Determination of the Average Muon Lifetime in a Polyvinyltoluene-Based Plastic Scintillator

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An experiment was carried out to measure the average lifetime of muon in a polyvinyltoluene-based plastic scintillator. The average muon lifetime was measured to be about $2.008^{+0.062}_{-0.120}$ microseconds, which in general agrees with the accepted muon lifetimes and our theoretical predictions.

1 Introduction

In 1934, Hideki Yukawa proposed a theory of the strong nuclear force that holds the protons and neutrons together in an atomic nucleus. In analogy with photons as the quanta of the electromagnetic interaction, Yukawa predicted a heavy particle that mediates the short-range strong nuclear force.[1] The mass of this mediator particle was estimated to be about 200 times the mass of the electron,[2] which means that the particle would have a mass between that of the electron and the proton.

Although Yukawa did not assume the correctness of his theory since no such particles were known at the time,[2] by 1937 two separate groups (Anderson and Neddermeyer at Caltech, and Street and Stevenson at Harvard) identified from cosmic rays a charged particle that has a mass similar to the particle predicted by Yukawa, although the connection between the cosmic ray particle and Yukawa's theory was actually drawn by Robert Oppenheimer.[3] This particle became known as the meson ("meso-" means middle). Although some discrepancies between the properties of the meson and the predictions by Yukawa were later revealed, this particle was believed to be the Yukawa particle for the next ten years until a decisive 1946 experiment was carried out by Conversi, Pancini, and Piccioni in Italy.[1,4] They separated the positive mesons from the negative ones and found that the rate of nuclear absorption of the negative mesons in carbon was much lower than predicted, which means that the meson interacts with the carbon atomic nuclei much more weakly than a strong nuclear force mediator should have been.[5,6] This difficulty was soon resolved in the following year by Powell and his collaborators by showing experimentally that there are indeed two kinds of mesons, which they called π and μ, and the π meson was later recognized to be the "true" Yukawa meson. As more and more intermediate-weight particles, i.e. mesons, were discovered, it gradually became clear that the μ meson actually resembled the electron rather than the other mesons. Therefore, the μ meson was reclassified, together with the electron, as a lepton, and was more widely referred to as the muon.[1]

In this experiment, we measured the average lifetime of the muon in a plastic scintillator. We intended to compare our experimental result with the accepted spontaneous decay lifetime of the muon.

2 Theory

This section describes theories that are relevant to this experiment.

2.1 The muon

The muon is an elementary particle, or more specifically, a lepton. It has a rest mass of about 106MeV/c^2 and is about 200 times more massive than the electron. A muon can be either negatively or positively charged, with a respective charge of $-e$ or $+e$. The negative and positive muons are antiparticles to each other.[7,8]

A muon is not a stable particle. A negative muon decays principally into an electron, an electron antineutrino, and a muon neutrino. A positive muon will decay into the corresponding antiparticles.[1] These decays are represented as follows

$$\begin{aligned} \mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu \\ \mu^+ &\rightarrow e^+ + \nu_e + \bar{\nu}_\mu \end{aligned} \tag{1}$$

Other types of decay for the muons are exceedingly rare.[9]

2.2 Our source of muon

The muons that we tried to detect in this experiment are traced to the cosmic rays, which are high-energy particles (primarily protons and heavier nuclei) that originate from the outer space and bombard the earth's atmosphere.[10] The collisions between these high-energy particles and the particles in the earth's atmosphere will produce a shower of additional particles, including pions (i.e. the aforementioned π meson). The pions are unstable, and the positive and negative pions (π^+ and π^-) will further decay and produce positive and negative muons.[11] A large number of these muons will be able to reach the sea level and be detected by us.[12]

2.3 Muon capture in nuclei

When a negatively charged muon passes through a material, it can be attracted and bound to an atomic nucleus just like an electron. Within a time scale of 10^{-13}s, the muon will cascade down to the 1s ground state, after which the muon can either decay or be captured by a proton in the nucleus.[13] For ^{12}C nuclei, for example, about 90 percent of the ground state muons will decay, while the rest of the ground state muons will be captured by the ^{12}C nuclei.[14] The dominant interaction for a muon capture is

$$\mu^- + p \rightarrow n + \nu_\mu \tag{2}$$

where p denotes proton and n denotes neutron. Although there are usually various ends that such a muon capture can end up with, the capturing nucleus will typically emit some form of energy before settling down to a stable state. The emitted energy can be gamma rays and/or kinetic energy carrying particles such as protons, neutrons, and/or electrons.[13]

