Causal Inference (6.S059/15.C08/17.C08)

Recitation, Week 1.

Topic: Potential Outcomes and Data Analysis.

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1/ Introduction

Presentation

- **TA: Benjamín Muñoz**
  - 4th Year PhD Candidate.
  - Fields: Comparative Politics + Methods.
- **TA: Licheng Liu**
  - 5th Year PhD Candidate.
  - Fields: Methods.+ Political Economy
- **Causal Inference:**
  - Design-based Inference, Potential Outcomes, Causal Graphs, Classic Identification Strategies, Machine Learning + Causal Inference.
  - Focus on intuition and empirical applications.
  - Hopefully a useful and fun experience!

Logistics

- **Recitation:** Fridays, 11-12 and 14-15 (ET). Room: 56-169.
    1. Provide big picture
    2. Fill in gaps.
    3. Math review.
    4. Share pset hints.
    5. Answer questions about the course content. $\rightsquigarrow$ Supplement lectures.
- **Office Hours:** Tuesdays 14:00-15:00 PM or by appointment (ET).
    1. Answer general pset questions.
    2. Answer specific pset questions.
       ★ **Licheng OH:** Wednesdays, 13:00-14:00 PM (or by appointment).
- **Canvas and Piazza**
    1. Faster response 9am–5pm.
    2. Try to answer each other's questions.
- **Problem Set 1**
    1. Release: February 15th.
    2. Due: February 28th, 15:00 PM (ET).

2/ Potential Outcomes

Potential Outcome Framework

- Inference about **counterfactuals**.
- **We're inside the urn!** There are (at least) two potential outcomes for every unit i, but only one is realized.
- Conditional phrases are important here, i.e. potential outcomes are the outcomes that would have been realized; had a unit i (not) received treatment.
- Holland (1986): "The key notion, however, is the potential (regardless of whether it can be achieved in practice or not) for exposing or not exposing each unit to the action of a cause."

Potential Outcomes (under SUTVA)

- D_i: Indicator of treatment intake for unit i, where $i = 1, \ldots, N$; for now,$D_i \in 0, 1$.
- Y_{1i} : Potential outcome for unit i with treatment.
- Y_{0i} : Potential outcome for unit i without treatment.
- Y_i: Variable of interest, whose value is observed, which may be affected by the treatment.

Switching Equation

Realized Outcome: $Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$.

> "The interpretation of these two values, Y_{1i} and Y_{0i} for a given unit i, is that Y_{1i} is the value of the response that would be observed if the unit were exposed to [treatment] and Y_{0i} is the value that would be observed on the same unit if it were exposed to [control]."

- Without SUTVA, potential outcomes can increase exponentially.

Causal Quantities

- **Causal Effect:** τ is the effect of the treatment on the outcome.

  $\rightsquigarrow$ We want the difference between potential outcomes Y_{1i}–Y_{0i}

Fundamental Problem of Causal Inference

- Intuition: we never observe both Y_{1i} and Y_{0i}.
- Implication: τ_i is unidentifiable without further assumptions.
- Solution: introduce assumptions that can help us impute unobserved potential outcomes.

- Focus on a different **Quantity of Interest:** averages!
- **Average Treatment Effect** (ATE): $\tau_{ATE} = \mathbb{E}[Y_{1i} - Y_{0i}]$
- Note that τ_{ATE} is still unidentified.
- **Other Quantities of Interest (QoI):** ATT, ATC, etc.
- When would $\tau_{ATT} \neq \tau_{ATE}$?

Causal Quantities

- **Causal Effect:** τ is the effect of the treatment on the outcome.

  $\rightsquigarrow$ We want the difference between potential outcomes $Y_{1i} - Y_{0i}$

Fundamental Problem of Causal Inference

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- Focus on a different **Quantity of Interest:** averages!
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- **Other Quantities of Interest (QoI):** ATT, ATC, etc.
- When would $\tau_{ATT} \neq \tau_{ATE}$? When D_i and Y_{di} are associated.

