

Team **Bozonghereh** Notebook

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Graph

2-SAT(SCC-Topological Sort)

```
//O(E) 1-base
//if you want use SCC only ignore
//last method booleans are 1 to n / ~i = i+n
//add edge with inverse! a or b == ~b -> a == == a -> ~b
int n,m,cnt = 1,verticesScc[MAX_N];
vector<int>g[MAX_N],g1[MAX_N], tSort, scc[MAX_N];
bool mark[MAX_N],ans[MAX_N],satisfy = true;
void topologicalSort(int v) {
    mark[v] = true;
    for(int u:g[v])
        if(!mark[u])
            topologicalSort(u);
    tSort.push_back(v);
}
void dfs(int v) {
    mark[v] = true;
    scc[cnt].push_back(v);
    verticesScc[v] = cnt;
    for(int u:g1[v])
        if(!mark[u])
            dfs(u);
}
void SCC() {
    for(int i = 1 ; i <= n ; i++)
        if(!mark[i])
            topologicalSort(i);
    reverse(tSort.begin(), tSort.end());
    memset(mark, false, sizeof mark);
    for(int v:tSort)
        if(!mark[v]) {
            cnt++;
            dfs(v);
        }
    cnt--;
}
int inverse(int t) {
    if(t <= n)
        return t+n;
    return t-n;
}
bool check() {
    for(int i = 1 ; i <= n ; i++)
        if(verticesScc[i] == verticesScc[i+n])
```

```
        return false;
    return true;
}
//one of the expressions are (x or y)
void addExpresion(int x,int y) {
    g[inverse(x)].push_back(y);
    g[inverse(y)].push_back(x);
    g1[y].push_back(inverse(x));
    g1[x].push_back(inverse(y));
}
void sat() {
    n *= 2;
    SCC();
    n/=2;
    if(!check()) {
        satisfy = false;
        return;
    }
    memset(mark, false, sizeof mark);
    reverse(tSort.begin(),tSort.end());
    for(int v:tSort) {
        if(v <= n && !mark[v]) {
            mark[v] = true;
            ans[v] = true;
        }
        if(v > n && !mark[v-n])
            mark[v-n] = true;
    }
}
```

Topological Sort(Second Kind)

```
//O(E log n) 1-base
for(int i = 1 ; i <= n ; i++)
    if(deg[i] == 0)
        s.insert(i);
while(s.size() > 0) {
    int v = *s.begin();
    s.erase(s.begin());
    tSort.push_back(v);
    for(int u:g1[v]) {
        deg[u]--;
        if(deg[u] == 0)
            s.insert(u);
    }
}
```

Bellman-Ford

```
//O(VE) 1-base
//Directional-we can solve inequalities like :  $x(i) - x(j) \leq c$  we make
//one node for each  $x(i)$  we make a source with edge weight 0 to all other
//nodes we put a edge from j to i with weight c bellman ford from source
//x[i] is dis[i]
ll n,m,dis[MAX_N];
//first Node, second Node, weight
vector<pair<pair<ll,ll>,ll > > edges;
bool negativeCycle;
void bellmanFord() {
    memset(dis, 31, sizeof dis);
    dis[1] = 0;
    for(int i = 1 ; i <= n ; i++)
        for(pair<pair<ll,ll>,ll > e:edges)
            if(dis[e.first.first] + e.second < dis[e.first.second])
                dis[e.first.second] = dis[e.first.first] + e.second;

    for(pair<pair<ll,ll>,ll > e:edges)
        if(dis[e.first.first] + e.second < dis[e.first.second])
            negativeCycle = true;
}
```

Floyd-Warshall

```
//O(V^3) 1-base
int n,m,dis[MAX_N][MAX_N],g[MAX_N][MAX_N];
//directional, g[i][j] = INF
void floydWarshall() {
    for(int i = 0 ; i < MAX_N ; i++)
        for(int j = 0 ; j < MAX_N ; j++)
            dis[i][j] = min(INF,g[i][j]);
    for(int i = 1 ; i <= n ; i++)
        dis[i][i] = 0;
    for(int k = 1 ; k <= n ; k++)
        for(int i = 1 ; i <= n ; i++)
            for(int j = 1 ; j <= n ; j++)
                if(dis[i][j] > dis[i][k] + dis[k][j])
                    dis[i][j] = dis[i][k] + dis[k][j];

    //for using minimax and maximin minimax :
    //dis[i][j] = min(dis[i][j],max(dis[i][k],dis[k][j]))
    //dis[i][j] + dis[j][i] < 0 -> negative cycle
}
```

DSU

```
//O(log(n)) no-base
```

```
int n, m, par[MAX_N], sz[MAX_N];
void create(int v) {
    par[v] = v;
    sz[v] = 1;
}
int find_par(int v) {
    if(par[v] == v)
        return v;
    par[v] = par[par[v]];
    return find_par(par[par[v]]);
}
void join(int v, int u) {
    u = find_par(u);
    v = find_par(v);
    if(u == v)
        return;
    if(sz[v] < sz[u])
        swap(u, v);
    par[u] = v;
    sz[v] += sz[u];
}
```

Flow (Edmonds-Karp)

```
//O((min(f|E|, |V||E|^2)) 1-base
int n,m,s,t,c[MAX_N][MAX_N],cf[MAX_N][MAX_N],par[MAX_N],maxFlow;
bool mark[MAX_N];
queue<int>q;
bool bfs() {
    memset(mark, false, sizeof mark);
    while(q.size()) q.pop();
    q.push(s);
    mark[s] = true;
    par[s] = -1;
    while(q.size() > 0) {
        int a = q.front();
        q.pop();
        for(int i = 1 ; i <= n ; i++) {
            if(!mark[i] && cf[a][i] > 0) {
                par[i] = a;
                mark[i] = true;
                q.push(i);
            }
        }
    }
}
```

```

    return mark[t];
}
void edmondsKarp() {
    while(bfs()) {
        vector<int>path;
        int tmp = t;
        while(tmp != -1) {
            path.push_back(tmp);
            tmp = par[tmp];
        }
        reverse(path.begin(),path.end());
        int MIN = INF;
        for(int i = 0 ; i < path.size() -1 ; i++) {
            int a = path[i];
            int b = path[i+1];
            MIN = min(MIN,cf[a][b]);
        }
        for(int i = 0 ; i < path.size() -1 ; i++) {
            int a = path[i];
            int b = path[i+1];
            cf[a][b] -= MIN;
            cf[b][a] += MIN;
        }
        maxFlow += MIN;
    }
}
void addDirectionalEdge(int u, int v, int cap) {
    c[u][v] += cap, cf[u][v] += cap;
}
void addBiDirectionalEdge(int u, int v, int cap) {
    c[u][v] += cap, c[v][u] += cap, cf[u][v] += cap, cf[v][u] += cap;
}

```

Flow (Dinic)

//O(V²*E) no-base

```

struct Edge {
    int to,reverseIndex,cap,flow;
};
int n,m,s,t,maxFlow,dis[MAX_N];
vector<Edge>g[MAX_N];

