    return temp

def cross_entropy(x, y, w):
    """
    Cross Entropy: log loss measure the converges
    Only works with binary class at the moment
    """
    temp = - np.sum (y * np.log(logistic_function(x,w)) + (1-y)*
np.log(1- logistic_function(x,w)))
    return temp

def gradient(x, y, w, eta):
    """
    Gradient f(x)f()
    """
    temp = np.dot(x.T, logistic_function(x,w) - y)
    return eta * temp


def sgd(xtr,ytr, T,eta):
    """
    SGD: Logistic Regression using Stochastic Gradiant Descent
    Input: xtr:training feature data, ytr:training class data
            T: iterations, eta: learning rate
    Output:  wnew: parameter vector, k: occurences cross enthropy
worked
    """
    wnew = np.zeros((xtr.shape[1],1))     #initialize parameter vector
w(4x1)
    n = len(ytr.iloc[:,]) # size of the training dataset
    ce = 1 #initialize cross enthropy val
    k = 0

    for i in range(T):
        xtr,ytr = shuffle_dataset(xtr,ytr) #shuffle data each
iteration
        for j in range(n):
            wold = wnew
            x_i = np.array([xtr.iloc[j]]) #set xtrain row to nparray
```

```
            y_i = np.array(ytr.iloc[j]) # set ytrain nparray
            wnew = wold - gradient(x_i,y_i,wold,eta) #get updated
parameters
            temp_ce = cross_entropy(x_i, y_i, wnew) #perform cross
etrophy
            ce_dif = abs(temp_ce - ce) #compare previous ce with new
ce
            if (ce_dif < 0.00001):  #break if minimum is found less
then 0.00001
                ce = 1
                k=k+1           #count occurences of ce
                break
            else:
                ce = temp_ce    #progress ce
    return wnew,k
```

Problem 2 Test Program

```
path = "/content/gdrive/MyDrive/Git/ECE523/HW2/acute_inflamation.csv"
dataset = pd.read_csv(path, header=None)

#split dataset into testing and traing
xtest,xtrain,ytest,ytrain = split_dataset(dataset,.6)

#train w parameters
wtr,k = sgd(xtrain,ytrain,1000,0.02)

#test w parameters against xtest feature data
yhat = logistic_function(xtest,wtr)
for i in range(len(yhat)):
    if yhat[i] >=0.5:
        yhat[i] = 1
    else:
        yhat[i] = 0


print("w parameters: \n", wtr)
print("k occurences using Cross Entropy: ",k)
dif_y = yhat[:,0] - ytest[:]
perc_correct = (1.0 - (float(np.count_nonzero(dif_y))/len(dif_y)))*100

print("Precent Accuracy with test data: ", perc_correct)

print("\nModify ytest[2] value (1 out of %s) to confirm SGD
working..." % ytest.size)
if ytest[2]==1:
    ytest[2]=0
else:
    ytest[2]=1
```

```
dif_y = yhat[:,0] - ytest[:]
perc_correct = (1.0 - (float(np.count_nonzero(dif_y))/len(dif_y)))*100

print("Precent Accuracy with altered test data: ", perc_correct)
print(ytest.size)
```

```
w parameters:
 [[-2.78368288]
 [ 1.99715886]
 [-2.0301456 ]
 [ 5.46981692]
 [ 3.22064441]
 [-0.13217954]]
k occurences using Cross Entropy:  610
Precent Accuracy with test data:  100.0

