Investigating the Hall Effect and the Magnetoresistance with a Semiconductor

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An experiment was carried out on a semiconductor to verify the $V_h \propto BI$ relation in the Hall effect as well as the constancy of the semiconductor's Hall coefficient. The experimental results agree with theoretical predictions very well. The magnetoresistance of the semiconductor was also examined, which is consistent with the semiconductor having more than one type of charge carriers.

1 Introduction

In 1879, Edwin Hall, while still a graduate student, discovered his eponymous effect at the newly founded Johns Hopkins University. Hall's motivation, as described by himself in his original paper, was due to a doubt on Maxwell's assertion that the magnetic force due to an external magnetic field acts on the conductor itself rather than the current inside the conductor. Under crucial guidance of his doctoral advisor Henry Rowland, Hall started off to prove the action of the external magnetic field on the current, and finally succeeded, despite some negative results in the beginning, in detecting a permanent potential difference transverse to the current through a gold leaf that unambiguously resulted from the external magnetic field. It is worth noting that Hall initially aimed to measure the change of resistance of a silver wire spiral due to the external magnetic field. Although he did observe changes of the resistance in some of his trials, the changes were in general too small to be regarded as conclusive by him.[1]

Although the Hall effect was discovered more than 130 years ago, it found very few technical utilizations until the 1960s,[2] when advances in integrated circuit vastly reduced the cost of amplifying the minuscule Hall voltage.[3,4] Since then, the application of the Hall effect expanded dramatically. The Hall effect has become essential in the studies of metals, semiconductors, and magnetic solids. Many devices such as keyboards, gaussmeters, automobile ignition systems, and satellites have incorporated components that employ the Hall effect.[5]

In this experiment, we would verify the proportionality between the Hall voltage and the product of the external magnetic field and the current with a semiconductor, and would thus verify the constancy of its Hall coefficient. We would also examine the magnetoresistance of the semiconductor to verify that it has more than one type of charge carriers.

2 Theory

This section describes the theories that are relevant to this experiment.

2.1 Semiconductors

A semiconductor is a material with electron energy levels grouped into two **energy bands**, where an energy band is a nearly continuous range of the allowed electron energy levels. The two energy bands are

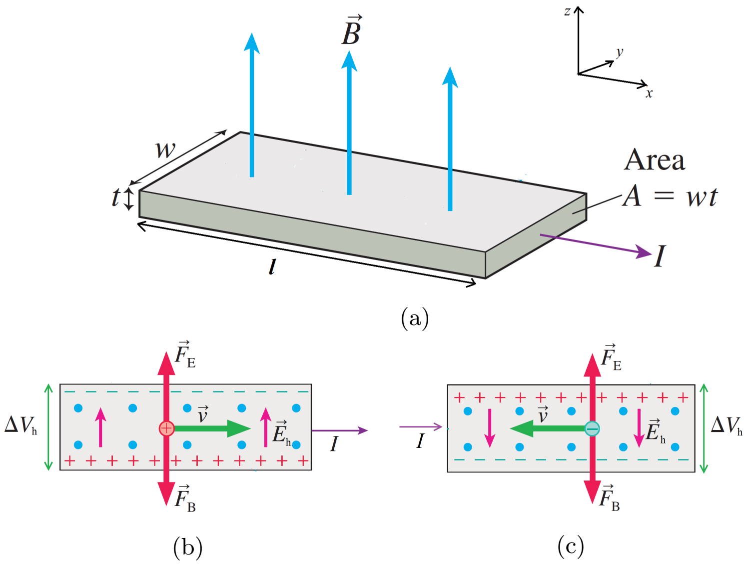


Figure 1: The Hall probe with (b) positive charge carriers (c) negative charge carriers.

