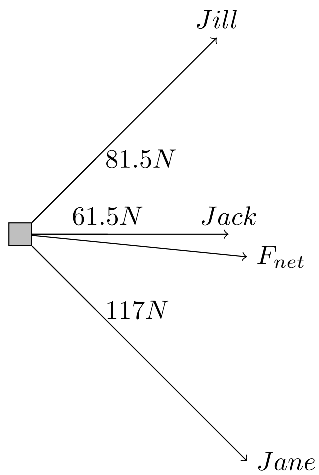
Problem 1:

Three people pull simultaneously on a stubborn donkey. Jack pulls eastward with a force of 61.5 N, Jill pulls with 81.5 N in the northeast direction, and Jane pulls to the southeast with 117 N. (Since the donkey is involved with such uncoordinated people, who can blame it for being stubborn?) Find the magnitude of the net force the people exert on the donkey.

We are prompted for a two dimensional free body diagram and thus omit the force of gravity and the normal force.
Free body diagram:



Force is equal to

$$(61.5 + cos(45°)81.5 + cos(45°)117)\hat{\imath} + (61.5 + sin(45°)81.5 + sin(45°)117)\hat{\jmath}$$

or

$$201.8607\hat{\imath} - 25.1023\hat{\jmath}$$

The magnitude of which is

$$\sqrt{201.8607^2 + -25.1023^2} = 203.4155$$

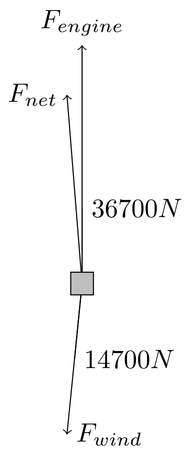
By taking the inverse trig functions of the respective normalized $\hat{\imath}$ or $\hat{\jmath}$ components we find the angle of direction of F_{net} as

$$\theta = sin^{-1}(\frac{-25.1023}{203.4155}) = -7.0886°$$

Problem 2:

A 17500 kg jet airplane is flying through some high winds. At some point in time, the airplane is pointing due north, while the wind is blowing from the north and east. If the force on the plane from the jet engines is 36700 N due north, and the force from the wind is 14700 N in a direction 80.0° south of west, what will be the magnitude and direction of the plane's acceleration at that moment? Enter the direction of the acceleration as an angle measured from due west (positive for clockwise, negative for counterclockwise).

Free body diagram in two dimensions:



The magnitude and direction of the plane can by found by considering west as 0 degrees and north as 90 degrees

$$cos(-80°)14700\hat{i} + (sin(-80°)14700 + 36700)\hat{j}$$

or

$$2552.6282\hat{i} + 22223.3260\hat{j}$$

The magnitude of which is

$$\sqrt{-2552.6282^2 + 22223.3260^2} = 22369.45N$$

and the direction in terms of angle measured from due west is

$$\theta = 90° - cos^{-1}(\frac{22223.3260}{22369.45}) = 83.4475°$$

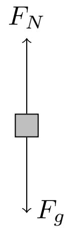
Thus the magnitude of the acceleration is found by the force divided by the mass, in terms of meters per second squared:

$$\frac{22369.45}{17500} = 1.2783m/s^2$$

Problem 3

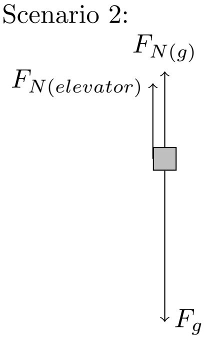
A simple elevator ride can teach you quite a bit about the normal force. There are three different scenarios given detailing the rider's experience in an unnamed hotel. For each scenario, calculate the normal force $F_{N,1-3}$ acting on the rider if his mass is $m = 73.6$ kg and the acceleration due to gravity $g = 9.81$ m/s^2.

Scenario 1:



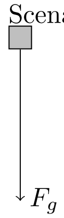
In this first example the elevator is exerting no force so we draw the free body diagram to only include the normal force and gravity. Because of this we know the normal force is exactly that of gravity times the weight of the rider.

Scenario 2:



In the second example the elevator is accelerating at 4.17 m/s^2. He has a net direction exactly opposite gravity, so we know the normal force is greater than gravity. We add together the reaction force of gravity and find the force associated with the additional acceleration. Thus the normal force is exerting $(9.81 + 4.17)73.6 = 415.104$N upon the rider.

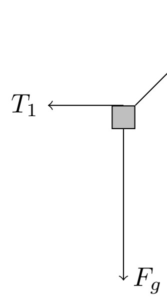
Scenario 3:



In the third example the elevator is falling, and thus so is the rider. Because he's not standing on anything there is no normal force acting on him. $F_N = 0$.