2.4 Time distribution of muon decay

Muon is an unstable particle and will follow an exponential decay law:

$$N(t) = N_0 e^{-\lambda t} \tag{3}$$

where N(t) is the number of muons present at time t, N_0 is the number of muons present at time 0, and λ is the exponential decay constant.[11] The accepted value for the **lifetime** of the muon, $\tau = \frac{1}{\lambda}$, is about 2.197μs.[7] Taking the differential of both sides of Equation 3, we obtain

$$-dN = (N_0 \lambda dt) e^{-\lambda t} \tag{4}$$

where -dN can be interpreted as the number of muons that decay within the short time interval dt. Therefore, we can see that for a fixed short time interval dt, the number of muons that decay between times t and t+dt is proportional to $e^{-\lambda t}$. However, we need to emphasize that -dN is only an expectation value of the number of muons that would actually decay within this time interval. Remember that for exponential decays, λdt can be interpreted as the probability that a muon present at time t would decay between t and t+dt. Therefore, for a fixed short time interval dt, the number of muons that would decay between times t and t+dt is really a binomial random variable of parameters N(t) and λdt,[15] and Equation 4 should be rewritten as

$$E[Count(t)] = (N_0 \lambda dt)e^{-\lambda t} \tag{5}$$

where E[Count(t)] denotes the expectation value of the number count of the muons that decay between times t and t+dt.

3 Experiment

3.1 Experimental setup and methodology

As shown in Figure 1, the apparatus of this experiment mainly consists of a detector box and an electronics box. The detector box mainly consists of a volume of polyvinyltoluene-based scintillator (polyvinyltoluene

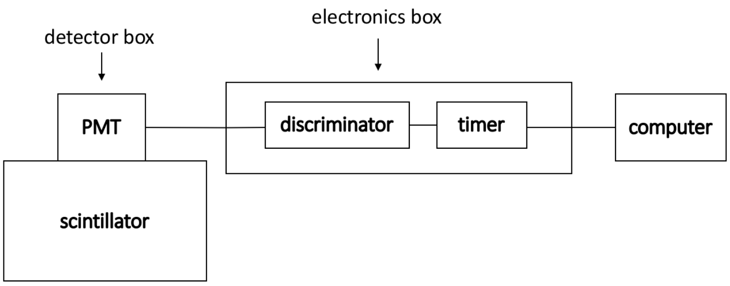

Figure 1: Experimental setup for the muon lifetime experiment.

is a type of plastic) and a photomultiplier tube (PMT). A scintillator is a material that scintillates, which means that it can absorb the kinetic energy of a passing-by charged particle by excitation or ionization of the scintillator molecules, and then re-emit part of the absorbed energy in the form of photons. The PMT will amplify the photoelectric current produced by the incident photons. Besides, the PMT is subject to an adjustable high voltage, where a higher voltage will cause a greater amplification of the photoelectric current. The amplified current is then transmitted to the electronics box, inside which there is a discriminator which converts the amplified current to a voltage pulse and then transmits this voltage pulse to trigger a timer only when the voltage pulse is higher than an adjustable threshold value. The timer is connected to a computer interface, which is able to record and display all the timer triggerings, called **detections**, in real time.

In this experiment, we are only interested in the muons that finally stop in the scintillator. Recall from Section 2.2 that we will receive a flux of muons produced in the upper atmosphere. As just mentioned, a muon that enters the scintillator with sufficient kinetic energy will trigger a detection. Besides, if the muon finally stops in the scintillator, then according to Section 2.1, the muon will spontaneously decay into an electron/positron, a neutrino, and an antineutrino. Since an electron/positron is much lighter than a muon, the electron/positron will inherit a large amount of kinetic energy from the decaying muon's rest mass. Therefore, the electron/positron will be able to excite/ionize the scintillator molecules again and trigger a second detection. Both detections (as instants of time) will be recorded by the computer and the time interval between these two detections can be regarded as the **decay time** of that particular muon with its stopping moment as time 0. Since we are only interested in the decay times of the muons at rest, we will only examine the pairs of successive detections that are close to each other. The computer interface will mark out these pairs of detections whose interval is less than 40000 clock cycles (about 800,000 nanoseconds) of the timer inside the detector box as **decay events**. Therefore, for the muons that finally settle at rest in the scintillator, we are able to obtain their decay times using our experimental apparatus.

