Math 401 - Homework #4
Least-Squares and Curve Fitting
due in class Monday, 6/11

Instructions: Work through the problems below. Write down your solutions on paper. You can use MATLAB for computations unless the problem says "Must do all computations by hand." For problems in which you use MATLAB to do computations, I do not want you to turn in any MATLAB code. You need to clearly present all steps of your solution so that I can follow your thought process without guessing what you are thinking (try writing in complete sentences.)

1. Do Exercise 4.1 from the text.
2. Do Exercise 4.5 from the text.
3. **(Must do all computations by hand.)** Do Exercise 4.10 from the text. Here are some remarks:
   - For (i), the suggestion given in the problem is really the Gram-Schmidt Process.
   - For (ii), use the formula for a projection in terms of an orthogonal basis that we mentioned in class (this is the same as the suggestion given in the problem.)
   - For (b), the "easy" method means the usual method of solving $A^T A\widehat{\mathbf{x}} = A^T\mathbf{b}$.
4. Do Exercise 4.13 from the text.
5. $$\begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \\ -2 & 2 & -2 \\ 1 & -3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$
   (a) Find all least-squares solutions to the above linear system (be careful, there is more than one least-squares soltuion.) Present your answers in parametric vector form.
   (b) Give two different least-squares solutions, and compute the least-squares error for each one.
6. Consider the following system of linear equations, which has n equations and just one variable x.
$$\begin{cases} x = b_1 \\ x = b_2 \\ \quad \vdots \\ x = b_n \end{cases}$$
   (a) Express this linear system in the form $A\mathbf{x} = \mathbf{b}$. Clearly identify A, $\mathbf{x}$, and $\mathbf{b}$.
   (b) Clearly, this is an inconsistent linear system (unless all b_i's are equal). Find the least-squares solution. (Does the answer make sense intuitively? The least-squares solution is the value of x which tries its best to satisfy all equations at once.)
7. **(Must do all computations by hand.)** Do Exercise 5.1 from the text.

8. Suppose we want to find a line $y = \beta_0 + \beta_1 x$ that best fits the following data:

x	y
0	1.15
1	1.56
2	2.69
3	3.45
4	4.10
5	5.00
6	5.89
7	7.03
8	7.77
9	8.17

   (a) Write down the system of equations that we need to find a least-squares solution to in order to find the best fit line.

   (b) Find the line $y = \beta_0 + \beta_1 x$ that best fits the data.

   (c) Use your line to predict the value of y when $x = 15$.

9. Consider the following data set:

x	y
-7	69.4
-5	42.2
-3	22.3
-1	10.4
1	6.0
3	9.0
5	20.1
7	38.5

   (a) i. Find the least-squares line $y = \beta_0 + \beta_1 x$ that best fits this data.
   ii. Find the associated least-squares error.

   (b) i. Find the least-squares parabola $y = \gamma_0 + \gamma_1 x + \gamma_2 x^2$ that best fits this data.
   ii. Find the associated least-squares error.

   (c) Provide a picture of a graph of the data points, the least-squares line, and the least-squares parabola (all in the same graph). You can either print out a plot made in MATLAB (or something else), or you can create the graph using technology and then sketch it yourself on paper (it doesn't have to be perfect).