separated by an **energy gap** which consists of energy levels that are forbidden. The lower band is called the **valence band**, while the upper band is called the **conduction band**. A fully-occupied band cannot carry any electric current. For a pure semiconductor, all the electron energy states (called the **orbitals**) in the valence band are occupied when the temperature reaches 0 Kelvin. When an electron in the valence band gains enough energy and moves into the conduction band, it will leave behind an unfilled orbital, called a **hole**. Electrons in the conduction band are called **conduction electrons**. Both the conduction electrons and the holes are charge carriers in a semiconductor, carrying a charge of $-e$ and $+e$ respectively. In a pure semiconductor, the number of electrons and the number of holes should balance each other. However, it is possible to add a small amount of another material, called the **dopant**, to a pure semiconductor. A dopant that can donate electrons to the original semiconductor is called a **donor**, while one that can accept electrons from the original semiconductor is called an **acceptor**. Therefore, adding a dopant can change the original balance between the numbers of the conduction electrons and the holes. A semiconductor with more conduction electrons than holes is called an **n-type semiconductor**, while one with more holes than conduction electrons is called a **p-type semiconductor**.[6,7]

2.2 Hall Effect

The Hall effect allows us to determine whether the charge carriers in an electric current are positive or negative, by supplying an external magnetic field to a current carrying device. Figure 1(a) shows a flat, cuboid-shaped device, whose width, thickness, and length are w, t and l, respectively. For our purpose we will call it a conductor (or a **Hall probe**), which does not necessarily imply a high conductivity. Suppose there is only one type of charge carriers in this conductor, either positive or negative, whose motion is described by the Drude model[8]. As shown in Figure 1(a), the current I is moving toward the +x direction, and there is a uniform external magnetic field B pointing at the +z direction across the conductor. Then as shown in Figure 1(b) and 1(c), positive charge carrier would be deflected by the magnetic field toward the -y direction, while negative charge carriers would also be deflected toward the -y direction. Therefore, as more and more charge carriers get deflected by the magnetic field and accumulate at the bottom (-y) surface of the conductor, the top (+y) surface will accumulate an excess of the opposite charge. As a result, the top and bottom surfaces will act like a parallel-plate capacitor and create a potential difference ΔV_h (called the **Hall voltage**) transverse to the two surfaces. Meanwhile, an electric field $E_h = \frac{\Delta V_h}{w}$ will be established between the top and bottom surfaces, and the electric force on a moving charge carrier due to this electric field will eventually balance the magnetic force on the same charge carrier so that the charge carrier will stop being deflected by the external magnetic field and the Hall voltage will reach equilibrium. This voltage can be measured, whose sign can be used to determine whether the charge carrier is positive or negative.

Now, suppose each charge carrier carries a charge q and the drift speed of the charge carrier is v_d. Then, the magnetic force on a charge carrier between the top and bottom surfaces is $F_B = qv_dB$, while the electric force due to E_h on the same charge carrier is $F_E = qE_h$. At equilibrium, we have $F_B = F_E$, so $v_dB = E_h = \frac{\Delta V_h}{w}$. Therefore, the steady-state Hall voltage is

$$\Delta V_h = wv_dB \tag{1}$$

Notice that

$$v_d = \frac{J}{\rho} = \frac{I/wt}{\rho} \tag{2}$$

where J is the current density, ρ is the charge density of the charge carriers, wt is the cross-sectional area of the conductor, and I is the current. Substituting Equation 2 into Equation 1, we obtain

$$\Delta V_h = \frac{IB}{t\rho} \tag{3}$$

Therefore, we can see that the steady-state Hall voltage is proportional to the product of the current and the external magnetic field, assuming that the thickness of the conductor and the charge density of the charge carriers remain constant with respect to the varying current and magnetic field.[9] The **Hall coefficient**, R_H, is defined to be

$$R_H \equiv \frac{V_h}{IB} \times t \tag{4}$$

Then according to Equation 3, we can see that $V_h \propto IB$ is equivalent to the Hall coefficient of the conductor being constant. Although our derivation is based on a conductor with a single carrier type, it turns out that, for a fixed temperature, the constancy of the Hall coefficient R_H and the relation $V_h \propto IB$, is generally true for materials including metals and semiconductors.[10]