Basic Example

i	D_i	Y_i	Y_{1i}	Y_{0i}	τ_i
1	1	0			
2	1	0			
3	0	2			
4	0	2			
$\mathbb{E}[Y_{1i}]$					
$\mathbb{E}[Y_{0i}]$					
$\mathbb{E}[Y_{1i} - Y_{0i}]$					
$\mathbb{E}[Y_{1i} - Y_{0i} \mid D_i = 1]$					

Basic Example

i	D_i	Y_i	Y_{1i}	Y_{0i}	τ_i
1	1	0	0		
2	1	0	0		
3	0	2		2	
4	0	2		2	
$\mathbb{E}[Y_{1i}]$					
$\mathbb{E}[Y_{0i}]$					
$\mathbb{E}[Y_{1i} - Y_{0i}]$					
$\mathbb{E}[Y_{1i} - Y_{0i} \| D_i = 1]$					

Basic Example

i	D_i	Y_i	Y_{1i}	Y_{0i}	τ_i
1	1	0	0	1	-1
2	1	0	0	1	-1
3	0	2	0	2	-2
4	0	2	0	2	-2
$\mathbb{E}[Y_{1i}]$			0		
$\mathbb{E}[Y_{0i}]$				1.5	
$\mathbb{E}[Y_{1i} - Y_{0i}]$					-1.5
$\mathbb{E}[Y_{1i} - Y_{0i} \mid D_i = 1]$					-2

More on SUTVA

- **No interference between units:** Potential outcomes for a unit must not be affected by treatment for any other units.
  - Intuition: Y_{di} is of treatment status of any other unit.
  - Violation: Spillover or contagion.
- **No different versions of treatment** (a.k.a. stabiliy, consistency): Nominally identical treatments are in fact identical.
  - Intuition: Treatments are identical for all units.
  - Violation: Variable levels of treatment (e.g., more/less money).
- Or: $Y_{(D_1, D_2, \ldots, D_N)i} = Y_{(D'_1, D'_2, \ldots, D'_N)i}$ if $D_i = D'_i$

Without SUTVA

i	K_i	D_i	Y_i	Y_{110i}	Y_{100i}	Y_{010i}	Y_{001i}	Y_{011i}	Y_{101i}	Y_{000i}	Y_{111i}
1	1	1	0	0	0	0	1	0	0	1	0
2	1	1	0	0	0	0	1	0	0	1	0
3	2	0	1	1	1	1	0	0	0	1	0

Potential Outcomes Review

What are the following quantities?

- Y_{di} :
- $Y_i(1)$:
- $Y_i(D_i, D_{i-1}, D_{i+1})$:
- $Z_i(1)$:

Potential Outcomes Review

What are the following quantities?

- Y_{di} : Potential outcomes for i under given set of treatments d.
- $Y_i(1)$: Potential outcome for i given $D_i = 1$ (i.e. treatment if binary)
- $Y_i(D_i, D_{i-1}, D_{i+1})$: Potential outcome for unit i, given the treatment status of units i, $i-1$, and $i+1$
- $Z_i(1)$: Potential outcome (of Z) for unit i given $D_i = 1$

Potential Outcomes Review

- $\mathbb{E}[Y_i(0)|D_i = 1]$:
- $\mathbb{E}[Y_i(1)|D_i = 1]$:
- $\mathbb{E}[Y_i(1)|D_i = 0]$:
- $\mathbb{E}[Y_i(0)|D_i = 0]$:

Potential Outcomes Review

- $\mathbb{E}[Y_i(0)|D_i = 1]$: Expected value of the outcome if they were not treated for those in the treatment group. $\rightsquigarrow$ **Not Observed**
- $\mathbb{E}[Y_i(1)|D_i = 1]$: Expected value of the outcome if they were treated for those in the treatment group. $\rightsquigarrow$ **Observed**
- $\mathbb{E}[Y_i(1)|D_i = 0]$: Expected value of the outcome if they were treated for those in the control group. $\rightsquigarrow$ **Not Observed**
- $\mathbb{E}[Y_i(0)|D_i = 0]$: Expected value of the outcome if they were not treated for those in the control group. $\rightsquigarrow$ **Observed**

Potential Outcomes Review

Are the following equalities always true?