```

```

queue<int>q;
void addEdge(int u, int v, int cap) {
    Edge x,y;
    x.to = v, y.to = u;
    x.cap = cap, y.cap = 0;
    x.flow = y.flow = 0;
    x.reverseIndex = g[v].size();
    y.reverseIndex = g[u].size();
    g[u].push_back(x);
    g[v].push_back(y);
}
bool bfs() {
    memset(dis, 31, sizeof dis);
    while(q.size()) q.pop();
    q.push(s);
    dis[s] = 0;
    while(q.size() > 0) {
        int v = q.front();
        q.pop();
        for(Edge x:g[v]) {
            int u = x.to;
            if(dis[u] == INF && x.flow < x.cap) {
                dis[u] = dis[v]+1;
                q.push(u);
            }
        }
    }
    return (dis[t] != INF);
}
int dfs(int v,int f) {
    if(v == t)
        return f;
    for(int i = 0 ; i < g[v].size() ; i++) {
        Edge &x = g[v][i];
        int u = x.to;
        if(x.cap <= x.flow) continue;
        if(dis[u] == dis[v]+1) {
            int tmp = dfs(u,min(f,x.cap-x.flow));
            if(tmp > 0) {
                x.flow += tmp;
                g[u][x.reverseIndex].flow -= tmp;
            }
        }
    }
}

```

```

        return tmp;
    }
}
return 0;
}
void dinic() {
    while(bfs())
        while(int tmp = dfs(s,INF))
            maxFlow += tmp;
}

```

Flow(Push-Relabel)

```

//O(V^3) 0-base
struct Edge {
    int from, to, cap, flow, index;
    Edge(int from, int to, int cap, int flow, int index) :
        from(from), to(to), cap(cap), flow(flow), index(index) {}
};
int n, m, s, t, maxFlow, dist[MAX_N], active[MAX_N], CNT[2*MAX_N];
vector<Edge> g[MAX_N];
ll excess[MAX_N];
queue<int> q;
void CLEAR() {
    maxFlow = 0;
    while(q.size()) q.pop();
    memset(CNT, 0, sizeof CNT);
    for(int i = 0 ; i < MAX_N ; i++) {
        g[i].clear();
        dist[i] = active[i] = excess[i] = 0;
    }
}
void AddEdge(int from, int to, int cap) {
    g[from].push_back(Edge(from, to, cap, 0, g[to].size()));
    if (from == to) g[from].back().index++;
    g[to].push_back(Edge(to, from, 0, 0, g[from].size() - 1));
}
void Enqueue(int v) {
    if (!active[v] && excess[v] > 0) { active[v] = true; q.push(v); }
}

```

```

void Push(Edge &e) {
    int amt = int(min(excess[e.from], ll(e.cap - e.flow)));
    if (dist[e.from] <= dist[e.to] || amt == 0) return;
    e.flow += amt;
    g[e.to][e.index].flow -= amt;
    excess[e.to] += amt;
    excess[e.from] -= amt;
    Enqueue(e.to);
}
void Gap(int k) {
    for (int v = 0; v < n; v++) {
        if (dist[v] < k) continue;
        CNT[dist[v]]--;
        dist[v] = max(dist[v], n+1);
        CNT[dist[v]]++;
        Enqueue(v);
    }
}
void Relabel(int v) {
    CNT[dist[v]]--;
    dist[v] = 2*n;
    for (int i = 0; i < g[v].size(); i++)
        if (g[v][i].cap - g[v][i].flow > 0)
            dist[v] = min(dist[v], dist[g[v][i].to] + 1);
    CNT[dist[v]]++;
    Enqueue(v);
}
void Discharge(int v) {
    for (int i = 0; excess[v] > 0 && i < g[v].size(); i++)
        Push(g[v][i]);
    if (excess[v] > 0) {
        if (CNT[dist[v]] == 1)
            Gap(dist[v]);
        else
            Relabel(v);
    }
}
void push_relabel() {
    CNT[0] = n-1;
    CNT[n] = 1;
    dist[s] = n;
}

```

```

    active[s] = active[t] = true;
    for (int i = 0; i < g[s].size(); i++) {
        excess[s] += g[s][i].cap;
        Push(g[s][i]);
    }
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        active[v] = false;
        Discharge(v);
    }
    ll totflow = 0;
    for (int i = 0; i < g[s].size(); i++) totflow += g[s][i].flow;
    maxFlow = totflow;
}

```

Min Cost Max Flow

```

//O(Unknown) 0-base
struct Edge {
    int to, f, cap, cost, rev;
};
int n,m,s,t;
int prio[MAX_N], curflow[MAX_N], prevedge[MAX_N], prevnode[MAX_N],
q[MAX_N], pot[MAX_N];
bool inqueue[MAX_N];
vector<Edge> graph[MAX_N];
void CLEAR() {
    for(int i = 0 ; i < MAX_N ; i++) {
        prio[i] = curflow[i] = prevedge[i] = prevnode[i] = q[i] =
pot[i] = inqueue[i] = 0;
        graph[i].clear();
    }
}
void addEdge(int s, int t, int cap, int cost) {
    Edge a = {t, 0, cap, cost, graph[t].size()};
    Edge b = {s, 0, 0, -cost, graph[s].size()};
    graph[s].push_back(a);
    graph[t].push_back(b);
}
void bellmanFord(int s, int dist[]) {

```

```

    for(int i = 0 ; i < MAX_N ; i++)
        dist[i] = INF;
    dist[s] = 0;
    int qt = 0;
    q[qt++] = s;
    for (int qh = 0; (qh - qt) % n != 0; qh++) {
        int u = q[qh % n];
        inqueue[u] = false;
        for (int i = 0; i < (int) graph[u].size(); i++) {
            Edge &e = graph[u][i];
            if (e.cap <= e.f) continue;
            int v = e.to;
            int ndist = dist[u] + e.cost;
            if (dist[v] > ndist) {
                dist[v] = ndist;
                if (!inqueue[v]) {
                    inqueue[v] = true;
                    q[qt++ % n] = v;
                }
            }
        }
    }
}

```

```

pair<int, int> minCostFlow(int s, int t, int maxf) {
    // bellmanFord can be safely commented if edges costs are non-negative
    //bellmanFord(s, pot);
    int flow = 0;
    int flowCost = 0;
    while (flow < maxf) {
        priority_queue<ll, vector<ll>, greater<ll> > q;
        q.push(s);
        for(int i = 0 ; i < MAX_N ; i++)
            prio[i] = INF;
        prio[s] = 0;
        curflow[s] = INF;
        while (!q.empty()) {
            ll cur = q.top();
            int d = cur >> 32;
            int u = cur;
            q.pop();

```

```

    if (d != prio[u])
        continue;
    for (int i = 0; i < (int) graph[u].size(); i++) {
        Edge &e = graph[u][i];
        int v = e.to;
        if (e.cap <= e.f) continue;
        int nprio = prio[u] + e.cost + pot[u] - pot[v];
        if (prio[v] > nprio) {
            prio[v] = nprio;
            q.push(((ll) nprio << 32) + v);
            prevnode[v] = u;
            prevedge[v] = i;
            curflow[v] = min(curflow[u], e.cap - e.f);
        }
    }
}
if (prio[t] == INF)
    break;
for (int i = 0; i < n; i++)
    pot[i] += prio[i];
int df = min(curflow[t], maxf - flow);
flow += df;
for (int v = t; v != s; v = prevnode[v]) {
    Edge &e = graph[prevnode[v]][prevedge[v]];
    e.f += df;
    graph[v][e.rev].f -= df;
    flowCost += df * e.cost;
}
}
return make_pair(flow, flowCost);
}