2.3 Magnetoresistance in Hall Effect

For most types of materials, we have

$$\boldsymbol{J} = \sigma \boldsymbol{f} \tag{5}$$

where $\boldsymbol{J}$ is the current density through the material, σ is an empirical constant for the material called the conductivity, and $\boldsymbol{f}$ is force per unit charge. Typically, the force that drives the charge carriers in a current is the electric force due to the applied voltage, so Equation 5 becomes

$$\boldsymbol{J} = \sigma \boldsymbol{E} \tag{6}$$

which is Ohm's law.[8]

Now consider a Hall probe with a single charge carrier type (as shown in Figure 1), then divide both sides of Equation 6 by ρ, the charge carrier charge density, we obtain

$$\boldsymbol{v}_d = \frac{\sigma}{\rho}\boldsymbol{E} = \mu \boldsymbol{E} \tag{7}$$

where $\mu \equiv \frac{v_d}{E}$ is called the **mobility** of the charge carrier. From Equation 7, we can see that $\mu \propto \sigma$. Suppose the Hall probe has resistance R_p, then

$$R_p \propto \frac{1}{\sigma} \propto \frac{1}{\mu} \tag{8}$$

Now, consider a uniform external magnetic field $\boldsymbol{B}$ applied on the Hall probe (see Figure 1). Then, according to Equation 5 and the Lorentz' force law, Ohm's law should be modified to be

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{9}$$

where $\boldsymbol{v}$ is the velocity of a charge carrier. Let V_p be the applied voltage across the Hall probe. Then, due to the extra term $\boldsymbol{v} \times \boldsymbol{B}$ in Equation 9 compared to Equation 6, the resistance of the Hall probe $R_p = \frac{V_p}{I} = \frac{El}{Jwt}$ will change. Therefore, applying an external magnetic field on the Hall probe will change the Hall probe's resistance. Using the Drude model and Equation 9, the resistance as a result of the magnetic field can be derived to be[10]

$$R_p(B) = \frac{l}{wt} \times \frac{1 + \mu^2 B^2}{\rho\mu} \tag{10}$$

Therefore, $R_p(B) \propto (1 + \mu^2 B^2)$. Notice that this relationship applies only when there is a single charge carrier type. If the Hall probe is a semiconductor with both conduction electrons and holes as charge carriers, then the relation between the resistance of the Hall probe and the magnetic field will be more complicated.[11]

3 Experiment

3.1 Experimental setup

The apparatus of this experiment mainly consists of a main circuit and a pair of electromagnets. Figure 1 gives a schematic diagram of the Hall probe, which is a semiconductor of a flat cuboid shape.

As shown in Figure 2, the main circuit of our apparatus is comprised of the Hall probe and a 2000 Ohm resistor connected in series, subject to an adjustable total voltage for the entire circuit. Both the Hall probe and the 2000 Ohm resistor are monitored by their respective voltmeters, where the Hall voltage (transverse to the direction of the current through the Hall probe) as well as the longitudinal voltage across the 2000 Ohm resistor (along the direction of the current) are measured. Besides, the Hall probe is in contact with a heating device, whose temperature is adjustable by another power supply and readable from a thermometer. With this, we will be able to read the temperature of the Hall probe.

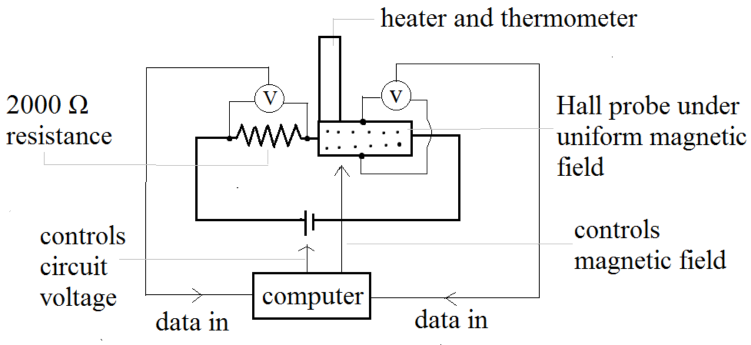


Figure 2: Apparatus setup for the Hall effect experiment.