Notice that the electronics box might be triggered to alarm a first detection without catching up a second detection within the 40000 clock-cycle limit. This might be due to other passing-by particles that also lose their kinetic energies to the scintillator, muons that do not end up stopping in the scintillator, or muons that decay while still moving in the scintillator. For a decay-while-moving muon, there will effectively be only one detection event since the duration of a discriminator pulse is larger than the typical time for a muon to travel through the scintillator and therefore the second discriminator pulse triggered by the electron as a result of the decay will not be distinguished from the first pulse by the timer. So, the aforementioned **rule of two successive detections** allows us to filter through almost only the muons that decay at rest in the scintillator.

According to Section 2.3, negative muons might be captured by the nuclei in the scintillator before they spontaneously decay. Since the capturing nuclei will eventually end up in a stable state by transfering energy to the scintillator, this process will also be able to trigger a detection alarm. Therefore, together with the first detection alarm caused by the moving muon, this detection alarm will suffice to cause the computer interface to also mark it up as a **"decay event"**. Therefore, our data of "decay events" will in fact include both spontaneous decays as well as muon captures.

3.2 Experimental theory

According to Section 2.4, for a short time interval dt, the expectation value of the number of muons that decay between times t and $t + dt$ will be

$$E[Count(t)] = (N_0 \lambda dt) e^{-\lambda t} \tag{6}$$

Therefore, we have

$$\log E[Count(t)] = -\lambda t + \log (N_0 \lambda dt) \tag{7}$$

where "log" always denotes the natural logarithm in this article. Therefore, there is a linear relation between the time t and the logarithm of the expectation value of the number of muons that decay between times t and $t + dt$, if dt is short. According to Section 3.1, our experimental data will be a set of muon decay times. Therefore, we can partition these decay times into a histogram with a bin width of a short time interval. In this way, we can obtain a list of data of Count(t) vs. t, and hence of $\log Count(t)$ vs. t. After this, we can apply least squares linear regression to find a best fitting line and determine the muon lifetime from the inverse of the parameter λ.

However, we need to emphasize that we will only determine an average lifetime of the positive and the negative muons that come to a rest in the scintillator. Recall from Section 3.1 that negative muons can be captured by the scintillator nuclei before they decay, and such events will also be recorded by the computer with a "decay time". Since the nuclei in the plastic scintillator are mostly carbon and hydrogen and the carbon nuclei have much stronger attractions to the negative muons than the hydrogen nuclei do, the mean lifetime for negative muons (considering both spontaneous decay and muon capture) in the scintillator is essentially the same as that in carbon, whose accepted value is about 2.043μs.[16] Meanwhile, the lifetime of a muon, either positive or negative, in free space is about 2.197μs. Therefore, the muon lifetime determined in this experiment should ideally lie between these two values.

3.3 Procedure

Using the experimental apparatus described in Section 3.1, we obtained the muon decay time data in three separate trials. The first trial lasted for about two days, the second trial lasted for about 5 days, and the third trial lasted for about a week. Since our detecting machine was somehow very sensitive to subtle changes in the environment, we adjusted the high voltage on the photomultiplier tube and the threshold of the discriminator between these trials, necessarily, in order to prevent our machine from reporting a surge of unrealistic decay events.

4 Data analysis

According to Section 3.3, we obtained the decay time data in three separate trials. Since a large set of data typically favors a more accurate determination of the average muon decay time, we will merge all the three sets of data into one for the purpose of data analysis. The merged set of data has 12920 decay events in total (when we say decay events, we really mean **the recorded decay events**, which include events of muon capture), with a minimum recorded decay time of 40 nanoseconds (ns) and a maximum recorded decay time of 19880ns. Remember from Section 3.2 that we want to partition our data into a histogram with a small bin width of time so that we can take the logarithm of the decay event number count for each bin and fit the logarithm data with a straight line. Notice that the number count of the muon decay events between times t and t+dt is a binomial random variable, whose variance (square of standard deviation) is approximately equal to its mean when the probability of a muon to decay in this time interval is small.[15] Therefore, for Count(t), we will take $\pm\sqrt{Count(t)}$ as its experimental error. Hence, we will have