Problem 4:

A 27.2 kg mass is suspended by the cable assembly as shown in the figure. The cables have no mass of their own. The cable to the left (T_1) of the mass makes an angle of 0.00° with the horizontal, and the cable to the right (T_2) makes an angle θ_2 of 40.6°. If the mass is at rest, what is the tension in each of the cables, T_1 and T_2? The acceleration due to gravity is g = $9.81 m/s^2$.



Force of gravity is $9.81 \times 27.2 = 266.832$N. Because the cable T_1 is completely horizontal it can only exert force in that direction, and doesn't resist gravity. Thus the full force of gravity is reacted against by only T_2 to give us zero acceleration. Thus we have that

$$sin(\theta_2) \times T_2 = 266.832 \rightarrow T_2 = \frac{266.832}{sin(\theta_2)} = 410.0223N$$

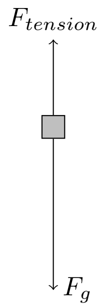
To determine the tension force from T_1 we need to figure out the force being exerted horizontally by T_1, which is

$$cos(\theta_2) \times 410.0223 = 311.3182N$$

So T_1 must be the opposite force. Because it's not clear which direction is positive this can serve as the force of T_1.

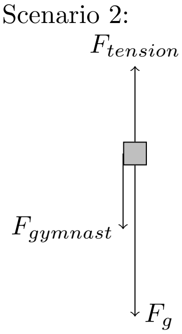
Problem 5: Suppose a 68.5kg gymnast climbs a rope. What is the tension in the rope if she climbs at a constant speed?

Scenario 1:



Acceleration is zero, so the tension is $F = 9.81 \times 68.5 = 671.985$.

Scenario 2:



Acceleration is 1.75, so the tension is increased by the acceleration times her mass, and $F = 9.81 \times 68.5 = 671.985 + 1.75 \times 68.5 = 791.86N$.

Problem 6:

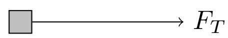
An overnight rainstorm has caused a major roadblock. Three massive rocks of mass $m_1 = 556kg, m_2 = 884kg$, and $m_3 = 355kg$ have blocked a busy road. The rocks are lined up from left to right in the order m_1, m_2, m_3. The city calls a local contractor to use a bulldozer to clear the road. The bulldozer applies a constant force to m_1 in order to slide the rocks off the road. Assuming the road is a flat, frictionless surface and the rocks are all in contact, what force, must be applied to m_1 to slowly accelerate the group of rocks from the road at 0.250 m/s^2?

$$F_{m_{21}} \longleftarrow \blacksquare \longrightarrow F_{bulldozer}$$

The bulldozer needs to exert .25 $\times$ 556N to move boulder 1, but as seen in the diagram there will be an opposing force on each boulder from the one in front. So the sum of the masses in effect will be the total mass the bulldozer needs to move. So we compute $0.25 \times 556 + 884 + 355$, which is $448.75N$. In turn the force of boulder one on two is the total force minus the force of boulder one, ie $884 + 355 \times .25$N, or 309.75.

$$F_{m_{32}} \longleftarrow \blacksquare \longrightarrow F_{m_{12}}$$

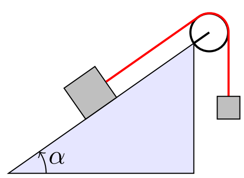
Three crates with various contents are pulled by a force $F_{pull} = 3677N$ across a horizontal, frictionless roller-conveyor system. The group of boxes accelerates at $1.430m/s^2$ to the right. Between each adjacent pair of boxes is a force meter that measures the magnitude of the tension in the connecting rope. Between the box of mass m_1 and the box of mass m_2, the force meter reads $F_{12} = 1397N$ Between the box of mass m_2 and the box of mass m_3, the force meter reads $F_{23} = 2351N$. Assume that the ropes and force meters are massless.

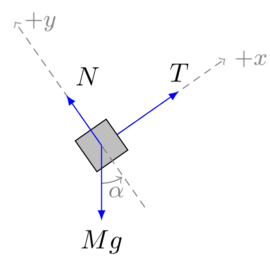


Clearly, there is force acting in one direction from the initial force, and then from each box to each rope behind it, and in turn each rope to the box behind it. Because the total force is 3677N and gives an acceleration of $1.430m/s^2$ we know the mass by $F = ma \rightarrow m = \frac{F}{a} = \frac{3677}{1.43} = 2571.3287kg$. To find the mass of each box we look first at the force meter F_{12} which shows that a force of 1397N gives us the acceleration, so $m_1 = \frac{1397N}{1.43} = 976.9231kg$. Then we take the reading of F_{23} in the same way to find $m_1 + m_2 = \frac{2351N}{1.43} = 1644.0559kg$ This is, however, the force to accelerate both box 1 and box 2, so we subtract m_1 to find $m_2 = 667.1328$. Finally we subtract the two masses from the total mass to find $m_3 = M - m_1 - m_2 = 927.2728$

Problem 8:
As a hiker in Glacier National Park, you are looking for a way to keep the bears from getting at your supply of food. You find a campground that is near an outcropping of ice from one of the glaciers. Part of the ice outcropping forms a $51.5°$ slope up to a vertical cliff. You decide that this is an ideal place to hang your food supply as the cliff is too tall for a bear to reach it. You put all of your food into a burlap sack, tie an unstretchable rope to the sack, and tie another bag full of rocks to the other end of the rope to act as an anchor. You currently have 19.5 kg of food left for the rest of your trip so you put 19.5 kg of rocks in the anchor bag to balance it out.