```

Bipartite Matching

```

//O(VE) 1-base
int n1,n2,match[MAX_N];
bool mark[MAX_N];
vector<int>g[MAX_N];
bool dfs(int v) {
    if(mark[v]) return false;
    mark[v] = true;

```

```

    for(int u:g[v]) {
        if(match[u] == -1 || dfs(match[u])) {
            match[u] = v, match[v] = u;
            return true;
        }
    }
    return false;
}
void optimize() {
    for(int i = 1 ; i <= n1 ; i++) {
        for(int v:g[i]) {
            if(match[v] == -1) {
                match[i] = v, match[v] = i;
                break;
            }
        }
    }
}
void MATCH() {
    memset(match, -1, sizeof match);
    optimize();
    for(int i = 1 ; i <= n1 ; i++) {
        if(match[i] != -1) continue;
        memset(mark, false, sizeof mark);
        dfs(i);
    }
}

```

Weighted Bipartite Matching

```

//O(Unknown) 0-base
//if we want max weight perfect matching then the edges that are not in
the //graph should have -INF weight if we want just max weight matching
then //the edges that are not in the graph should have 0 weight if we want
min //weight matching then we negative the edges weight
int a[MAX_N][MAX_N], uable[MAX_N], dlable[MAX_N], n,m;
int umatch[MAX_N], dmatch[MAX_N], umark[MAX_N], dmark[MAX_N];
bool dfs(int k){
    umark[k]=1;
    for(int i=0; i<n; i++) if(dmark[i]==0 &&
uable[k]+dlable[i]==a[k][i]){
        dmark[i]=1;

```

```

bool done=0;
if(dmatch[i]==-1){
    done=1;
}else{
    if(dfs(dmatch[i])) done=1;
}
if(done){
    umatch[k]=i;
    dmatch[i]=k;
    return 1;
}
}
return 0;
}

void mwmatching(){
    for(int i = 0 ; i < MAX_N ; i++)
        ulable[i] = dlable[i] = 0;
    for(int i=0; i<n; i++)
    for(int j=0; j<n; j++)
        ulable[i]=max(ulable[i], a[i][j]);
    for(int i = 0 ; i < MAX_N ; i++)
        umatch[i] = dmatch[i] = -1;
    for(int size=0; size<n; ){
        bool done=1;
        while(done){
            done=0;
            for(int i = 0 ; i < MAX_N ; i++)
                umark[i] = dmark[i] = 0;
            for(int i=0; i<n; i++) if(umark[i]==0 && umatch[i]==-1)
                if(dfs(i)){
                    done=1;
                    size++;
                }
        }
        int eps=(int)(1e9);
        for(int i=0; i<n; i++) if(umark[i])
        for(int j=0; j<n; j++) if(!dmark[j])
            eps=min(eps, ulable[i]+dlable[j]-a[i][j]);
        for(int i=0; i<n; i++)
            if(umark[i]) ulable[i]-=eps;
        for(int i=0; i<n; i++)

```

```

        if(dmark[i]) dlable[i]+=eps;
    }
}
int main(){
    for(int i=0; i<m; i++){
        cin >>x >>y >>w;
        a[x][y]=max(w, a[x][y]);
    }
    mwmatching();
    int ans=0;
    for(int i=0; i<n; i++)
        ans+=a[i][umatch[i]];
    cout <<ans <<endl;
}

```

Cut Vertex(Biconnected Component)

```

//O(E) no-base
ll n,m, par[MAX_N],low[MAX_N], height[MAX_N],markV[MAX_N],cnt;
vector<ll>v[MAX_N];
vector<pair<ll,ll>>bcc[MAX_N], tmp_find;
bool mark[MAX_N];
vector<ll>articulationPoints;
set<ll>what;
void CLEAR() {
    tmp_find.clear(); cnt = 1; bridge.clear();
    for(int i = 0 ; i < MAX_N ; i++)
        bcc[i].clear(); v[i].clear() low[i] = par[i] = height[i] = mark[i]
= markV[i] = 0
}
void FIND(pair<ll,ll>x) {
    while(tmp_find.size() > 0) {
        pair<ll,ll>y = tmp_find[tmp_find.size()-1];
        tmp_find.pop_back();
        bcc[cnt].push_back(y);
        if(y == x || (y.first == x.second && y.second == x.first))
            break;
    }
    cnt++;
}
void dfs(int u, int h) {
    mark[u] = true;

```



```

low[u] = h;
height[u] = h;
int childCount = 0;
bool isArticulation = false;
for(int i = 0 ; i < v[u].size() ; i++) {
    int node = v[u][i];
    if(!mark[node]) {
        tmp_find.push_back(make_pair(u,node));
        par[node] = u;
        dfs(node, h+1);
        childCount++;
        if(low[node] >= height[u]) {
            FIND(make_pair(u,node));
            isArticulation = true;
        }
        low[u] = min(low[u], low[node]);
    }
    else if(node != par[u] && height[node] < height[u]) {
        tmp_find.push_back(make_pair(u,node));
        low[u] = min(low[u], height[node]);
    }
}
if((par[u] != 0 && isArticulation) || (par[u] == 0 && childCount > 1))
{
    articulationPoints.push_back(u);
    markV[u] = true;
}

```

Cut Edge

```

//O(E) no-base
int n, m, par[MAX_N], low[MAX_N], height[MAX_N] ;
bool mark[MAX_N];
vector<int>g[MAX_N];
vector<pair<int,int> >cutEdges;
//dfs(1,0)
void dfs(int v, int h) {
    mark[v] = true;
    low[v] = h;
    height[v] = h;
    for(int u:g[v]) {

```

```

        if(!mark[u]) {
            par[u] = v;
            dfs(u, h+1);
            if(low[u] > height[v])
                cutEdges.push_back({v, u});
            low[v] = min(low[v], low[u]);
        }
        else if(u != par[v] && height[u] < height[v])
            low[v] = min(low[v], height[u]);
    }
}

```

LCA

```

//O(n log n) no-base
//dfs(0, 0)
const int MAXN=1e5+10,MAXL=20;
vector<int> g[MAXN];
int par[MAXN][MAXL],h[MAXN];
void dfs(int v,int p) {
    par[v][0]=p;
    for(int i=1;i<MAXL;i++)
        par[v][i]=par[par[v][i-1]][i-1];
    for(int u:g[v])
        if(u!=p) {
            h[u]=h[v]+1;
            dfs(u,v);
        }
}
int get_par(int v,int h) {
    for(int i=0;i<MAXL;i++)
        if(h&(1<<i))
            v=par[v][i];
    return v;
}
int LCA(int v,int u) {
    if(h[v]>h[u])
        swap(v,u);
    u=get_par(u,h[u]-h[v]);
    if(v==u)
        return v;
}

```

```

for(int i=MAXL-1;i>=0;i--)
    if(par[v][i]!=par[u][i]) {
        v=par[v][i];
        u=par[u][i];
    }
return par[v][0];
}

```

Eulerian Tour

```

//O(E) no-base
//if the odd degree vertex is 2 or all degrees are even
//directional two vertex one out and one in
//edges are numbered from 1 to m
//g[i][j] = {k, l} means i-th vertex is connected with k-th vertex through
l-th edge
int n, m;
bool mark[MAX_N];
vector<pair<int, int> >g[MAX_N];
vector<int>ans;
void Euler(int v) {
    while(g[v].size() > 0) {
        pair<int, int> u = g[v].back();
        g[v].pop_back();
        if(!mark[u.second]) {
            mark[u.second] = 1;
            Euler(d);
        }
    }
    ans.push_back(u);
}

