

Mandatory Assignment 1

Submission Deadline: 8th February, 2022

Exercise 1. Comparing Likelihoods

Let $P: \mathcal{A} \rightarrow \mathbb{R}^+$ be a probability measure, as defined in the lecture, for an algebra \mathcal{A} over events Ω . Show that the relation \succeq with

$$A \succeq B \text{ if and only if } P(A) \geq P(B) \text{ for } A, B \in \mathcal{A}$$

satisfies the following desired properties for comparing likelihoods of random events:

1. For $A, B \in \mathcal{A}$, either $A \succeq B$ or $B \succeq A$. (Completeness)
2. For $A, B, C \in \mathcal{A}$, if $A \succeq B$ and $B \succeq C$ then $A \succeq C$. (Transitivity)
3. For $X_1, X_2, Y_1, Y_2 \in \mathcal{A}$, if $X_i \succeq Y_i$ for $i = 1, 2$, then $X_1 \cup X_2 \succeq Y_1 \cup Y_2$. (Consistency)
4. For $X, Y \in \mathcal{A}$, if $X \succ Y$, then $X^C \preceq Y^C$.
5. For $X, Y \in \mathcal{A}$ with $Y \subseteq X$, we have $X \succeq Y$.

Exercise 2. Transitive Relations

- a) Find an example of an intransitive binary relation (other than the one from the second lecture).
- b) Consider the relation over a set of rewards R , that is induced by a utility function $U: \mathcal{R} \rightarrow \mathbb{R}$ such that for $a, b \in \mathcal{R}$ we have $a \succeq b$ iff $U(a) \geq U(b)$. Argue, why such a relation is always transitive.

Exercise 3. Risk Taking

Argue for the following utility functions whether they induce risk neutral, risk affine, risk averse, or undefined risk behaviour.

- a) $U_1(r) = r^2$
- b) $U_2(r) = \sqrt{r}$

c) $U_3(r) = 0.3 \cdot r - 10$

d) $U_4(r) = 5 \cdot r^3$

Exercise 4. Saint Petersburg Paradox Revisited

Consider the following game. Repeatedly toss a fair coin (50% heads, 50% tails). Let x_i (random variable) be the i -th coin toss. Stop once the outcome of x_i is heads with a reward of 2^i NOK. Assume that your utility for money is given by the function $U(r) = 1/r$. What is the expected utility of playing the game? Does this reflect the amount of money you would pay for playing the game (why/why not)?