Modify ytest[2] value (1 out of 72) to confirm SGD working...
Precent Accuracy with altered test data:  98.61111111111111
72
```

Problem 3 Code

3 Density Estimation [20pts]

The ECE523 Lecture notes has a function for generating a checkerboard data set. Generate checkerboard data from two classes and use any density estimate technique we discussed to classify new data using

$$\widehat{p}_{Y|X}(y|x) = \frac{\widehat{p}_{X|Y}(x|y)\widehat{p}_Y(y)}{\widehat{p}_X(x)}$$

where $\widehat{p}_{Y|X}(y|x)$ is your estimate of the posterior given you estimates of $\widehat{p}_{X|Y}(x|y)$ using a density estimator and $\widehat{p}_Y(y)$ using a maximum likelihood estimator. You should plot $\widehat{p}_{X|Y}(x|y)$ using a pseudo color plot (see https://goo.gl/2SDJPL). Note that you must model $\widehat{p}_X(x)$, $\widehat{p}_Y(y)$, and $\widehat{p}_{X|Y}(x|y)$. Note that $\widehat{p}_X(x)$ can be calculated using the Law of Total Probability.

Problem 3 Functions

```
def gen_checkerboard(N,a,alpha):
    """
    N: number of point on board
    a: width of the board
    alpha: rotation of the checkerboard (radians)
    """
    d = np.random.rand(N,2).T
    d_transformed = np.array([d[0]*np.cos(alpha)-d[1]*np.sin(alpha),
                              d[0]*np.sin(alpha)+d[1]*np.cos(alpha)]).T
    s = np.ceil(d_transformed[:,0]/a)+np.floor(d_transformed[:,1]/a)
```

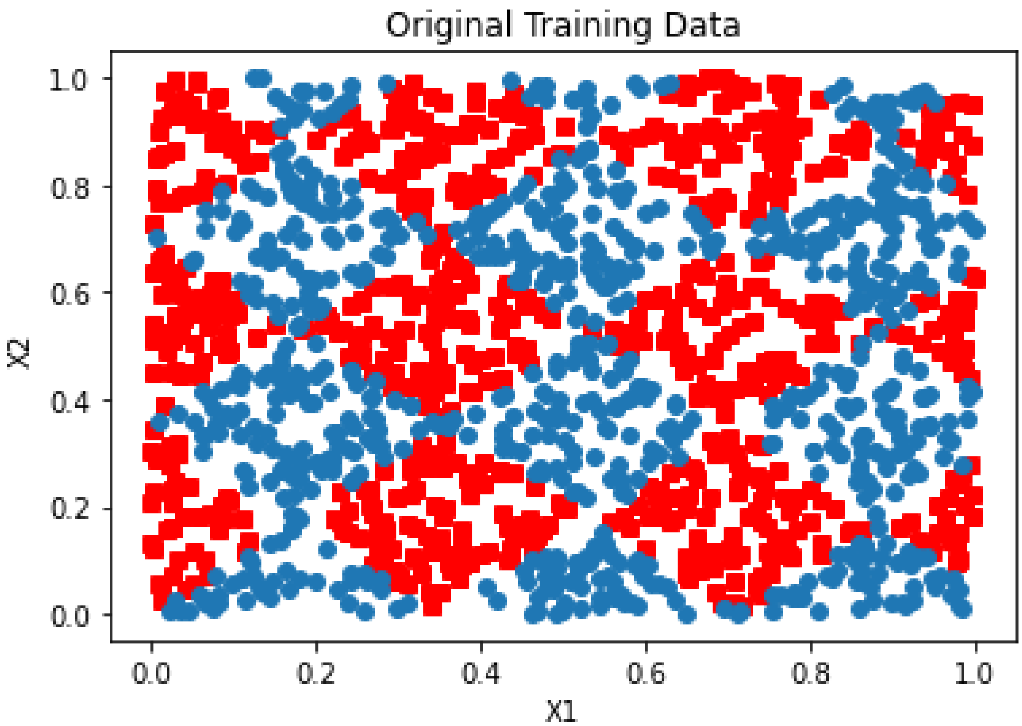
```
    lab = 2 - (s%2)
    data = d.T
    return data, lab

def algorithm_k_NN(k,point,dataset):
    """
    k = k number of point to compare to
    xt_point = test data point
    x = x1,x2 feature data
    y = classifiers
    """
    #Add distance from test point to training points in datasets to 
datasets
    dataset['distance'] = np.sqrt((dataset['x1'] - point[0]) ** 2 + 
                                     (dataset['x2'] - point[1]) ** 
2)

