Math 401 - Groupwork #6
Discrete Dynamical Systems

Instructions: Work through the problems below in order. Write down all of your answers on paper, to be turned in at the end of class. I do not want you to turn in any MATLAB code. Make sure that the name of every student in the group is on the paper. Some problems ask you to save images and email them to me (`ayashins@math.umd.edu`) at the end. Do as many of the problems as you can, and do not stress about finishing all of them. You will receive a grade based on the correctness of what you've turned in, **provided that everyone is actively working on it during class.**

1. In ELMS, download the file `dynamics.m` to your computer, and then open the script in MATLAB. The script enters a matrix A and an initial vector $\mathbf{x}_0$ for a discrete dynamical system $\mathbf{x}_{k+1} = A\mathbf{x}_k$. It then repeatedly multiplies by A and stores the resulting vectors $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots$ as the column of the matrix `x`. Then a scatter plot of these vectors is produced.

   (a) Take a look at the script and make sure you understand what it is doing. The script is initially set up for the predator-prey system we encountered previously in class:

   $$\mathbf{x}_{k+1} = \begin{bmatrix} 0.4 & 0.5 \\ -0.2 & 1.2 \end{bmatrix} \mathbf{x}_k, \qquad \mathbf{x}_0 = \begin{bmatrix} 500 \\ 250 \end{bmatrix}.$$

   Play around with the script by changing the initial vector $\mathbf{x}_0$. Find a choice of $\mathbf{x}_0$ in which both values are positive, but both species end up dying out. You may need to change your axes to get a better view of what is happening in the graph. Write down your $\mathbf{x}_0$ and save an image of your graph as `dyn1a.bmp`.

   (b) (If you changed your axes, revert them to `[0 500 0 500]`) The command `eig(A)` will return the eigenvalues of a matrix A and the command `[P,D] = eig(A)` will diagonalize A by creating the matrix P of eigenvectors and the diagonal matrix D of eigenvalues. Add the command `[P,D] = eig(A)` to your script to get the eigenvalues and eigenvectors. Then use `fplot` to plot the two eigenspaces (the lines through the origin that point in the directions of the two eigenvectors.) Note that you can compute the slope of each line by extracting entries from P. If `m` is a slope, you can plot the line $y = mx$ with `fplot(@(x) m*x)`. Don't forget to use `hold on` and `hold off`. You should do this in such a way that if we change A, then it will plot the eigenspaces of the new A. Save an image of your plot with the eigenspaces as `dyn1b.bmp`.

   (c) Your eigenspaces should divide the picture into three regions. Choose an initial condition from each region and plot them all (along with the eigenspaces) in the same graph. To do this, I recommend you add variables `y0` and z0 below `x0` (and similarly `y` and `z` below `x`) and create a matrix of the "trajectory" for each initial condition, by mimicing what was done for `x`. Then plot each using `scatter`. Save this image as `dyn1c.bmp`.

   (d) Now change the original matrix to $A = \begin{bmatrix} 0.4 & 0.5 \\ -0.25 & 1.2 \end{bmatrix}$ as we did in class. Run your script to produce an image just like in the previous part, but for this different matrix. Make sure the eigenspaces have been adjusted. It should look qualitatively different than the previous one. Save your image as `dyn1d.bmp`.

   (e) Now suppose that the owls eat more rats than before, but the rats reproduce faster in such a way that the matrix becomes $A = \begin{bmatrix} 0.4 & 0.5 \\ -0.4 & p \end{bmatrix}$. Approximately find the threshold p that is necessary for both species to survive. Then round p up to the next tenth and save a plot `dyn1e.bmp` with three trajectories and the eigenspaces as before. Write down the eigenvalues for this matrix, and briefly explain, in terms of the eigenvalues, why the populations will grow.

2. For this part, change your axes to `[-500 500 -500 500]`, so you can see all four quadrants. Use your script to plot three trajectories and the eigenspaces for $\mathbf{x}_{k+1} = A\mathbf{x}_k$ for each of the given A. Choose your initial conditions to come from different areas, say different quadrants. We are no longer interpreting this as a predatory-prey system, so we are not disregarding negative values. For each part, write down the eigenvalues of A, and save an image of your plot as `dyn2x.bmp` where `x` is the part you are on.

   (a) $A = \begin{bmatrix} 0.8 & 0.5 \\ -0.1 & 1.0 \end{bmatrix}$ (There will be an issue with the eigenspaces of this one. Do you see why?)

   (b) $A = \begin{bmatrix} 0.3 & 0.4 \\ -0.3 & 1.1 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 0.96 & 0.06 \\ 0.04 & 0.94 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$ (Again, there will be an issue with the eigenspaces here.)