```

Data Structures

Ordered Set

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

```

```

typedef
tree<int,null_type,less<int>,rb_tree_tag,tree_order_statistics_node_update
> Tree;
Tree t;
int main() {
    int n;
    cin>>n;
    for(int i=0;i<n;i++) {
        int x;
        cin>>x;
        t.insert(x);
    }
    int x;
    while(cin>>x) {
        int index=t.order_of_key(x);
        int value=*t.find_by_order(x);
        cout<<index<<endl;
    }
}

```

Heavy Light

```

vector<int> g[MAXN];
int par[MAXN],h[MAXN],sz[MAXN],up[MAXN],st[MAXN],en[MAXN],a[MAXN];
int T=0,n, seg[MAXN*4];
void dfs_make(int v,int p=0) {
    par[v]=p;
    if(v!=0) h[v]=h[p]+1;
    sz[v]=1;
    int ind=0,Max=0,pind=-1;
    for(int i=0;i<g[v].size();i++) {
        int u=g[v][i];
        if(u!=p) {
            dfs_make(u,v);
            sz[v]+=sz[u];
            if(sz[u]>Max)ind=i,Max=sz[u];
        }
        else pind=i;
    }
    if(pind!=-1) {
        swap(g[v][pind],g[v][g[v].size()-1]);
        g[v].pop_back();
    }
}

```

```

    }
    if(g[v].size()) swap(g[v][0],g[v][ind]);
}
void dfs_hld(int v) {
    st[v]=T++;
    if(g[v].empty()==0) {
        up[g[v][0]]=up[v];
        dfs_hld(g[v][0]);
        for(int i=1;i<g[v].size();i++){
            int u=g[v][i];
            up[u]=u;
            dfs_hld(u);
        }
    }
    en[v]=T;
}
inline bool cont(int v,int u) {
    return (st[u]>=st[v] and st[u]<en[v]);
}
int lca(int v,int u) {
    if(cont(v,u)) return v;
    if(cont(u,v)) return u;
    int ans1=par[up[v]];
    while(!cont(ans1,u)) ans1=par[up[ans1]];
    int ans2=par[up[u]];
    while(!cont(ans2,u)) ans2=par[up[ans2]];
    if(h[ans1]<h[ans2]) return ans2;
    else return ans1;
}
void add(int s,int e,int ind,int i,int val){
    if(s>i or e<=i) return;
    if(e==s+1) {
        seg[ind]+=val;
        return;
    }
    int mid=(s+e)/2;
    add(s,mid,left(ind),i,val), add(mid,e,right(ind),i,val);
    seg[ind]=seg[left(ind)]+seg[right(ind)];
}
int fin(int s,int e,int ind,int x,int y) {
    if(s>=y or e<=x) return 0;

```

```

    if(x<=s and e<=y) return seg[ind];
    int mid=(s+e)/2;
    return fin(s,mid,left(ind),x,y) +fin(mid,e,right(ind),x,y);
}
int calc(int Par,int v) {
    if(Par==v) return a[v];
    int ret=0,last=-1;
    while(st[v]>st[Par]) {
        if(st[up[v]]>st[Par]) ret+=fin(0,n,1,st[up[v]],st[v]+1);
        else break;
        v=par[up[v]];
    }
    ret+=fin(0,n,1,st[Par],st[v]+1);
    return ret;
}
int main() {
    cin>>n;
    for(int i=0;i<n;i++) cin>>a[i];
    for(int i=1;i<n;i++) {
        int v,u;
        cin>>v>>u;
        g[v].push_back(u);
        g[u].push_back(v);
    }
    dfs_make(0);
    up[0]=0;
    dfs_hld(0);
    for(int i=0;i<n;i++) add(0,n,1,st[i],a[i]);
    int m; cin>>m;
    for(int i=0;i<m;i++) {
        int v,u;
        cin>>v>>u;
        int l=lca(v,u);
        cout<<calc(l,v)+calc(l,u)-a[v]<<endl;
    }
}

```

Math

Power Lemma

Lemma

For all n and m , and $e \geq \log_2(m)$ it holds that

$$n^e \bmod m = n^{\phi(m) + e \bmod \phi(m)} \bmod m.$$

($\phi(m)$ = Euler's totient function.)

Wilson

$$(p-1)! \bmod p = -1$$

AKS

$$(x+a)^n \equiv (x^n + a) \pmod{n} \quad (n \text{ prime and } \gcd(a, n) = 1)$$

Newton's Method

Solve equations with one variable start from a random guess

By solve it means find x where $f(x) = 0$

$$x_0 = 1, x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \text{for square root } \sqrt{A} \quad f(x) = x^2 - A$$

Extended Euclid- Chinese Remainder

//O(log(n)) no-base

ll x,y,d;

//check for a == 0 & b == 0 separately

//the answer is less for a and more for b

//Solve $a*x + b*y = d$ where $d = \gcd(a,b)$

//answers of $X = x + (b/d)*n$, answers of $Y = y - (a/d)*n$

void extendedEuclid(ll a, ll b) {

if (b == 0) x = 1; y = 0; d = a; return;

extendedEuclid(b, a % b);

ll x1 = y; ll y1 = x - (a / b) * y; x = x1; y = y1;

```

}
int inv(int a, int m) {
    int m0 = m, t, q; int x0 = 0, x1 = 1; if (m == 1) return 0;
    while (a > 1) q = a / m; t = m; m = a % m, a = t; t = x0; x0 = x1 - q * x0;
    x1 = t;
    if (x1 < 0) x1 += m0; return x1;
}
//O(k) k is size of num[] and rem[]. Returns the smallest
// number x such that:
// x % num[0] = rem[0],
// x % num[1] = rem[1],...
// x % num[k-1] = rem[k-1]
// Assumption: Numbers in num[] are pairwise coprime
int chinese_remainder(int num[], int rem[], int k) {
    int prod = 1;
    for (int i = 0; i < k; i++)
        prod *= num[i];
    int result = 0;
    for (int i = 0; i < k; i++) {
        int pp = prod / num[i];
        result += rem[i] * inv(pp, num[i]) * pp;
    }
    return result % prod;
}
// finds all solutions to ax = b (mod n)
vector<int> modular_linear_equation_solver (int a, int b, int n){
    int x, y;
    vector<int> solutions;
    int d = extended_euclid (a, n, x, y);
    if (b%d == 0){
        x = mod (x*(b/d), n);
        for (int i = 0; i < d; i++)
            solutions.push_back (mod (x + i*(n/d), n));
    }

    return solutions;
}

```

Miller

//O(iteration) no-base

```

ll modulo(ll x, ll y, ll Mod){
    ll ret=1;
    for(; y!=0; y/=2){
        if(y%2) ret=(ret*x)%Mod;
        x=(x*x)%Mod;
    }
    return ret;
}

bool Miller(ll p,int iteration){
    if(p<2) return 0 if(p==2) return 1; if(p%2==0) return 0;
    ll s=p-1; while(s%2==0) s/=2;
    for(int i=0; i<iteration; i++){
        ll a=rand()%(p-1)+1, temp=s;
        ll mod=modulo(a, temp, p);
        while(temp!=p-1 && mod!=1 && mod!=p-1){
            mod=(mod*mod)%p;
            temp*=2;
        }
        if(mod!=p-1 && temp%2==0)
            return 0;
    }
    return 1;
}

```

Linear System

