Math 401 - Homework #1
Leontif Input-Output Model
due in class Friday, 6/1

Instructions: Work through the problems below. Write down your solutions on paper. You can use MATLAB for computations unless the problem says "Must do all computations by hand." For problems in which you use MATLAB to do computations, I do not want you to turn in any MATLAB code. You need to clearly present all steps of your solution so that I can follow your thought process without guessing what you are thinking (try writing in complete sentences.)

1. Suppose an economy has three sectors producing Product 1, Product 2, and Product 3 respectively.

   - To produce 1 unit of Product 1 it takes 0.02 units of Product 1, 0.06 units of Product 2, and 0.10 units of Product 3.
   - To produce 1 unit of Product 2 it takes 0.40 units of Product 2 and 0.10 units of Product 3 (no units of Product 1).
   - To produce 1 unit of Product 3 it takes 0.18 units of Product 1, 0.01 units of Product 2, and 0.10 units of Product 3.

   (a) Write down the consumption matrix M.

   (b) Determine how many units of each product need to be produced in order to satisfy an external demand of 200 units of Product 1, 180 units of Product 2, and 175 units of Product 3?

   (c) Suppose that the external demand for Product 3 increases by 1 unit. Determine how many additional units of each product need to be produced in order to satisfy this increased demand.

2. Suppose the consumption matrix for four sectors is given by

$$M = \begin{bmatrix} 0.02 & 0 & 0.02 & 0.04 \\ 0 & 0.01 & 0.05 & 0.04 \\ 0.03 & 0 & 0.01 & 0.02 \\ 0.02 & 0.05 & 0.01 & 0 \end{bmatrix}$$

   (a) Find the smallest integer i such that $I + M + ... + M^i$ is the same as $I + M + ... + M^{i-1}$ to the fourth decimal digit (use `format short` in MATLAB).

   (b) Compute $(I - M)^{-1}$.

   (c) Describe what the entry in row 3 and column 2 of $(I - M)^{-1}$ represents (economically speaking).

3. Do Exercise 2.11 from the text. (It should say "greater than or equal to 1")

4. **(Must do all computations by hand.)** Suppose an economy with two sectors has the consumption matrix $M = \begin{bmatrix} 0.75 & 0.1 \\ 0.25 & 0.9 \end{bmatrix}$.

   (a) Show that $I - M$ is not invertible.

   (b) Consider the case of a closed economy (this means there is no external demand: $\mathbf{d} = \mathbf{0}$), so that $\mathbf{p} = M\mathbf{p}$. Find nonzero positive production values p_1, p_2 that satisfy this system.

(c) evaluate the ratio p_1/p_2 of your production levels from the previous part.

5. **(Must do all computations by hand.)** Suppose an economy with two sectors has the consumption matrix $M = \begin{bmatrix} 0.6 & 0.7 \\ 0.5 & 0.5 \end{bmatrix}$.

   (a) Compute $(I - M)^{-1}$. (You can use the formula for the inverse of a 2×2 matrix.) Simplify your answer.

   (b) Suppose $\mathbf{d} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$. Find the unique vector $\mathbf{p}$ such that $\mathbf{p} = M\mathbf{p} + \mathbf{d}$.

   (c) Something is very messed up with the economics of this situation. Describe an undesirable property of the original matrix M which is related to the fact that we aren't getting economically feasible results. (More precisely, point out a good property that M does not have which would guarantee this situation couldn't happen.)

6. Suppose two consumption matrices for an economy with two sectors are given by

$$M_1 = \begin{bmatrix} 0.06 & 1.02 \\ 0.05 & 0.10 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 1.02 & 0.06 \\ 0.05 & 0.10 \end{bmatrix}$$

   (a) Find $(I - M_1)^{-1}$ and $(I - M_2)^{-1}$.

   (b) If $\mathbf{d} = \begin{bmatrix} 100 \\ 200 \end{bmatrix}$ find $(I - M_1)^{-1}\mathbf{d}$ and $(I - M_2)^{-1}\mathbf{d}$.

   (c) Which of these is realistic and which is not? For the unrealistic one, explain economically what is wrong with its consumption matrix. Make sure that your argument couldn't be applied to the other matrix as well.

7. Do Exercise 2.17 from the text.

8. **(Must do all computations by hand.)** Do Exercise 2.18 from the text.