Mandatory Assignment 1

Bendik Nyheim

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Exercise 1. Comparing Likelihoods

1. Completeness

If both $A \succcurlyeq B$ and $B \succcurlyeq A$, that is $A \succcurlyeq B \succcurlyeq A$, it implies $P(A) \geq P(B) \geq P(A)$, which only holds for the special case $P(A) = P(B)$.

2. Transitivity

Assuming $A \succcurlyeq B$ and $B \succcurlyeq C$, we can input the first relation into the second, getting $B \succcurlyeq C \implies A \succcurlyeq C$. This holds if we have completeness.

3. Consistency

Assuming disjoint events, we have that $P(X_1 \cup X_2) = P(X_1) + P(X_2)$ and $P(Y_1 \cup Y_2) = P(Y_1) + P(Y_2)$. Thus, we have consistency, as $X_1 \cup X_2 \succcurlyeq Y_1 \cup Y_2$ implies $P(X_1) + P(X_2) \geq P(Y_1) + P(Y_2)$, and we easily see that the inequality holds since we can compare each term $P(X_1) \geq P(Y_1)$ and $P(X_2) \geq P(Y_2)$.

4.

This is easily proven with algebra. We know that $P(A) + P(A^C) = 1$. Using this, we get

$$\begin{aligned} P(X) &> P(Y) \\ 1 - P(X^C) &> 1 - P(Y^C) \\ -P(X^C) &> -P(Y^C) \\ P(X^C) &< P(Y^C) \\ \implies X^C &\preccurlyeq Y^C \end{aligned}$$

5.

We have that $Y \subseteq X$. The set X can be written as

$$X = \{Y, X \cap Y^C\}$$

The probability can be written as

$$\begin{aligned} P(X) &= P(Y \cup (X \cap Y^C)) \\ P(X) &= P(Y) + P(X \cap Y^C) - P(X \cap (X \cap Y^C) \\ \implies P(X) &\geq P(Y) \end{aligned}$$

In the last line, I used that $P(X \cap (X \cap Y^C) = 0$ and $P(X \cap Y^C) \geq 0$.

Exercise 2. Transitive Relations

a)

An example of an intransitive binary relation is "is a parent of". If Ola is the parent of Ida, and Per is the parent of Ola, that does not mean Per is the parent of Ida.

b)

It is always transitive because it is defined from the utility function, and the only thing that matter then is which utility is greater.

Exercise 3. Risk Taking

a)

$U_1(r) = r^2$ is a convex utility function, which induces a risk affine behaviour. That means a lottery is preferred over a certain outcome.

b)

$U_2(r) = \sqrt{r}$ is a concave function, which induces a risk averse behaviour. That means a certain outcome is preferred over a lottery.

c)

$U_3(r) = 0.3 \cdot r - 10$ is a linear function, which induces a risk neutral behaviour. Neither a lottery or certain outcome is preferred over the other.

d)

$U_4(r) = 5 \cdot r^3$ is neither a linear, convex or concave function, so the risk behaviour is undefined.

Exercise 4. Saint Petersburg Paradox Revisited

The expected utility of playing this game with the utility function $U(r) = 1/r$ is

$$\mathbb{E}[U(r)] = \sum_{i=1}^{\infty} U(r)P(x_i = \text{heads}) = \sum_{i=1}^{\infty} \frac{1}{2^i}\frac{1}{2^i} = \frac{1}{3} \tag{1}$$

This game is not something I would play to make money. I would however pay maybe 20NOK just for the thrill of it.