```

//O(n^3) 0-base
// Gauss-Jordan elimination with full pivoting.
// Uses:
// (1) solving systems of linear equations (AX=B)
// (2) inverting matrices (AX=I)
// (3) computing determinants of square matrices
// INPUT:  a[][] = an nxn matrix
//          b[][] = an nxm matrix solve m different equation
// OUTPUT: X      = an nxm matrix (stored in b[][])
//          A^{-1} = an nxn matrix (stored in a[][])
//          returns determinant of a[][]
const double EPS = 1e-10;
typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;

```

```

typedef vector<VT> VVT;
T GaussJordan(VVT &a, VVT &b) {
    const int n = a.size();
    const int m = b[0].size();
    VI irow(n), icol(n), ipiv(n);
    T det = 1;
    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++) if (!ipiv[j])
            for (int k = 0; k < n; k++) if (!ipiv[k])
                if (fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
        if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl;
            return 0; }
        ipiv[pk]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        if (pj != pk) det *= -1;
        irow[i] = pj;
        icol[i] = pk;
        T c = 1.0 / a[pk][pk];
        det *= a[pk][pk];
        a[pk][pk] = 1.0;
        for (int p = 0; p < n; p++) a[pk][p] *= c;
        for (int p = 0; p < m; p++) b[pk][p] *= c;
        for (int p = 0; p < n; p++) if (p != pk) {
            c = a[p][pk];
            a[p][pk] = 0;
            for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
            for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
        }
    }
    for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
        for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
    }
    return det;
}

int main() {
    double A[MAX_N][MAX_N], B[MAX_N][MAX_N];
    n = 2, m = 1;
    VVT a(n), b(n);
    for (int i = 0; i < n; i++) {

```

```

    a[i] = VT(A[i], A[i] + n);
    b[i] = VT(B[i], B[i] + m);
}
double det = GaussJordan(a, b);
cout << "Determinant: " << det << endl;
cout << "Inverse: " << endl;
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++)
        cout << a[i][j] << ' ';
    cout << endl;
}
cout << "Solution: " << endl;
for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++)
        cout << b[i][j] << ' ';
    cout << endl;
}
}

```

Catalan Number

$$C_n = \frac{1}{n+1} C(2n, n)$$

$$C_{n+1} = \sum_{i=0}^n C_i C_{n-i}$$

FFT

```

//O(n log n) 0-base
typedef complex<double> ftype;
const double pi = acos(-1);
const int maxn = 1 << 17;
ftype w[maxn];
void init() {
    for(int i = 0; i < maxn; i++) {
        w[i] = polar(1., 2 * pi / maxn * i);
    }
}
template<typename T>
void fft(T *in, ftype *out, int n, int k = 1) {
    if(n == 1) {
        *out = *in;

```

```

        return;
    }
    int t = maxn / n;
    n >>= 1;
    fft(in, out, n, 2 * k);
    fft(in + k, out + n, n, 2 * k);
    for(int i = 0, j = 0; i < n; i++, j += t) {
        ftype t = w[j] * out[i + n];
        out[i + n] = out[i] - t;
        out[i] += t;
    }
}
vector<ftype> evaluate(vector<int> p) {
    while(__builtin_popcount(p.size()) != 1) {
        p.push_back(0);
    }
    vector<ftype> res(p.size());
    fft(p.data(), res.data(), p.size());
    return res;
}
vector<int> interpolate(vector<ftype> p) {
    int n = p.size();
    vector<ftype> inv(n);
    fft(p.data(), inv.data(), n);
    vector<int> res(n);
    for(int i = 0; i < n; i++) {
        res[i] = round(real(inv[i]) / n);
    }
    reverse(begin(res) + 1, end(res));
    return res;
}
void align(vector<int> &a, vector<int> &b) {
    int n = a.size() + b.size() - 1;
    while(a.size() < n) {
        a.push_back(0);
    }
    while(b.size() < n) {
        b.push_back(0);
    }
}
vector<int> poly_multiply(vector<int> a, vector<int> b) {

```

```

align(a, b);
auto A = evaluate(a);
auto B = evaluate(b);
for(int i = 0; i < A.size(); i++) {
    A[i] *= B[i];
}
return interpolate(A);
}

const int base = 10;
vector<int> normalize(vector<int> c) {
    int carry = 0;
    for(auto &it: c) {
        it += carry;
        carry = it / base;
        it %= base;
    }
    while(carry) {
        c.push_back(carry % base);
        carry /= base;
    }
    return c;
}

//multiple of two number
vector<int> multiply(vector<int> a, vector<int> b) {
    return normalize(poly_multiply(a, b));
}

int main() {
    init(); //coef of x^0 x^1 ...
    vector<int> a, b, ans;
    ans = poly_multiply(a, b);
    while(ans.back() == 0) ans.pop_back();
}

```

Geometry

Simplex Algorithm

//O(unknown) 0-base
 //This is a simplex solver. Given m x n matrix A, m-vector b, n-vector c,

```

//finds n-vector x such that
//A x <= b (component-wise)
//maximizing
//< x , c >
//where <x,y> is the dot product of x and y.
typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
const DOUBLE EPS = 1e-9;
struct LPSolver {
    int m, n;
    VI B, N;
    VVD D;
    LPSolver(const VVD &A, const VD &b, const VD &c) :
        m(b.size()), n(c.size()), N(n+1), B(m), D(m+2, VD(n+2)) {
        for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] =
A[i][j];
        for (int i = 0; i < m; i++) { B[i] = n+i; D[i][n] = -1; D[i][n+1] =
b[i]; }
        for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m+1][n] = 1;
    }
    void Pivot(int r, int s) {
        for (int i = 0; i < m+2; i++) if (i != r)
            for (int j = 0; j < n+2; j++) if (j != s)
                D[i][j] -= D[r][j] * D[i][s] / D[r][s];
        for (int j = 0; j < n+2; j++) if (j != s) D[r][j] /= D[r][s];
        for (int i = 0; i < m+2; i++) if (i != r) D[i][s] /= -D[r][s];
        D[r][s] = 1.0 / D[r][s];
        swap(B[r], N[s]);
    }
    bool Simplex(int phase) {
        int x = phase == 1 ? m+1 : m;
        while (true) {
            int s = -1;
            for (int j = 0; j <= n; j++) {
                if (phase == 2 && N[j] == -1) continue;
                if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] <
N[s]) s = j;
            }

```

```

    if (D[x][s] >= -EPS) return true;
    int r = -1;
    for (int i = 0; i < m; i++) {
        if (D[i][s] <= 0) continue;
        if (r == -1 || D[i][n+1] / D[i][s] < D[r][n+1] / D[r][s] ||
            D[i][n+1] / D[i][s] == D[r][n+1] / D[r][s] && B[i] < B[r]) r =
i;
    }
    if (r == -1) return false;
    Pivot(r, s);
}
}
DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] <= -EPS) {
        Pivot(r, n);
        if (!Simplex(1) || D[m+1][n+1] < -EPS) return
-numeric_limits<DOUBLE>::infinity();
        for (int i = 0; i < m; i++) if (B[i] == -1) {
            int s = -1;
            for (int j = 0; j <= n; j++)
                if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] <
N[s]) s = j;
            Pivot(i, s);
        }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n+1];
    return D[m][n+1];
}
};

int main() {
    const int m = 4;
    const int n = 3;
    DOUBLE _A[m][n] = {
        { 6, -1, 0 },
        { -1, -5, 0 },
        { 1, 5, 1 },
        { -1, -5, -1 } };

```