What happens when you lower the food bag over the edge and let go of the anchor bag? The weight of the bags and the rope are negligible. The ice is smooth enough to be considered frictionless.





T food bag is pulling down at $F = 19.5 \times 9.81 = 191.295N$ and the force on the anchor bag will be the same. The anchor bag is also being acted on by gravity, but also the normal force perpendicular to the slope. The normal force perpendicular to the slope is $cos(51.5) \times 191.295 = 119.084N$ while the force pulling the object down the slope is $sin(51.5) \times 191.295 = 149.709N$ downwards.

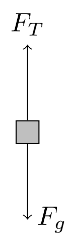
The anchor bag is being pulled by the full 191.295 N of the food bag but only exerting $149.709N$ in the opposite direction. Therefore the net force on the bags is 41.5860N, towards the food bag. Thus the acceleration is the force divided by the weight of the two bags together: $\frac{41.5859}{39.5} = 1.066$

Problem 9:

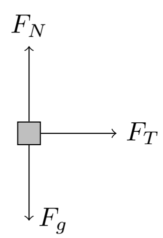
Using a simple pulley and rope system, a crewman on an Arctic expedition is trying to lower a crate of mass m_1=5.26 kg to the bottom of a steep ravine of height H=27.5 m. The crewman of mass m_2=45.0 kg is walking while holding the rope, being careful to lower the crate at a constant speed of v_0=1.50 m/s. Unfortunately, when the crate reaches a point h = 15.3 m above the ground, the crewman steps on a slick patch of ice and slips. The crate immediately accelerates toward the ground, dragging the hapless crewman across the ice and toward the edge of the cliff.

Assuming the ice is perfectly slick, so that there is no friction between the crewman and the ice once he slips and falls down, at what speed v_1 will the crate hit the ground? Assume also that the rope is long enough to allow the crate to hit the ground before the crewman slides over the side of the cliff, and that the pulley is frictionless.

A free body diagram for the crate shows gravity and rope pulled tense by the weight of the man acting upon it:



The man's diagram shows gravity, the normal force, and the taught rope pulling him, although once he leaves the cliff edge the normal force won't act on him until he hits the ground:



The force from the crate is the acceleration from gravity times its weight.

This is the tension on the rope, pulling the man along, as well. Meanwhile the man is resisting this force by adding his own weight to the weight total of the system of objects. So while the mass of the crate times gravity gives 51.6 N of force, the acceleration is this force divided by the total weight of the man and the crate, which is $\frac{51.6}{5.26+45} = 1.0267$. So, the acceleration of the box is slower than it would otherwise be. The crate hits the ground at a speed given by

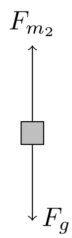
$v = \sqrt{v_0 + 2ay} = 5.8551$. The crewman's speed downward will be given by the typical projectile motion equation, with initial velocity zero and acceleration exactly that of gravity. He falls 27.5 meters so his velocity when he hits the ground is $v = \sqrt{v_0 + 2ay} = \sqrt{(}2 \times 9.81 \times 27.5) = 23.228 m/s$. In addition he is moving horizontally at the speed the rope was pulling him the moment the crate hit the ground, ie 5.8551 m/s. Using the pythagorean theorem we combine these velocities and get 23.9548 m/s

Problem 10

An Atwood machine consists of two masses hanging from the ends of a rope that passes over a pulley. The masses have the values $m_1 = 27.0$ kg and $m_2 = 13.0$ kg. Assume that the rope and pulley are massless and that there is no friction in the pulley.

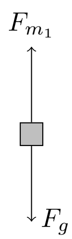
What is the magnitude of the masses' acceleration a?

A free body diagram for mass one shows gravity and the other mass acting



upon it

The other mass's free body diagram is the opposite:



We easily compute these forces as $F_{21} = 9.81 \times 27 = 264.87N$ and $F_{12} = 9.81 \times 13 = 127.53N$. We can easily now see the acceleration will be in the direction of the greater force, that of F_{21}, and $F_{net} = 264.87 - 127.53 = 137.34N$. Then we find acceleration based on $F = ma \rightarrow a = \frac{F}{m} = \frac{137.34}{40} = 3.4335$.

The tension is then found by subtracting that acceleration from gravity and multiplying by the mass of the heavier object: $(9.81 - 3.4335) \times 27 = 172.1655$, which should be the same as if we had added the acceleration to gravity and multiplied by the lighter object.