Math 401 - Groupwork #4
Least-Squares and Curve Fitting

Instructions: Work through the problems below in order. Write down all of your answers on paper, to be turned in at the end of class. I do not want you to turn in any MATLAB code. Make sure that the name of every student in the group is on the paper. Some problems ask you to save images and email them to me (`ayashins@math.umd.edu`) at the end. Do as many of the problems as you can, and do not stress about finishing all of them. You will receive a grade based on the correctness of what you've turned in, **provided that everyone is actively working on it during class.**

1. **(Do all computations for this problem by hand.)** Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \\ 3 & 1 \\ -1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 2 \end{bmatrix}$.

   (a) Find the least-squares solution of $A\mathbf{x} = \mathbf{b}$.

   (b) Something happened which made the solving of the normal equations very easy. Describe the nice property that the columns of A have which made everything nice. (Hint: each entry in A^TA can be interpreted as a dot product of two vectors.)

2. **(Use MATLAB from here onward.)** Consider the linear system

$$\begin{cases} x_1 + 2x_2 - x_3 + 3x_4 = 1 \\ 2x_1 + x_2 + x_3 + 2x_4 = 5 \\ -2x_1 + 3x_2 + 4x_3 + x_4 = -2 \\ 4x_1 + 2x_2 + x_3 = 1 \\ 2x_2 + x_3 + 3x_4 = 3 \\ x_1 - x_2 + 2x_3 = 5 \end{cases}$$

   (a) Find the least-squares solution $\widehat{\mathbf{x}}$ of this linear system. (Double check that you entered all matrices correctly!)

   (b) Write down the vector $A\widehat{x} - \mathbf{b}$ and the least-squares error $\|A\widehat{\mathbf{x}} - \mathbf{b}\|$. (Note: the command `norm(v)` will compute the norm of a vector `v` in MATLAB.)

   (c) Suppose instead that we try to approximate a solution to this system by solving the first four equations exactly for x_1, x_2, x_3, x_4. Find this solution. We'll refer to it as $\mathbf{x}_0$.

   (d) Write down the vector of errors $A\mathbf{x}_0 - \mathbf{b}$ for the full system (all six equations) and determine the error $\|A\mathbf{x}_0 - \mathbf{b}\|$. Is it greater than the error for the least-squares solution? Explain why we can know the answer to that question without performing any computations.

3. Suppose we want to find the least-squares line $y = \beta_0 + \beta_1 x$ that best fits the data

x	y
4	838
5	888
6	932
7	984
8	1041
9	1097
10	1171
11	1250

   (a) Write down the linear system that we need to find a least-squares solution to in order to find the least-squares line.

   (b) Find the least-squares line $y = \beta_0 + \beta_1 x$.

   (c) Use your line to predict the value of y when x is 12.

   (d) Plot the data points and graph the least squares line on the same graph. This can be accomplished by entering the code below (note you have to replace the letters `a` and `b` with your actual coefficients computed above.) Save an image of this figure as `ls3.bmp`

```
scatter([4 5 6 7 8 9 10 11], [838 888 932 984 1041 1097 1171 1250])
hold on
fplot(@(x) a + b*x, [4 11])
hold off
```

4. Find the least-squares parabola $y = \beta_0 + \beta_1 x + \beta_2 x^2$ that best fits the data

x	y
-2	-1.12
-1	-1.01
0	0.93
1	4.75
2	11.48

   Plot your parabola on the same graph as the data points, and save as `ls4.bmp`.