Not explicitly shown in Figure 2, the electromagnets are a pair of cylindrical coils that is able to supply a nearly uniform magnetic field in between. It is controlled by a power supply, whose voltage can be adjusted. During the experiment, we would insert the Hall probe between the electromagnets so that it would experience the nearly uniform magnetic field perpendicular to its largest surfaces.

In this experiment, there is a computer interface that controls both the **magnet voltage** and the total voltage of the circuit. Moreover, the readings of the **resistor voltage** and the Hall voltage (as mentioned above) are recorded on the same computer interface. The computer interface is programmed so that we are able to set up a range of desired magnet voltages and a range of desired resistor voltages. With these, the computer interface is able to run the circuit automatically at combinations of these two ranges of initial settings and record the total circuit voltage and the Hall voltage along with the set magnet voltage and resistor voltage. Notice that the computer is actually controlling the total circuit voltage instead of the resistor voltage. It will be treating the Hall probe as having a constant resistance (that is coded in the program) so that it can calculate and assign the corresponding total circuit voltages in response to the resistor voltage values set up by us. Therefore, since the resistance of the Hall probe can change (though not too much) during the experiment (due to the changing temperature or magnetic field), the actual measured resistor voltage will be close to but not equal to the resistor voltage values initially set up by us.

Although the computer interface can automate much of the experiment, the heater for the Hall probe should still be manually monitored by our watching the thermometer and adjusting the heater's power supply constantly in order to maintain a desired temperature for the Hall probe.

Lastly, we would use a magnetometer to calibrate the magnetic field between the two electromagnets with respect to the magnet voltage.

3.2 Limitations

Since this experiment was conducted in an undergraduate laboratory, we were not able to extensively manipulate some of the experimental apparatus. In particular, the Hall probe has all its connection wires sealed inside a non-transparent tape so that we were not able to determine in which direction the Hall voltage was measured. Moreover, we were not able to measure the Hall probe's dimensions (length, width, and thickness) due to the taping as well as the Hall probe's specific configuration. Lastly, the specific information about the Hall probe as a semiconductor was not available so that we were not able to know what material the Hall probe is made up of other than it being a semiconductor.

3.3 Measurement methodology

In this experiment, we desired to verify the $V_h \propto BI$ relation and examine the magnetoresistance of the Hall probe. Therefore, we will need to collect values of the Hall voltage V_h, the magnetic field B, the current I through the Hall probe, and the resistance R_p of the Hall probe. According to Section 3.1, the computer interface will record the values of B and V_h directly. Since it will also record the voltage V_r across the resistor and we know the resistor has a fixed resistance of R=2000 Ohm, the current through the resistor (and thus through the Hall probe) can be calculated as $I = \frac{V_r}{R}$ according to Ohm's law. Since the computer will also record the total circuit voltage V_tot and the Hall probe is connected in series with the resistor, the longitudinal voltage across the Hall probe will be $V_p = V_{tot} - V_r$. Since we know the current I, the resistance of the Hall probe can again be calculated with Ohm's law as $R_P = \frac{V_p}{I}$. Therefore, we can obtain the values of all the needed quantities for data analysis.

3.4 Procedure

First, we used the magnetometer to calibrate the magnetic field between the two electromagnets with respect to the magnet voltage. Then, we conducted three trials of measurements corresponding to three different temperatures, 22°C, 69°C, and 101°C, of the Hall probe (the temperature did fluctuate by around 1°C during each trial). As mentioned in Section 3.1 and 3.3, for each trial, we would measure the total circuit voltage, the resistor voltage, and the Hall voltage, with the Hall probe under various magnetic fields. For magnetic fields corresponding to 5, 10, 15, 20, and 25 volts of the magnet voltage, we inserted the Hall probe in between the electromagnets such that the magnetic fields would penetrate the largest surface of the Hall probe perpendicularly, as per Figure 1. For the measurement with respect to the zero magnetic field, we took the Hall probe outside the electromagnets and turned the magnets off. For each magnetic field including zero, we conducted measurements for the resistor voltage roughly from 0 volts to 5 volts, with a step of 0.5 volts. In summary, we conducted our measurements for three temperatures of the Hall probe, for each temperature we put the Hall probe under 6 different magnetic fields, and for each magnetic field we collected data for 11 resistor voltages. All the measurements were completed through the computer interface except that we read and monitored the temperatures with the thermometer.