- $Y_{di} = Y_i(d)$:
- $Y_{D_i i} = Y_i(D_i, D_{i-1}, D_{i+1})$:
- $\mathbb{E}[Y_i(0)|D_i = 1] = \mathbb{E}[Y_i(0)|D_i = 0]$:
- $\underbrace{E[Y_{1i} - Y_{0i}]}_{\tau_{ATE}} = \underbrace{\mathbb{E}[Y_{1i} - Y_{0i}|D_i = 1]}_{\tau_{ATT}}$:

Potential Outcomes Review

Are the following equalities always true?

- $Y_{di} = Y_i(d)$: Yes, this is just different notation.
- $Y_{D_i i} = Y_i(D_i, D_{i-1}, D_{i+1})$: No,no if SUTVA is violated.
- $\mathbb{E}[Y_i(0)|D_i = 1] = \mathbb{E}[Y_i(0)|D_i = 0]$: No, see previous slide.
- $\underbrace{E[Y_{1i} - Y_{0i}]}_{\tau_{ATE}} = \underbrace{\mathbb{E}[Y_{1i} - Y_{0i}|D_i = 1]}_{\tau_{ATT}}$: No, see next slide.

The ATT

$$
\begin{aligned}
\hat{\tau} &= \mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] \\
&= \mathbb{E}[Y_{1i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 0] \\
&= \mathbb{E}[Y_{1i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 0] \underbrace{- \mathbb{E}[Y_{0i}|D_i = 1] + \mathbb{E}[Y_{0i}|D_i = 1]}_{0} \\
&= \mathbb{E}[Y_{1i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 0] + \mathbb{E}[Y_{0i}|D_i = 1] \\
&= \underbrace{\mathbb{E}[Y_{1i} - Y_{0i}|D_i = 1]}_{\tau_{ATT}} + \underbrace{\mathbb{E}[Y_{0i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 0]}_{\text{Selection Bias}}
\end{aligned}
$$

When is the ATT going to be identified?

When $\mathbb{E}[Y_{0i}|D_i = 1] = \mathbb{E}[Y_{0i}|D_i = 0] = \mathbb{E}[Y_{0i}]$

The ATE

$$
\begin{aligned}
\hat{\tau} &= \mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] \\
&= \mathbb{E}[Y_{1i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 0] \\
&= \mathbb{E}[Y_{1i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 0] \underbrace{- \mathbb{E}[Y_{0i}|D_i = 1] + \mathbb{E}[Y_{0i}|D_i = 1]}_{0} \\
&= \mathbb{E}[Y_{1i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 0] + \mathbb{E}[Y_{0i}|D_i = 1] \\
&= \underbrace{\mathbb{E}[Y_{1i} - Y_{0i}|D_i = 1]}_{\tau_{ATT}} + \underbrace{\mathbb{E}[Y_{0i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 0]}_{\text{Selection Bias}}
\end{aligned}
$$

When is the ATE going to be identified?

When $\mathbb{E}[Y_{0i}|D_i = 1] = \mathbb{E}[Y_{0i}|D_i = 0] = \mathbb{E}[Y_{0i}]$

When $\mathbb{E}[Y_{1i}|D_i = 1] = \mathbb{E}[Y_{1i}|D_i = 0] = \mathbb{E}[Y_{1i}]$

Identification

What does it mean for something to be **identified**? A quantity is identified if it can be *calculated* using observed information.

Are the following quantities identified?

- $\tau_i = Y_i(1) - Y_i(0)$ No!
- $\tau = \mathbb{E}[Y_i(1) - Y_i(0)]$ No!
- $\hat{\tau} = \mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0]$ Yes!

What are the assumptions of experiments that make $\hat{\tau} = \tau$?

- $\{Y_i(1), Y_i(0)\} \perp\!\!\!\perp D_i$
- SUTVA

Inference

- What is statistical inference? The process of learning about underlying population parameters using data. Statistical inference is about estimation and involves uncertainty.
- Do we still have uncertainty about our estimate of $\hat{\tau}$, even if our assumptions hold? Yes. Because of sampling variation (potentially) and variation in D_i (treatment assignment).
- What if we have infinite data? Yes. Because of variation in D_i.

3/ Data Analysis with Python/R