    #Sort Distances in Dataset in ascending order
    dataset.sort_values(by=['distance'],inplace=True)

    #create a set of closest K distances
    k_set = dataset.iloc[:k]
    k_list = k_set['y1'].to_numpy()

    #Take largest distance for kmax and set to radius
    radius = k_set['distance'].max()

    #len of training data
    y1_len = len(dataset.y1)

    #Total Count of training where y == 1,2
    n1 = (dataset.y1.values == 1).sum()
    n2 = (dataset.y1.values == 2).sum()

    #Count when K == 1,2
    k1,k2 = 0 ,0
    if (1.0 in k_list) == True:
        k1 = k_set['y1'].value_counts()[1]
    if (2.0 in k_list) == True:
        k2 = k_set['y1'].value_counts()[2]

    #calculate volume
    volume = np.pi * (radius**2)
    #p(x|y=1)
    pxy1 = k1/(n1*volume)
    #p(x|y=2)
    pxy2 = k2/(n2*volume)
    #p(y=1)  NOTE: n1 equals total 1's in y1
    py1  = n1/y1_len
    #p(y=2)  NOTE: n2 equals total 2's in y1
```

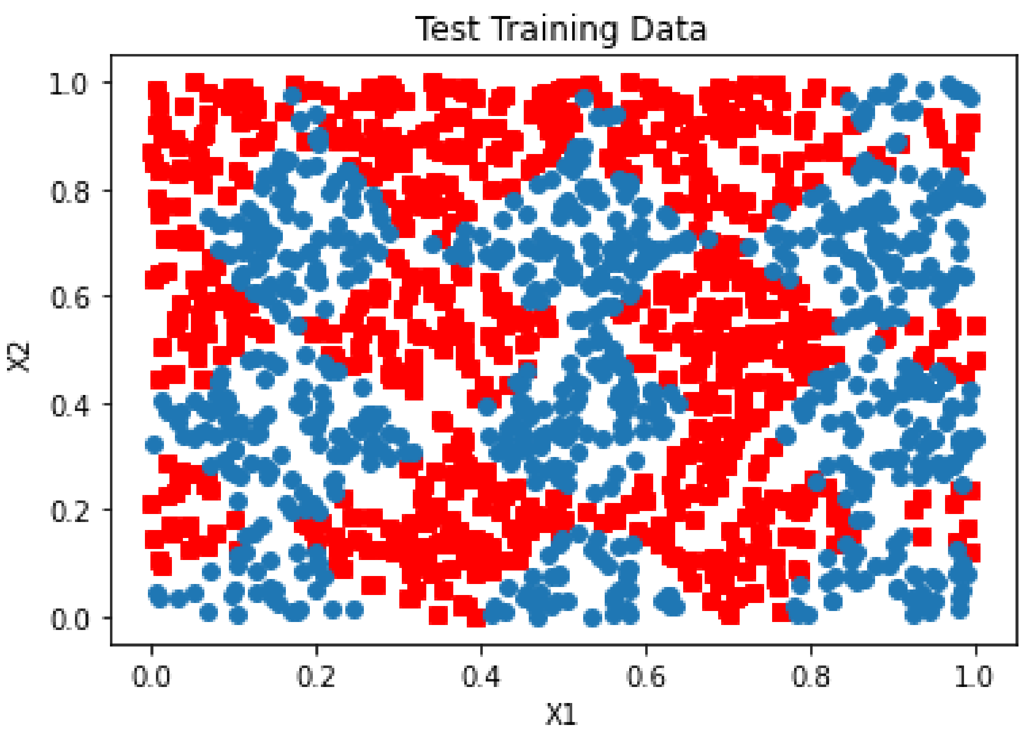
```
    py2  = n2/y1_len