```

    DOUBLE _b[m] = { 10, -4, 5, -3 };
    DOUBLE _c[n] = { 1, -1, 0 };
    VVD A(m);
    VD b(_b, _b + m);
    VD c(_c, _c + n);
    for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);
    LPSolver solver(A, b, c);
    VD x;
    DOUBLE value = solver.Solve(x);
    cerr << "VALUE: " << value << endl;
    cerr << "SOLUTION:";
    for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
    cerr << endl;
    return 0;
}

```

Closest Pair

```

//O(n logn) 1-base
struct point {
    ll x,y;
};
int t,n;
point p[MAX_N],p1[MAX_N];
vector<point>v;
bool cmp(point a,point b) {
    if(a.x < b.x) return true;
    if(a.x > b.x) return false;
    return a.y <= b.y;
}
bool cmp1(point a,point b) {
    if(a.y < b.y) return true;
    if(a.y > b.y) return false;
    return a.x <= b.x;
}
ll dis(point a,point b) {
    return (a.y-b.y)*(a.y-b.y)+(a.x-b.x)*(a.x-b.x);
}
pair<point,point>strip_closest() {
    ll MIN = INF;
    point a,b;

```



```

for(int i = 0 ; i < v.size() ; i++) {
    for(int j = i+1 ; j <= min(i+7,int(v.size())-1) ; j++) {
        if(dis(v[i],v[j]) < MIN) {
            MIN = dis(v[i],v[j]);
            a = v[i];
            b = v[j];
        }
    }
}
return make_pair(a,b);
}

pair<point,point> closest_pair(int l,int r) {
    if(l == r) return make_pair(p[1],p[2]);
    if(l == r-1) return make_pair(p[1],p[r]);
    int mid = (l+r)/2;
    pair<point,point>ret;
    pair<point,point>a = closest_pair(l,mid);
    pair<point,point>b = closest_pair(mid+1,r);
    ll d;
    if(dis(a.first,a.second) < dis(b.first,b.second)) {
        ret = a;
        d = dis(a.first,a.second);
    }
    else {
        ret = b;
        d = dis(b.first,b.second);
    }
    v.clear();
    for(int i = 1 ; i <= r ; i++)
        if(abs(p1[i].x -p1[mid].x) <= d) v.push_back(p1[i]);
    pair<point,point>c = strip_closest();
    if(dis(c.first,c.second) < dis(ret.first,ret.second)) ret = c;
    return ret;
}

int main() {
    scanf("%d",&n);
    for(int i = 1 ; i <= n ; i++) {
        scanf("%lld %lld",&p[i].x,&p[i].y);
        p1[i] = p[i];
    }
}

```

```

sort(p+1,p+n+1,cmp);
sort(p1+1,p1+n+1,cmp1);
pair<point,point> ans = closest_pair(1,n);
}

```

Areas and Angles

```

double INF = 1e100;
double EPS = 1e-8;
struct PT {
    double x, y;
    PT (){}
    PT (double x, double y) : x(x), y(y){}
    PT (const PT &p) : x(p.x), y(p.y){}
    PT operator- (const PT &p){ return PT(x-p.x,y-p.y); }
    PT operator+ (const PT &p){ return PT(x+p.x,y+p.y); }
    PT operator* (double c){ return PT(x*c,y*c); }
    PT operator/ (double c){ return PT(x/c,y/c); }
};

double dot (PT p, PT q){ return p.x*q.x+p.y*q.y; }
double dist2 (PT p, PT q){ return dot(p-q,p-q); }
double dist (PT p, PT q) { return sqrt( dist2(p, q) ); }
double cross (PT p, PT q){ return p.x*q.y-p.y*q.x; }
// rotate a point CCW or CW around the origin
PT RotateCCW90 (PT p){ return PT(-p.y,p.x); }
PT RotateCW90 (PT p){ return PT(p.y,-p.x); }
PT RotateCCW (PT p, double t){
    return PT(p.x*cos(t)-p.y*sin(t),
        p.x*sin(t)+p.y*cos(t));
}

// rotate p1 around p0 clockwise, by angle a
PT RotateC(PT p0, PT p1, double a) {
    p1 = p1-p0;
    return p0 + PT(cos(a)*p1.x-sin(a)*p1.y,
        sin(a)*p1.x+cos(a)*p1.y);
}

// p1->p2 line, reflect p3 to get r.
PT reflect(PT& p1, PT& p2, PT p3) {
    if(dist(p1, p3)<EPS) {return p3;}
    double a=dot(p2-p1,p3-p1)/(dist(p1,p2)*dist(p1,p3));
    a=acos(a);
}

```

```

        return RotateC(p1, p3, -2.0*a);
    }
    double SignedTriArea (PT a, PT b, PT c) {
        return( (a.x*b.y - a.y*b.x + a.y*c.x
        - a.x*c.y + b.x*c.y - c.x*b.y) / 2.0 );
    }
    double SignedArea (vector<PT> v){
        double area = 0;
        for (int i = 0; i < v.size(); i++){
            int j = (i+1) % v.size();
            area += v[i].x*v[j].y - v[j].x*v[i].y;
        }
        return area / 2.0;
    }
}

```

Lines Points Intersect

```

// Given three colinear points p, q, r, the function checks if
// point q lies on line segment 'pr'
bool onSegment(PT p, PT q, PT r) {
    if (q.x <= max(p.x, r.x) && q.x >= min(p.x, r.x) &&
        q.y <= max(p.y, r.y) && q.y >= min(p.y, r.y))
        return true;
    return false;
}
//returns true if line segment 'p1q1' and 'p2q2' intersect.
// orientation code is in convex hull ( change Point to PT first! )
bool segmentIntersect(PT p1, PT q1, PT p2, PT q2) {
    int o1 = orientation(p1, q1, p2);
    int o2 = orientation(p1, q1, q2);
    int o3 = orientation(p2, q2, p1);
    int o4 = orientation(p2, q2, q1);
    if (o1 != o2 && o3 != o4)
        return true;
    // p1, q1 and p2 are colinear and p2 lies on segment p1q1
    if (o1 == 0 && onSegment(p1, p2, q1)) return true;
    if (o2 == 0 && onSegment(p1, q2, q1)) return true;
    if (o3 == 0 && onSegment(p2, p1, q2)) return true;
    if (o4 == 0 && onSegment(p2, q1, q2)) return true;
    return false;
}

```

```

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine (PT a, PT b, PT c){
    return a + (b-a)*dot(c-a,b-a)/dot(b-a,b-a);
}
// project point c onto line segment through a and b
PT ProjectPointSegment (PT a, PT b, PT c){
    double r = dot(b-a,b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a,b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}
//Compute the distance from AB to C
//if isSegment is true, AB is a segment, not a line.
double LinePointDist(PT A, PT B, PT C, bool isSegment){
    double dd = cross(B-A,C-A) / dist(A,B);
    if(isSegment){
        int dot1 = dot(B-A,C-B);
        if(dot1 > 0)return dist(B,C);
        int dot2 = dot(A-B,C-A);
        if(dot2 > 0)return dist(A,C);
    }
    return abs(dd);
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane (double x, double y, double z,
    double a, double b, double c, double d){
    return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}
// determine if two lines are parallel or collinear
bool LinesParallel (PT a, PT b, PT c, PT d){
    return fabs(cross(b-a,c-d)) < EPS;
}
bool LinesCollinear (PT a, PT b, PT c, PT d){
    return LinesParallel(a,b,c,d) && fabs(cross(a-c,d-c)) < EPS;
}
// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique

```