$$\log\left[1-\frac{1}{\sqrt{Count(t)}}\right] \text{ and } \log\left[1+\frac{1}{\sqrt{Count(t)}}\right] \tag{8}$$

as the corresponding negative and positive errors for the logarithm of Count(t). Notice that when $Count(t) \leqslant 1$, this pair of errors for $\log Count(t)$ are not both well defined. Therefore, we will discard all the points with Count(t)$\leqslant$1 in our data analysis.

The data points of $\log Count(t)$ vs. t with the respective error bars are plotted in Figure 2. Remember that we have truncated all the data points with Count(t)$\leqslant$1, so there are some obvious gaps at the tail of the data points in Figure 2. From Figure 2, we can see that the neck of the data points exhibits a roughly consistent slope. In contrast, the first several points are wildly higher, while the points in the tail flatten out. Since we expect these data points to more or less follow a straight line, we will treat the first several data points and the points in the flattening tail as noise, and only fit the neck of the data points. As per Figure 2, different data points can possess different measurement errors, so we will conduct a weighted least squares linear regression with the weight for each data point being

$$\text{weight for log Count(t)} = \left[\frac{1}{2}(\text{positive error} - \text{negative error})\right]^{-2} \tag{9}$$

where the positive and negative errors are cited from (8). Therefore, the larger the measurement error for a data point, the smaller its weight.

The fitting result for the data points from 400ns to 6949ns, with a bin width of 50ns, is presented in Figure 3. The weighted least squares fitting line in Figure 3 is -(0.000497913ns^{-1})t+4.29835. The standard error for the slope is computed to be 0.0000148802ns^{-1} when only the inverse of the weight from (9) is used as the variance estimator for each data point. Therefore, the corresponding average muon lifetime τ is $\frac{1}{0.000497913\text{ns}^{-1}}$=2008ns, with the corresponding statistical upper and lower bounds computed from the standard error being 1950ns (-120ns) and 2070ns (+62ns).