    #p(x) both equate
    pxtot = ((pxy1*py1)+(pxy2*py2))
    px = k/((n1+n2)*volume)
    #p(y=1|x)
    py1x = (pxy1*py1)/px
    #p(y=2|x)
    py2x = (pxy2*py2)/px
    #compare and output
    if py1x > py2x:
        return 1, py1x, py2x
    else:
        return 2, py1x, py2x
```

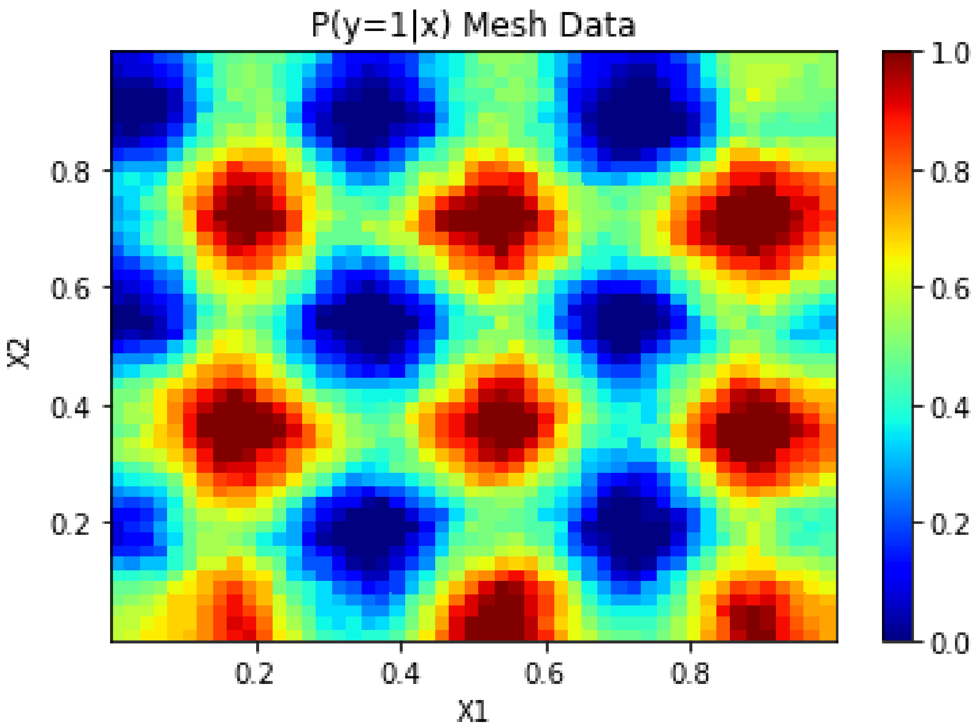
Problem 3 Test Program

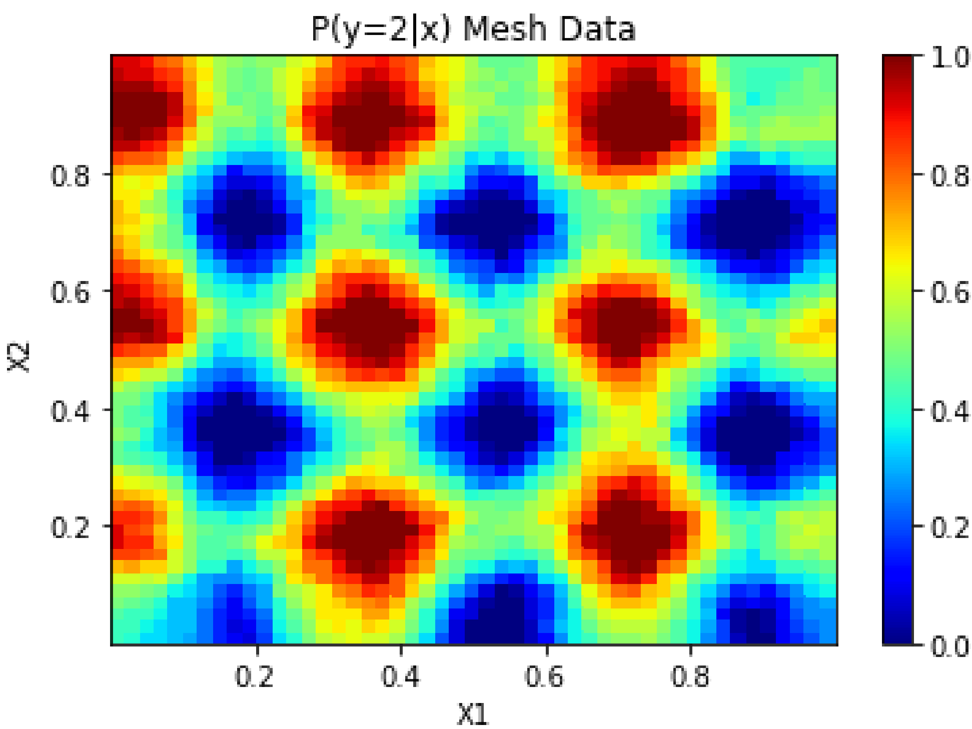
```
N=1500
x,y = gen_checkerboard(N,.25,3.14159/4)
xtest,yt = gen_checkerboard(N,.25,3.14159/4)
k = int(np.sqrt(N))