```

// intersection exists ( check collinear parallel )
PT ComputeLineIntersection (PT a, PT b, PT c, PT d){
    b=b-a; d=c-d; c=c-a;
    if (dot(b,b) < EPS) return a;
    if (dot(d,d) < EPS) return c;
    return a + b*cross(c,d)/cross(b,d);
}
// the relation of the point p and the segment p1->p2.
// 1 if point is on the segment; 0 if not on the line;
// -1 if on the line but not on the segment
int pAndSeg(PT& p1, PT& p2, PT& p) {
    double s=abs(SignedTriArea(p, p1, p2));
    if(s>EPS) return(0);
    double sg=(p.x-p1.x)*(p.x-p2.x);
    if(sg>EPS) return(-1);
    sg=(p.y-p1.y)*(p.y-p2.y);
    if(sg>EPS) return(-1);
    return(1);
}

```

Centers and Centroids

```

// compute center of circle given three points
PT ComputeCircleCenter (PT a, PT b, PT c){
    b=(a+b)/2;
    c=(a+c)/2;
    return ComputeLineIntersection (b,b+RotateCW90(a-b),
        c,c+RotateCW90(a-c));
}
// point generated by altitudes ( assuming it is triangle )
PT ComputeHcenter( PT p1, PT p2, PT p3 ) {
    PT a1 = ProjectPointLine( p2, p3, p1 );
    PT a2 = ProjectPointLine( p1, p3, p2 );
    return ComputeLineIntersection( p1, a1, p2, a2 );
}
// point generated by circumscribed circle ( assuming tri )
PT ComputeCenter( PT p1, PT p2, PT p3 ) {
    PT a1 = (p2+p3)*0.5;
    PT a2 = (p1+p3)*0.5;
    PT b1( a1.x - (p3.y-p2.y), a1.y + (p3.x-p2.x) );
    PT b2( a2.x - (p3.y-p1.y), a2.y + (p3.x-p1.x) );
}

```

```

return ComputeLineIntersection( a1, b1, a2, b2 );
}
PT ComputeCentroid (vector<PT> v){
    double cx = 0, cy = 0;
    double scale = 6.0 * SignedArea (v);
    for (int i = 0; i < v.size(); i++){
        int j = (i+1) % v.size();
        cx += (v[i].x+v[j].x)*(v[i].x*v[j].y-v[j].x*v[i].y);
        cy += (v[i].y+v[j].y)*(v[i].x*v[j].y-v[j].x*v[i].y);
    }
    PT res; res.x = cx/scale; res.y = cy/scale;
    return res;
}
// angle bisection ( assuming tri )
PT ComputeBcenter( PT p1, PT p2, PT p3 ) {
    double s1, s2, s3;
    s1 = dist( p2, p3 );
    s2 = dist( p1, p3 );
    s3 = dist( p1, p2 );
    double rt = s2/(s2+s3);
    PT a1 = p2*rt+p3*(1.0-rt);
    rt = s1/(s1+s3);
    PT a2 = p1*rt+p3*(1.0-rt);
    return ComputeLineIntersection( a1,p1, a2,p2 );
}

```

Point in Polygon

```

// 1 if p is in pv; 0 outside; -1 on the polygon
int PointInPolygon(vector<PT> pv, PT p)
{
    int n=pv.size(), j; pv.push_back(pv[0]);
    for(int i=0;i<n;i++){
        if(pAndSeg(pv[i], pv[i+1], p)==1) return(-1);
    }
    for(int i=0;i<n;i++) pv[i] = pv[i]-p;
    p.x=p.y=0.0; double a, y;
    while(1) {
        a=(double)rand()/10000.00;
        j=0;
        for(int i=0;i<n;i++) {
            pv[i] = RotateCCW(pv[i], a);
        }
    }
}

```

```

    if(abs(pv[i].x)<EPS) j=1;
}
if(j==0) {
    pv[n]=pv[0];
    j=0;
    for(int i=0;i<n;i++)
        if(pv[i].x*pv[i+1].x < -EPS) {
            y=pv[i+1].y-pv[i+1].x*(pv[i].y-pv[i+1].y)/(pv[i].x-pv[i+1].x);
            if(y>0) j++;
        }
    return(j%2);
}
}
return 1;
}

```

Big Intersections

```

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CLIntersection (PT a, PT b, PT c, double r){
    vector<PT> ret;
    PT d = b-a;
    double D = cross(a-c,b-c);
    double e = r*r*dot(d,d)-D*D;
    if (e < 0) return ret;
    e = sqrt(e);
    ret.push_back
(c+PT(D*d.y+(d.y>=0?-1:1)*d.x*e,-D*d.x+fabs(d.y)*e)/dot(d,d));
    if (e > 0)
        ret.push_back
(c+PT(D*d.y-(d.y>=0?-1:1)*d.x*e,-D*d.x-fabs(d.y)*e)/dot(d,d));
    return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CCIntersection (PT a, PT b, double r, double R){
    vector<PT> ret;
    double d = sqrt(dist2(a,b));
    if (d > r+R || d+min(r,R) < max(r,R)) return ret;
    double x = (d*d-R*R+r*r)/(2*d);

```

```

    double y = sqrt(r*r-x*x);
    PT v = (b-a)/d;
    ret.push_back (a+v*x + RotateCCW90(v)*y);
    if (y > 0)
        ret.push_back (a+v*x - RotateCCW90(v)*y);
    return ret;
}

vector<PT> PPIntersection(vector<PT>& p1, vector<PT>& p2) {
    vector<PT> pts;
    PT pp; pts.clear();
    int m=p1.size(), n=p2.size();
    for(int i=0;i<m;i++)
        if(PointInPolygon(p2, p1[i])!=0) pts.push_back(p1[i]);
    for(int i=0;i<n;i++)
        if(PointInPolygon(p1, p2[i])!=0) pts.push_back(p2[i]);
    if(m>1 && n>1)
        for(int i=0;i<m;i++)
            for(int j=0;j<n;j++)
                if( !LinesParallel(p1[i], p1[(i+1)%m], p2[j], p2[(j+1)%n]) ) {
                    pp = ComputeLineIntersection(p1[i], p1[(i+1)%m], p2[j],
p2[(j+1)%n]);
                    if(pAndSeg(p1[i], p1[(i+1)%m], pp)!=1) continue;
                    if(pAndSeg(p2[j], p2[(j+1)%n], pp)!=1) continue;
                    pts.push_back(pp);
                }
    if(pts.size()<=1)
        pts.clear();
    return pts;
}

// cut the convex polygon pol along line p1->p2;
// pol1 are the resulting polygon on the left side, pol2 on the right.
void cutPoly(vector<PT>& pol, PT& p1, PT& p2, vector<PT>& pol1,
vector<PT>& pol2) {
    pol1.clear(); pol2.clear();
    int i, sg, n=pol.size();
    PT q1,q2,r;
    for(i=0;i<n;i++) {
        q1=pol[i]; q2=pol[(i+1)%n];
        sg=orientation(p1, p2, q1);
        if(sg==0 || sg==2) pol1.push_back(q1);
        if(sg==0 || sg==1) pol2.push_back(q1);
    }
}

```