4 Data analysis

4.1 Magnetic field calibration

The magnetic fields at the magnet voltages of 5, 10, 15, 20, and 25 volts were measured by the magnetometer to be 0.107, 0.205, 0.294, 0.378, and 0.457 tesla, respectively.

4.2 Verifying the $V_h \propto BI$ relation and the constancy of the Hall coefficient

According to Section 3.4, we have obtained data for the Hall voltage V_h for different combinations of the temperature T, the magnetic field B, and the resistor voltage V_r. As described in Section 3.3, we can obtain the corresponding values of current I from the resistor voltage V_r and the resistor's resistance $R = 2000\Omega$. The plots for V_h vs. BI are shown in Figure 3, with the respective least squares linear fitting functions shown on top of the corresponding plots. For 22°C, 69°C, and 101°C, $\frac{V_h}{BI}$ obtained from the linear regression are $192.596 \pm 0.258 \mathrm{m}^2\mathrm{C}^{-1}$, $189.879 \pm 0.268 \mathrm{m}^2\mathrm{C}^{-1}$, and $186.949 \pm 0.275 \mathrm{m}^2\mathrm{C}^{-1}$, where the uncertainties come from the linear regression standard errors and the unit C denotes Coulomb. In this experiment, however, the magnetic field measurements are only accurate to three significant digits. Therefore, the fitting estimators for $\frac{V_h}{BI}$ should only have an accuracy up to three significant figures as $193\mathrm{m}^2\mathrm{C}^{-1}$, $190\mathrm{m}^2\mathrm{C}^{-1}$, and $187\mathrm{m}^2\mathrm{C}^{-1}$ respectively with inherent uncertainties of about $\pm 1\mathrm{m}^2\mathrm{C}^{-1}$.

As we can see in Figure 3(b), 3(c), and 3(d), at each temperature, the Hall voltage V_h is highly proportional to BI. Such a high proportionality is also supported by the relatively low standard errors (even lower than the inherent uncertainties due to the measurement precision of B) for the estimators of $\frac{V_h}{BI}$ obtained from the least squares fitting. Therefore, at the range of magnetic field and current that we examined, $\frac{V_h}{BI}$ is well constant for each temperature. Remember from Section 2.2 that $\frac{V_h}{BI} = \frac{R_H}{t}$, where R_H is the Hall coefficient of the semiconductor and t is its thickness along the direction of the magnetic field. In this experiment, since the thickness of the Hall probe will not change at a fixed temperature, the constancy of $\frac{V_h}{BI}$ at each temperature indicates the constancy of the Hall coefficient R_H at each temperature. This result is consistent with the theoretical descriptions in Section 2.2.

From Figure 3(a) as well as our fitting results for $\frac{V_h}{BI}$, we can see that across the three temperatures, the Hall coefficient, though not changing much, decreases slightly as the temperature increases.