#organize training dataset
dataset = pd.DataFrame(x,columns=['x1','x2'])
dataset['y1'] = y

#Create y test output set
ytest = np.zeros(len(yt))
#Go through each test point and run Knn
i=0
for xp in xtest:
    ytest[i],pnan,qnan = algorithm_k_NN(k,xp,dataset)
    i+=1

# Using the bounds of the test feature vector,
# generate 50x50 set for each possible point
xmin, xmax = np.min(xtest[:,0]),np.max(xtest[:,0])
ymin, ymax = np.min(xtest[:,1]),np.max(xtest[:,1])
xm, ym = np.mgrid[xmin:xmax:50j, ymin:ymax:50j]

#Generate empty posterior sets same shape as xm,ym (50x50)
pxy1 = np.zeros(xm.shape)
pxy2 = np.zeros(xm.shape)

#Iterate through each point on grid, and collect the posterior for 1
or 2
for i in range(len(xm[0])):
    for j in range(len(xm[1])):
        ynan, pxy1[i,j],pxy2[i,j] = algorithm_k_NN(k,
[xm[i,j],ym[i,j]],dataset)
```

```python
#Plot
plt.figure()
plt.plot(x[np.where(y==2)[0],0],x[np.where(y==2)[0],1],'s',c = 'r')
plt.plot(x[np.where(y==1)[0],0],x[np.where(y==1)[0],1],'o')
plt.xlabel("X1")
plt.ylabel("X2")
plt.title('Original Training Data')
plt.show()
plt.plot(xtest[np.where(ytest==2)[0],0],xtest[np.where(ytest==2)
[0],1],'s',c = 'r')
plt.plot(xtest[np.where(ytest==1)[0],0],xtest[np.where(ytest==1)
[0],1],'o')
plt.title('Test Training Data')
plt.xlabel("X1")
plt.ylabel("X2")
plt.show()

#Plot posterior probabilities for each point in grids
plt.figure()
plt.pcolormesh(xm,ym,pxy1,cmap='jet')
plt.colorbar()
plt.title('P(y=1|x) Mesh Data')
plt.xlabel("X1")
plt.ylabel("X2")
plt.show()

plt.figure()
plt.pcolormesh(xm,ym,pxy2,cmap='jet')
plt.colorbar()
plt.title('P(y=2|x) Mesh Data')
plt.xlabel("X1")
plt.ylabel("X2")
plt.show()
```


Original Training Data
X2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
X1


Test Training Data
X2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
X1


P(y=1|x) Mesh Data
X2
X1
0.2
0.4
0.6
0.8
1.0
0.0


P(y=2|x) Mesh Data
X2
X1
0.2
0.4
0.6
0.8
1.0
0.0