```

if( !LinesParallel(p1, p2, q1, q2) ) {
    r = ComputeLineIntersection(p1, p2, q1, q2);
    if(pAndSeg(q1, q2, r)==1) {
        pol1.push_back(r);
        pol2.push_back(r);
    }
}
}
if(pol1.size()<=2) pol1.clear();
if(pol2.size()<=2) pol2.clear();
}

```

Convex Hull

```

//O(n log n) 0-base
struct PT { int x; int y; };
vector<PT> p; //PTs of the Polygon to be processed
vector<PT> S; //Contains the convex hull
const double PI = 2.0*acos(0.0);
const double EPS = 1e-9; //too small/big????
int orientation(PT p1, PT p2, PT p3) {
    int val = (p2.y - p1.y) * (p3.x - p2.x) -
              (p2.x - p1.x) * (p3.y - p2.y);
    if (abs(val) < EPS) return 0; // colinear
    return (val > 0)? 1: 2; // clock(1) or counterclockwise(2)
}
//Returns the square of distance
int distSq(PT p1, PT p2) {
    return (p1.x - p2.x)*(p1.x - p2.x) +
           (p1.y - p2.y)*(p1.y - p2.y);
}
bool cmp (PT p1, PT p2) {
    int o = orientation(p1, p[0], p2);
    if (o==0) return (distSq(p[0], p1) <= distSq(p[0], p2));
    return (o==1);
}
void convexHull (int n=p.size()) {
    // Find the bottommost-leftmost PT
    int ymn = p[0].y, mn = 0;
    for (int i = 1; i < n; i++) {
        int y = p[i].y;

```

```

        if ((y < ymn) || (ymn == y && p[i].x < p[mn].x))
            ymn = p[i].y, mn = i;
    }
    swap(p[0], p[mn]);
    sort(p.begin()+1, p.end(), cmp);
    int m=1; //Removing collinears and same PTs
    for (int i=1; i<n; i++) {
        while (i < n-1 && orientation(p[0], p[i], p[i+1]) == 0)
            i++;
        p[m] = p[i]; m++;
    }
    if (m<3) return; // Go for the convex hull
    S.push_back(p[0]); S.push_back(p[1]); S.push_back(p[2]);
    for (int i = 3; i < m; i++) {
        // Keep removing top while the turn is not ccw
        while (orientation(S[S.size()-2], S[S.size()-1], p[i]) != 2)
            S.pop_back();
        S.push_back(p[i]);
    }
}
// return 0 if not convex, 1 if strictly convex,
// 2 if convex but there are points unnecessary
// this function does not work if the polygon is self intersecting
// in that case, compute the convex hull of v, and see if both have the
// same area
int isConvex( vector<PT>& v ) {
    int c0=0, c1=0, c2=0, n=v.size();
    for ( int i=0; i<n; i++ ) {
        int j=(i+1)%n; k=(i+2)%n;
        int s=orientation(v[i], v[j], v[k]);
        if (s==0) c0++;
        if (s==2) c1++;
        if (s==1) c2++;
    }
    if(c1 && c2) return 0;
    if(c0) return 2;
    return 1;
}

```

Misc

```
//(PI/3) * (H *H) * (3*R-H)
//volume of part of sphere H is height fromt the buttom
//R is the radius of sphere
//*Lattice Polygons and Pick's Theorem: A(P) = I(P) + B(P)/2 - 1
//    Where A(P) is the area of Polygon P, I(P) is the number of
//    lattice points inside P and B(P) num of points on boundary.
//*Check whether a polygon is convex: all three consecutive
//    points in the polygon must make left-turns
//    if visited in counter clockwise order.
//*Van Goh's algorithm not mentioned. Keep its idea in mind.
//    (ear-cutting and trianulation)
//*The idea of using binary search instead of complex formulas.
```

Dynamic Programming

LIS

```
//O(n log n) 0-base
//number of decreasing bags is size of LIS
//CAUTION: JUST THE SIZE NOT THE ORDER FOR ORDER USE VECTOR AND PAR
multiset<int>s;
int a[MAX_N], n;
void LIS() {
    for(int i=0 ; i<n ;i++) {
        int x=a[i];
        //for increasing lower to upper
        multiset<int>::iterator it=s.lower_bound(x);
        if(it==s.end())
            s.insert(x);
        else {
            s.erase(it);
            s.insert(x);
        }
    }
}
```

Matrix DP

$$dp_i = Adp_{i-1} + Bdp_{i-2} + Cdp_{i-3}$$

$$\begin{bmatrix} A & B & C \end{bmatrix} \begin{bmatrix} dp_{i-1} \\ dp_{i-2} \\ dp_{i-3} \end{bmatrix} = \begin{bmatrix} dp_i \end{bmatrix}$$

Works for 2D DP as well if $dp[i][j]$ only be filled with $dp[i-1][k]$
 Second dimension = n, build an n*n matrix how to fill j from different k

String

Hash

```
//O(n) 0-base
//be carefull of mod ! log MOD is mutlplied
const ll MOD = 999998727899999LL;
const ll P = 37;
ll h[MAX_N], po[MAX_N];
ll Hash(string s) {
    h[0]=(s[0]-'a'+1);
    for(int i=1;i<s.length();i++)
        h[i]=(h[i-1]*P+(s[i]-'a'+1))%MOD;
    return h[s.length()-1];
}
ll mul(ll a, ll b) {
    if(b == 0) return 0;
    ll x = mul(a, b/2);
    if(b%2) return (x+x+a)%MOD;
    else return (x+x)%MOD;
}
ll calc(int s,int e){
    if(s==0) return h[e];
    ll ans=h[e];
    ans-=mul(h[s-1], po[e-s+1]);
    if(ans < 0) ans += MOD;
    return ans;
}
```

```

}
void init() {
    po[0]=1;
    for(int i=1;i<MAX_N;i++)
        po[i]=(po[i-1]*P)%MOD;
}

```

KMP

```

//O(K) 0-base
//Searches for the string w in the string s (of length k). Returns the
//0-based index of the first match (k if no match is found). Algorithm
void buildTable(string& w, vector<int>& t){
    t = vector<int>(w.length());
    int i = 2, j = 0;
    t[0] = -1; t[1] = 0;

    while(i < w.length()) {
        if(w[i-1] == w[j]) { t[i] = j+1; i++; j++; }
        else if(j > 0) j = t[j];
        else { t[i] = 0; i++; }
    }
}

int KMP(string& s, string& w) {
    int m = 0, i = 0;
    vector<int> t;
    buildTable(w, t);
    while(m+i < s.length()) {
        if(w[i] == s[m+i]) {
            i++;
            if(i == w.length()) return m;
        } else {
            m += i-t[i];
            if(i > 0) i = t[i];
        }
    }
    return s.length();
}

```

Misc

Ternary Search

```

//O(logn) no-base
//first increase then decrease
ll low = 0, high = INF;
ld ans = 0;
while(low<=high) {
    ll lm = low+(high-low)/3;
    ll rm = high-(high-low)/3;
    ld lmval = check(lm);
    ld rmval = check(rm);
    if(lmval < rmval) {
        ans = max(ans, lmval);
        low = lm+1;
    }
    else {
        ans = max(ans, rmval);
        high = rm-1;
    }
}

```

Grid

