Math 401 - Groupwork #7
Markov Chains

Instructions: Work through the problems below in order. Write down all of your answers on paper, to be turned in at the end of class. I do not want you to turn in any MATLAB code. Make sure that the name of every student in the group is on the paper.

1. (Taken from Lay, Chapter 10) The weather in Charlotte, North Carolina can be classified as sunny, cloudy, or rainy on a given day. Climate data from 2003 reveal that

   - If a day is sunny, then the next day will be sunny with probability .65, cloudy with probability .10, and rainy with probability .25.
   - If a day is cloudy, then the next day will be sunny with probability .25, cloudy with probability .25, and rainy with probability .50.
   - If a day is rainy, then the next day will be sunny with probability .25, cloudy with probability .15, and rainy with probability .60.

   (a) Draw a diagram for this Markov chain (with nodes and arrows between the nodes labelled with probabilities).

   (b) Write down the transition matrix T for this Markov chain.

   (c) Is T regular? Briefly say how you know.

   (d) Suppose it is rainy on Wednesday. What is the probability that it will be sunny on Friday?

   (e) Suppose we want to schedule an outdoor soccer match with friends, but we need to book it three days in advance. We hope to avoid playing on a rainy day, but we don't care about whether it is sunny or cloudy. What should the weather be on the day we book it to minimize the probability of a rainy day when we play? Give the probability of rain in this case.

   (f) Consider the previous question again, but suppose we have to book it 7 days in advance instead. Do you think it is worthwhile to consider the current weather as a factor in our decision to book a date? Back up your answer with facts.

   (g) Use MATLAB to find the eigenvalues of T.

   (h) Give the steady-state vector $\mathbf{v}$ of T. (Make sure that your answer is a probability vector.)

   (i) Over the course of a year, about how many days in Charlotte are sunny? cloudy? rainy?

More on the back!

2. (Also from Lay) Suppose that whether it rains in Charlotte tomorrow depends on the weather conditions for today and yesterday. Climate data from 2003 show that

   - If it rained yesterday and today, then it will rain tomorrow with probability .58.
   - If it rained yesterday but not today, then it will rain tomorrow with probability .29.
   - If it rained today but not yesterday, then it will rain tomorrow with probability .47.
   - If it did not rain yesterday or today, then it will rain tomorrow with probability .31.

   Even though the weather depends on the last two days in this case, we can create a Markov chain model using the following four states

   1 it rained yesterday and today

   2 it rained yesterday but not today

   3 it rained today but not yesterday

   4 it did not rain yesterday or today

   So the "state" of any given day actually describes what happened that day as well as the previous day. The "state" of tomorrow actually describes what happens tomorrow as well as what happened today (The yesterday of tomorrow is today!) This means that once we know we are in a certain state, then half of the next state is already determined. This will make it impossible to transition directly between certain states.

   So, for example, the probability of a transition of state 1 to state 1 is .58, and the probability of a transition from state 1 to state 3 is 0. Let me know if you are confused.

   (a) Carefully draw a diagram for this Markov chain (with nodes, arrows, probabilities, etc.)

   (b) Write down the transition matrix T for this Markov chain.

   (c) Is T regular? Justify your answer.

   (d) If it rained on Friday but not on Thursday, what is the probability that we have no rain on the weekend (Sat/Sun)?

   (e) What is the ideal weather combination to have on Thursday and Friday to maximize the possibility of no rain on the weekend?

   (f) Find a steady-state vector for T.

   (g) What is the probability that two consecutive days in Charlotte are both rainy?

   (h) Consider a randomly chosen weekend (Sat/Sun) in Charlotte. Which is more likely: (1) neither day has rain, or (2) it rains on exactly one of the two days.