Math 401 - Homework #8
Google Pagerank and Absorbing Markov Chains
due in class Friday, 6/22

Instructions: Work through the problems below. Write down your solutions on paper. You can use MATLAB for computations unless the problem says "Must do all computations by hand." For problems in which you use MATLAB to do computations, I do not want you to turn in any MATLAB code. You need to clearly present all steps of your solution so that I can follow your thought process without guessing what you are thinking (try writing in complete sentences.)

1. Do Exercise 8.4 from the text.

2. Do Exercise 8.8 from the text.

3. Consider the absorbing Markov chain for a tied tennis game from class. It had five states

   1 Deuce (tie)

   2 Advantage A (A has 1 more point than B)

   3 Advantage B (B has 1 more point than A)

   4 A wins.

   5 B wins.

   In this scenario, A is generally a better player than B, but B is more of a "clutch" player, which means that B performs better when the game is on the line. Assume that when the game is tied in Deuce, A has a .7 probability of scoring. But when the game is in either Advantage A or Advantage B, then B has a .6 probability of scoring.

   (a) Draw the diagram for this Markov chain.

   (b) Write down the transition matrix T.

   (c) Assume the game begins in Deuce. Determine which player is more likely to win, and give the probabilities that each player wins.

   (d) Determine the expected number of scores (time steps) that will occur before the game ends.

4. Do problem #3 from Groupwork #8. For part (f), you need to know the answers to #2 part (c), which are given below

   i. If A is serving, then A wins with probability 0.5873 and B wins with probability 0.4127.

   ii. If B is serving, then A wins with probability 0.5065 and B wins with probability 0.4935.

5. Now we're going to model (a very simplified version of) the popular childrens' board game Snakes & Ladders as a Markov chain. For this model, we'll use a 3×3 board, with squares numbered 1-9. There will only be one player, who begins the game on square 1. The goal of the game is to land on square 9. The game progresses as follows: on each turn, the player rolls a die and randomly gets a number 1, 2, or 3. The player advances that many spaces. Here are the exceptions: if you land on the base of a ladder, you must climb the ladder to advance to the square on which it ends. If you land on the top of a snake, you must slide down the snake to its bottom. You must land on the exact square to

climb/slide the ladder/snake. We will assume there is one ladder going from square 4 to square 7 and there is one snake from square 6 to square 3. To win, you must land exactly on square 9. So if you are on square 7 and roll a 3, you remain on square 7. If you are on square 8 and roll a 2 or 3, you remain on square 8.

(a) First create a Markov chain for this game without the snake or the ladder. Be clear about what your transition matrix is. Determine the expected number of turns before the game ends.

(b) Now create a Markov chain for this game with the snake and ladder. Determine the expected number of turns before the game ends. (Hint: "being on square 4" and "being on square 6" shouldn't be states in your Markov chain, since it is impossible to remain on those squares.)

(c) Move the snake so that it goes from square 8 to square 5 (instead of from square 6 to square 3). Note it is the same penalty of -3 spaces as the previous snake. First make a prediction as to whether this increase or decrease the expected number of turns before the game ends. Then compute the expected number of turns.

Remark: The real Snakes & Ladders has a 10×10 board with 100 spaces and several snakes/ladders. To move, the player rolls a 6-sided die.