Math 401 - Groupwork #8
Google Pagerank and Absorbing Markov Chains

Instructions: Work through the problems below in order. Write down all of your answers on paper, to be turned in at the end of class. I do not want you to turn in any MATLAB code. Make sure that the name of every student in the group is on the paper.

1. Point a browser to the address:

   `http://www.math.umd.edu/~immortal/MATH401/google/page-cat.html`

   You'll find a page with some links. This webpage is interlinked with several (not too many) other pages.

   (a) Explore the links until you've figured out the structure of this little internet. Draw a diagram with arrows indicating how the pages are linked together.

   (b) Write down the Googe Pagerank transition matrix T. Be sure to account for the 15% chance that the random websurfer goes to a randomly chosen page. You can leave your written answer "unsimplified" (as a sum of two matrices, with scalar factors in front.)

   (c) Let $\mathbf{w}$ be a row vector of all 1's with the same number of columns as T. Compute $\mathbf{w}T$. Are you confident that you entered T correctly into MATLAB? Briefly explain what it is we are checking with this computation, and how it could indicate to us that we made a mistake.

   (d) Calculate the Google pagerank vector $\mathbf{p}$ of the pages using eigenvalue/eigenvector methods. Make sure your answer in a probability vector. Then list the animals in terms in order according to their pageranks.

   (e) Compute the pagerank vector using 70/30 instead of 85/15. Does it change the ranking values? Does it change the order of the pages?

2. In volleyball (as in tennis), you must win by at least two points. A *rally* begins with a serve by one of the teams and ends when one of the teams scores. The team that scored gets to serve the next rally. We'll ignore the beginning portion of a volleyball game and assume we've gotten to the point that it is tied, and the next team to go up by 2 points wins.

   Assume that when team A is serving, then team A has probability 0.60 of scoring. Similarly, assume that when team B is serving, then team B has a probability 0.55 of scoring. Model the progress of a tied volleyball game using an absorbing Markov chain with the following six states:

   1 tied – A serving

   2 tied – B serving

   3 A ahead by 1 point – A serving

   4 B ahead by 1 point – B serving

   5 A wins the game

   6 B wins the game

   (a) Draw a diagram for this Markov chain. Include all probabilities for possible transitions.

(b) Write down the transition matrix T. It should have the "block" form $T = \begin{bmatrix} Q & \mathbf{0} \\ R & I \end{bmatrix}$. Identify Q and R.

(c) i. Suppose the game is tied, and A is serving. Find the probabilities that each team wins.

ii. Suppose the game is tied, and B is serving. Find the probabilities that each team wins.

(d) i. Suppose the game is tied, and A is serving. Determine the expected number of rallies which will take place before the game ends.

ii. Suppose the game is tied, and B is serving. Determine the expected number of rallies which will take place before the game ends.

(e) i. Suppose A is ahead by 1 point (so A is serving). Find the probability that B wins.

ii. Suppose B is ahead by 1 point (so B is serving). Find the probability that A wins.

3. The scoring system for volleyball described in the previous problem is called *rally point scoring*. In this problem, we explore a different scoring system called *side out scoring*. In side out scoring, a team only gets a point if it wins a rally in which it was the server. For example, if team B wins a rally that team A served, then no one gets a point, but team B gets to right to serve next. If team A wins a rally that team A served, then team A gets a point and team A gets to serve again. We can model a tie game with side out scoring as a Markov chain with 8 states:

1 tied – A serving

2 tied – B serving

3 A ahead by 1 point – A serving

4 A ahead by 1 point – B serving

5 B ahead by 1 point – A serving

6 B ahead by 1 point – B serving

7 A wins the game

8 B wins the game

As before, assume that when team A is serving, then team A has probability 0.60 of scoring, and when team B is serving, then team B has a probability 0.55 of scoring.

(a) Draw a diagram for this Markov chain. Include all probabilities for possible transitions.

(b) Write down the transition matrix T. It should have the "block" form $T = \begin{bmatrix} Q & \mathbf{0} \\ R & I \end{bmatrix}$.

(c) i. Suppose the game is tied, and A is serving. Find the probabilities that each team wins.

ii. Suppose the game is tied, and B is serving. Find the probabilities that each team wins.

(d) i. Suppose the game is tied, and A is serving. Determine the expected number of rallies which will take place before the game ends.

ii. Suppose the game is tied, and B is serving. Determine the expected number of rallies which will take place before the game ends.

(e) i. Suppose A is ahead by 1 point and A is serving. Find the probability that B wins.

ii. Suppose B is ahead by 1 point and B is serving. Find the probability that A wins.

(f) Suppose the teams agree that one team gets to pick the scoring style and the other team gets to serve first (with the game starting in a tie).

i. Suppose team A has the option to either serve first or pick the scoring style. What is the best choice for team A to make?

ii. Suppose team B has the option to serve first or pick the scoring style. What is the best choice for team B to make?