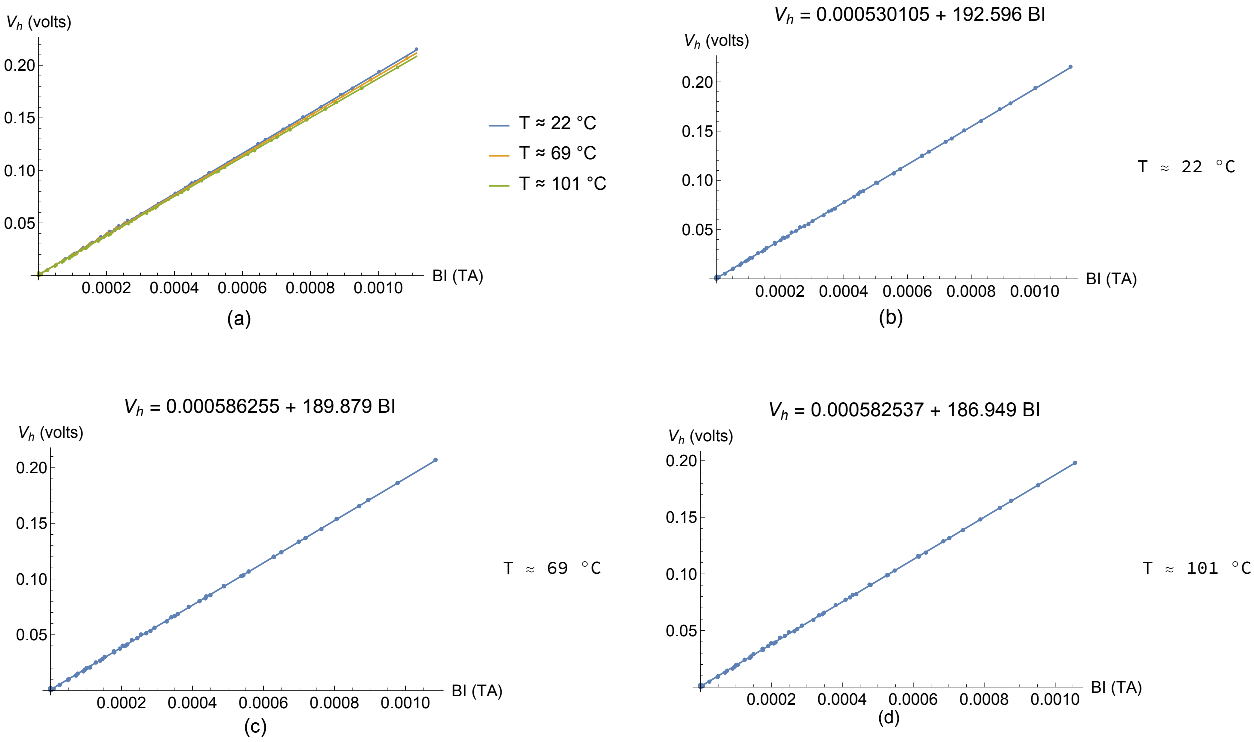


Figure 3: The Hall voltage (V_h) vs. magnetic field (B)×current (I) at (a) all three temperatures (b) 22°C (c) 69°C (d) 101°C. The linear regression lines for (a), (b), and (c) are drawn on top of the respective plots without indicating the units explicitly.

4.3 Examining the magnetoresistance of the Hall probe

According to Section 3.3, we were able to obtain the longitudinal Hall probe voltage V_p as well as the current I straightforwardly from the collected data. Since we want to examine the relation between B and the Hall probe's resistance R_p for each temperature, we will find the Hall probe resistance by conducting linear fits on the data of V_p vs. I at all the measured magnetic fields and temperatures. This process is essentially the same as what we did previously for finding $\frac{V_h}{BI}$ at different temperatures, and the slope of a V_p vs. I fitting line will be the estimator for R_p at a specific magnetic field and temperature.

The calculated resistances of the Hall probe are shown in Figure 4 as well as their uncertainties obtained from the least squares regression standard errors. The resistances of the Hall probe around 22°C, 69°C, and 101°C are around 750Ω, 830Ω, and 900Ω respectively. As we can see from the figure, compared to the change of the Hall probe resistance across different temperatures, the resistance of the Hall probe does not vary much around the same temperature as the magnetic field increases. For the varying magnetic fields between 0T and about 0.5T, the standard deviations of the measured Hall probe resistances around the three temperatures are all less than 10Ω.

