Math 401 - Groupwork #1
Leontif Input-Output Model

Instructions: Work through the problems below in order. Write down all of your answers on paper, to be turned in at the end of class. I do not want you to turn in any MATLAB code. Make sure that the name of every student in the group is on the paper. Do as many of the problems as you can, and do not stress about finishing all of them. You will receive a grade based on the correctness of what you've turned in, **provided that everyone is actively working on it during class.**

1. (Do all computations for this problem by hand.) Suppose an economy has two sectors producing Product 1 and Product 2 respectively.

   - To produce 1 unit of Product 1 it takes 0.1 units of Product 1 and 0.3 units of Product 2.
   - To produce 1 unit of Product 2 it takes 0.4 units of Product 1 and 0.2 units of Product 2.

   (a) Write down the consumption vectors for each product, and then write the consumption matrix M.

   (b) Find $(I - M)^{-1}$.

   (c) What should the total production be set to in order to satisfy an external demand of 60 units of Product 1 and 90 units of Product 2?

   (d) How many of the units produced in the previous part were consumed to satisfy internal demand?

   (e) Suppose the value 0.1 in the original problem description is replaced by α. Find the nonnegative value(s) of α for which $I - M$ is not invertible. (Recall a matrix is not invertible if and only if its determinant is 0.)

2. (Use MATLAB. The data in this problem was taken from Lay, §2.6.) The consumption matrix M and external demand $\mathbf{d}$ below is based on data from the 1958 U.S. economy, with data for 81 sectors grouped into 7 larger sectors: (1) nonmetal household and personal products, (2) final metal products (such as motor vehicles), (3) basic metal products and mining, (4) basic nonmetal products and agriculture, (5) energy, (6) services, and (7) entertainment and miscellaneous products. (Units are in millions of dollars.)

$$M = \begin{bmatrix} .1588 & .0064 & .0025 & .0304 & .0014 & .0083 & .1594 \\ .0057 & .2645 & .0436 & .0099 & .0083 & .0201 & .3413 \\ .0264 & .1506 & .3557 & .0139 & .0142 & .0070 & .0236 \\ .3299 & .0565 & .0495 & .3636 & .0204 & .0483 & .0649 \\ .0089 & .0081 & .0333 & .0295 & .3412 & .0237 & .0020 \\ .1190 & .0901 & .0996 & .1260 & .1722 & .2368 & .3369 \\ .0063 & .0126 & .0196 & .0098 & .0064 & .0132 & .0012 \end{bmatrix}, \qquad \mathbf{d} = \begin{bmatrix} 74,000 \\ 56,000 \\ 10,500 \\ 25,000 \\ 17,500 \\ 196,000 \\ 5,000 \end{bmatrix}$$

   The MATLAB code to enter this big matrix M into MATLAB is posted on the main page in ELMS. You can copy/paste it into MATLAB.

   (a) Find the production levels needs to satisfy the external demand $\mathbf{d}$.

   (b) What percentage of services (sixth sector) produced is consumed in satisfying internal demand?

(c) Suppose an additional 1,000 units of energy (fifth sector) is demanded. By how much does the production for all other sectors increase?

(d) Let $\mathbf{v} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$. Convince yourself that the matrix product $\mathbf{v}M$ computes all column sums of M. Write down all column sums of M. Are they all less than 1?

(e) A column sum of $(I - M)^{-1}$ for columns j represents the total number of units (from all sectors) that need to be produced to satisfy 1 unit of external demand from sector j. Determine which sector requires the most total production in this sense, and give the total number of units required for that sector.

3. Suppose the consumption matrix for two sectors is given by:

$$M = \begin{bmatrix} 0.10 & 0.06 \\ 0.05 & 0.12 \end{bmatrix}$$

(a) Find the smallest integer i such that $I + M + ... + M^i$ is the same as $I + M + ... + M^{i-1}$ to the fourth decimal digit (use `format short` in MATLAB).

(b) Write down the matrix $I + M + ... + M^i$ using your i from (a).

(c) Use MATLAB to evaluate $(I - M)^{-1}$ and write down the result.

4. Suppose the consumption matrix for two sectors is given by:

$$M = \begin{bmatrix} 0.1 & 0.6 \\ 0.6 & 0.6 \end{bmatrix}$$

(a) Give an example of nonzero production levels that satisfy $\mathbf{p} = M\mathbf{p}$. (Here, this is a closed economy, which means $\mathbf{d} = \mathbf{0}$.) Determine the ratio p_1/p_2 of your two production levels.

(b) Give an example of a nonzero external demand $\mathbf{d}$ (with positive entries) for which $\mathbf{p} = M\mathbf{p} + \mathbf{d}$ has no solutions.

(c) Is it possible to have a nonzero external demand $\mathbf{d}$ (with positive entries) for which $\mathbf{p} = M\mathbf{p} + \mathbf{d}$ has solutions? If so, give an example of such a $\mathbf{d}$. If not, explain why.