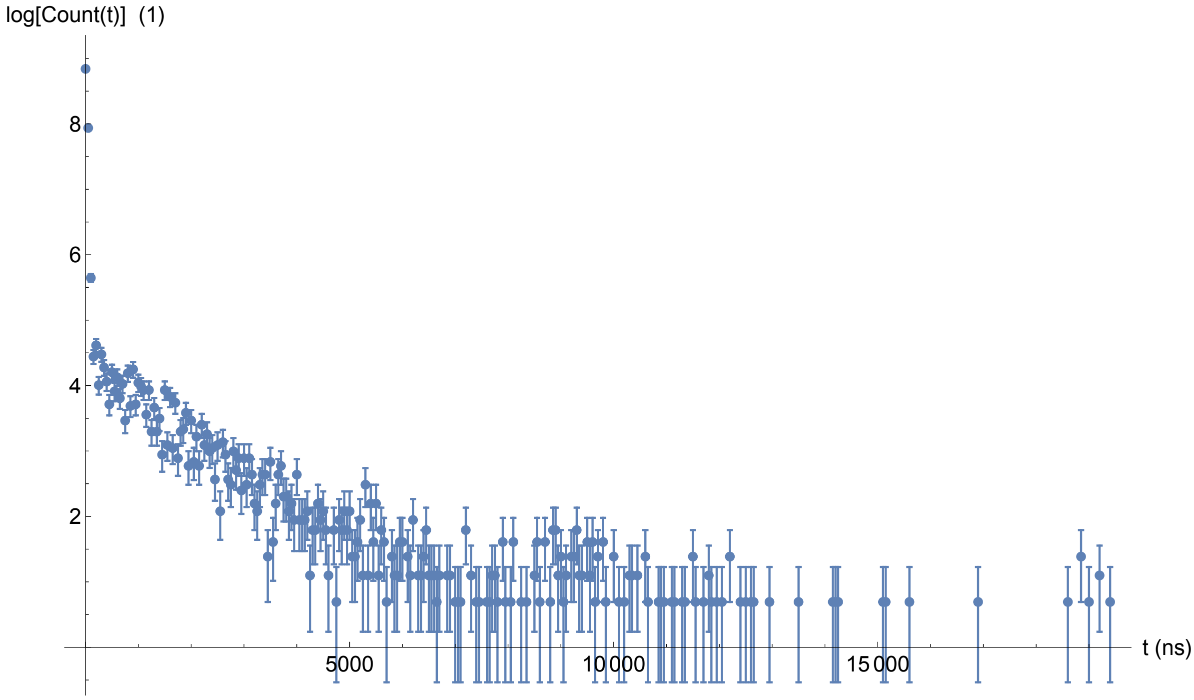

Figure 2: log Count(t) vs. t for the entire data set, with a bin width of 50 nanoseconds.

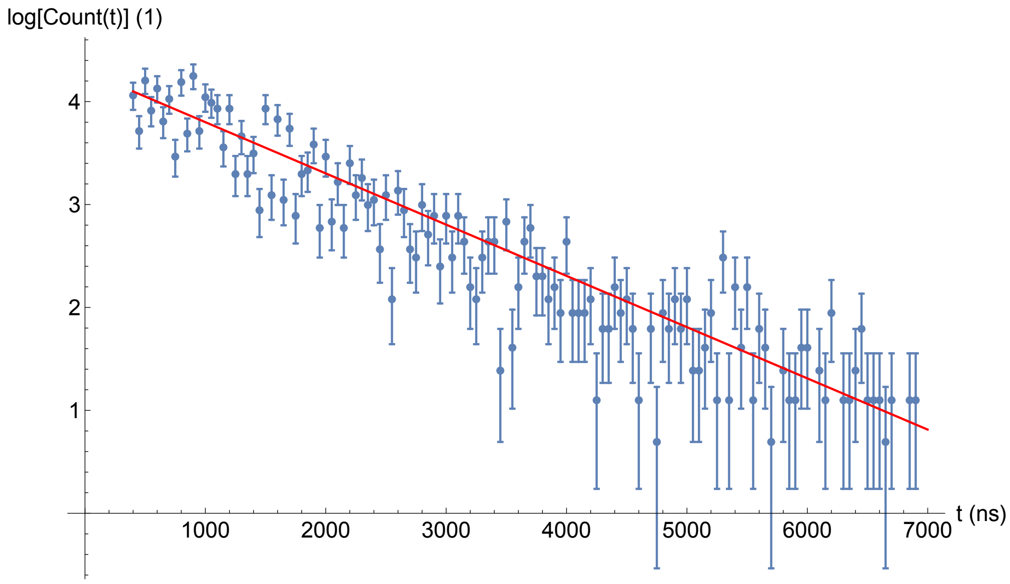

Figure 3: Weighted linear regression for log Count(t) vs. t for the data points from 400 nanoseconds to 6949 nanoseconds, with a bin width of 50 nanoseconds.

5 Conclusion

In this experiment, we intended to measure the average muon lifetime using a scintillator-based detector. The average muon lifetime was measured to be 2008ns, with the statistical upper and lower bounds (from the least squares standard error) being 2070ns and 1950ns. According to Section 3.2, we are expecting an average muon lifetime between 2043 and 2197 nanoseconds. Therefore, although the experimentally measured average muon lifetime of 2008ns does not lie in this expected range, the statistical upper bound of the measured value, 2070ns, stays in this range. So, our experimentally determined average muon lifetime in general seems reasonable, although a slightly larger value might be preferable. This might be due to the fact that we are using $\sqrt{Count(t)}$ as the measurement error for E[Count(t)], which is only a rough approximation. To obtain more accurate statistical uncertainties, the measurement error should be analyzed with more precise methods such as confidence intervals. Since a higher confidence level, say 90 percent, will typically correspond to a measurement uncertainty larger than $\sqrt{Count(t)}$, the corresponding statistical errors for the measured average muon lifetime will also become larger. Besides, our data set is still relatively small. In performing the linear regression, we were considering only 2220 decay events in total, with an average of 18 events per bin. Therefore, the average measurement error for a bin is roughly $\frac{\sqrt{18}}{18} = 24$ percent, which is relatively large. To improve our measurement result, we should try to record more muon decay events in order to minimize the statistical errors. After we have a clearer picture of the impact of the statistical errors, we can investigate into the systematic errors more closely.

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