Remember from Section 2.3, a material with a single charge carrier type will have a resistance R_p proportional to $1 + \mu^2 B^2$, where μ is the mobility of the charge carrier. If this were the case for our Hall probe, then we should have

$$\Delta R_p \equiv R_p(B) - R_p(B = 0) \propto B^2 \tag{11}$$

where $R_p(B)$ is the resistance of the Hall probe at a magnetic field B. To examine whether such a relation

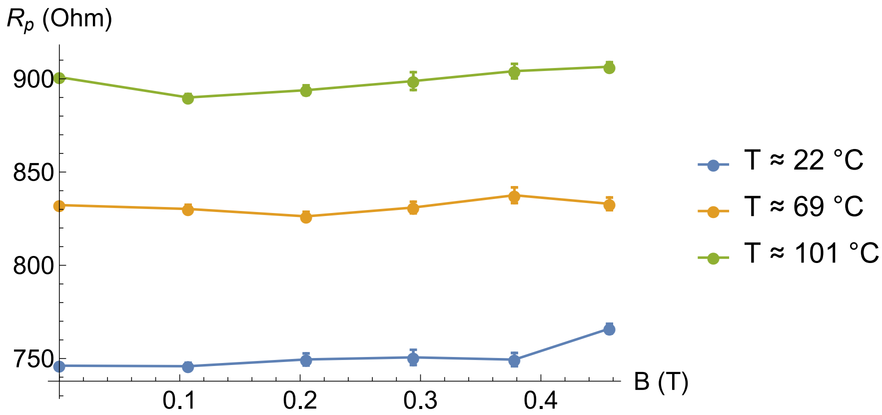
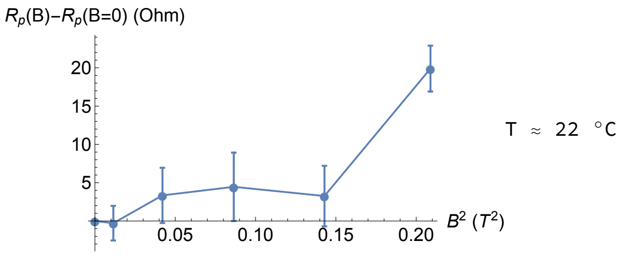
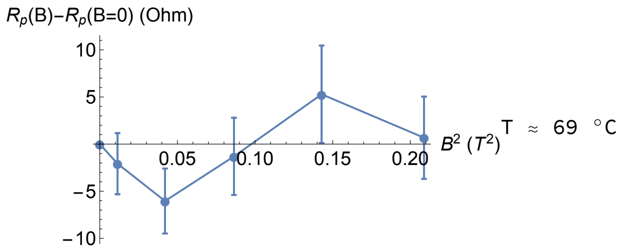


Figure 4: The resistances R_p of the Hall probe at various magnetic fields and different temperatures.

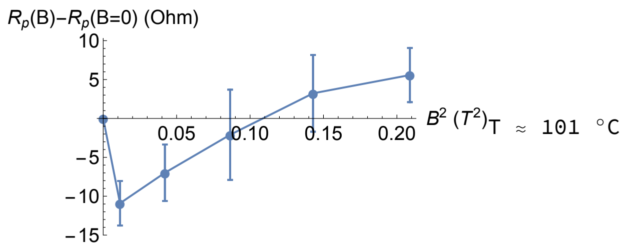
exists, plots of ΔR_p vs. B^2 at the three temperatures are given in Figure 5. The error bars for ΔR_p in each plot are derived from the uncertainties for R_p in Figure 4.



(a)



(b)



(c)

Figure 5: $R_p(B) - R_p(B = 0)$ vs. B^2 at (a) 22°C (b) 69°C (c) 101°C

As we can see in Figure 5, ΔR_p does not seem to be proportional to B^2 at our measured magnetic field range for all three temperatures. However, remember that the temperature of the Hall probe did vary by about 1°C for the measurements at each temperature. To calculate the effect of such a temperature variation on the Hall probe resistance, we will first find an approximate functional relation between the temperature and the Hall probe resistance. Figure 6 shows the Hall probe resistance vs. temperature, taking the Hall probe's resistance to be 750Ω, 830Ω, and 900Ω at the three temperatures.

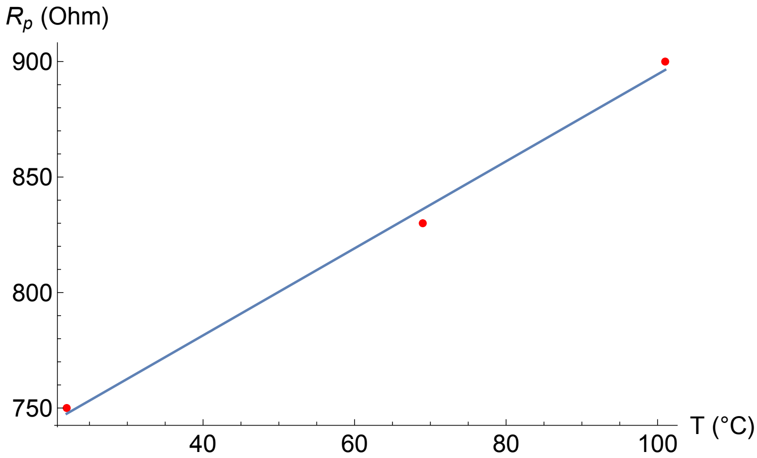


Figure 6: Hall probe resistance R_p vs. temperature T

As we can see, although there are only three data points, they are well fit by a straight line. Therefore, for our purpose, it should be good enough to approximate the Hall probe resistance as a linear function of the temperature T. Then, for a 1°C temperature variation, the corresponding resistance variation is $\frac{900-750}{101-22}\Omega \approx 2\Omega$. Re-examining Figure 5, we can see that such a 2Ω variation of the Hall probe resistance is comparable to the change of ΔR_p between two nearly magnetic fields. In this regard, our data of ΔR_p vs. B^2, as presented in Figure 5, are not clean enough to isolate the relation between ΔR_p and B^2. Nevertheless, even after considering the effect of the slightly changing temperature, the plots in Figure 5 still clearly rule out a proportionality relation between ΔR_p and B^2, because all the three plots contain relatively large fluctuations or jumps of ΔR_p that are inconsistent with a proportionality relation and cannot be completely accounted for by the 2Ω effect of the slightly-changing temperature and the respective uncertainties of ΔR_p. Therefore, we conclude that the Hall probe does not exhibit a simple magnetoresistance relation as in Equation 10. This is expected for a semiconductor who does not necessarily have an over-dominant charge carrier type.

5 Conclusion

In this experiment, we tried to verify the relation of $V_h \propto BI$ and the constancy of the Hall coefficient in the Hall effect with a semiconductor whose specific type is unknown. We also examined the magnetoresistance of the semiconductor and used that to deduce whether the semiconductor has an over-dominant charge carrier type. For temperatures of around 22°C, 69°C, and 101°C, $\frac{V_h}{BI}$ is found to be $193\text{m}^2\text{C}^{-1}$, $190\text{m}^2\text{C}^{-1}$, and $187\text{m}^2\text{C}^{-1}$ respectively, each with an uncertainty of about $1\text{m}^2\text{C}^{-1}$ that is dominantly due to the accuracy of the measured magnetic fields instead of the respective standard errors from least squares regression. Therefore, V_h is well proportional to BI, which is consistent with our theoretical prediction in Section 2.2. This result is equivalent to the constancy of the Hall coefficient of the semiconductor at a given temperature. As for the magnetoresistance of the semiconductor, we did not find a $R_p \propto 1 + (\mu B)^2$ relation, which means that our semiconductor does not have a single over-dominant charge carrier type. This is reasonable for a semiconductor.

As described in Section 4.3, the major source of error in this experiment is part of the uncertainty of the Hall probe resistance due to the slightly changing temperature while we varied the magnetic field. To minimize this effect, we could use a heater that has a temperature control function so that we could keep the Hall probe at a rather constant temperature when varying the magnetic fields.

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