Problem Set 1

Problem 1-1 Sort the functions in increasing order

a) $f_1(n) = n^{01999999} \log n$

$f_2(n) = 10000000n$ $\quad f_1 \leq f_2 < f_4 < f_3$

$f_3(n) = 1,000001^n$

$f_4(n) = n^2$

b) $f_1(n) = 2^{2^{1000000}} \rightarrow O(1)$

$f_2(n) = 2^{100000n} \rightarrow \Theta(2^n)$

$f_3(n) = \binom{n}{2} \rightarrow \frac{n!}{(n-2)!\,2!} = \frac{n(n-1)}{2} \rightarrow O(n^2)$

$f_4(n) = n\sqrt{n} \rightarrow n \times n^{\frac{1}{2}} = n^{3/2} \rightarrow O(n^{1/5})$

$f_1 < f_4 < f_3 < f_2$

c) $f_1(n) = n^{\sqrt{n}}$

$f_2(n) = 2^n \longrightarrow O(2^n)$

$f_3(n) = n^{10} \cdot 2^{n/2}$

$f_4(n) = \sum_{i=1}^{n} (i+1) \longrightarrow \sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$

$\frac{n(n+1)}{2} + n \leq \frac{n(n+1)+2n}{2} = \frac{n^2+3n}{2}$

$\hookrightarrow f_4 = O(n^2)$

$f_1 \longrightarrow n^{\sqrt{n}} \rightarrow 2^{\log n^{\sqrt{n}}} \longrightarrow 2^{n^{1/2}\log n}$ ↘ Less than Linear

$f_3 \longrightarrow n^{10} \cdot 2^{n/2} \Rightarrow 2^{\log n^{10}} \cdot 2^{n/2} \Rightarrow 2^{10\log n} \cdot 2^{n/2}$

$\longrightarrow 2^{10\log n + n/2} \rightarrow 2^{n/2 + 10\log n}$ → Linear $\rightarrow (2^{\frac{1}{2}})^n$

$f_4 < f_1 < f_3 < f_2$

Problem 1-2 correct asymptotic complexity where $T(n,n)$

a) $T(x,c) = \Theta(x) \quad : c \leq 2$

$T(c,y) = \Theta(y) \quad : c \leq 2$

$T(x,y) = \Theta(x+y) + T(x/2, y/2)$

$\rightarrow T(x,y) = c(x+y) + T(x/2, y/2)$

$\rightarrow T(x,y) = c(x+y) + \frac{c(x+y)}{2} + T(x/4, y/4)$

$\rightarrow c(x+y) + \frac{c(x+y)}{2} + \frac{c(x+y)}{4} + \cdots$ up bound $\rightarrow 2c(x+y)$

factor $(x+y)$ $(c + \frac{c}{2} + \frac{c}{4} + \cdots)$ for n,n $\longrightarrow (n+n) = 2n$

constant

$\longrightarrow T(n,n) = \Theta(n)$

b) $T(x,c) = \Theta(x) \quad : c \leq 2$

$T(c,y) = \Theta(y) \quad : c \leq 2$

$T(x,y) = \Theta(x) + T(x, y/2)$

تاریخ: / /

موضوع:

$T(x,y) = \Theta(x) + T(x, \frac{y}{2})$

$\hookrightarrow T(x,y) = cx + T(x, y/2)$

$\rightarrow T(x,y) = cx + cx + T(x, y/4)$

$\rightarrow \underbrace{cx + cx + cx + \cdots + cx}_{\log y}$

$\rightarrow T(x,y) = x \log y \rightarrow T(n,n) = \Theta(n \log n)$

c) $T(x,c) = \Theta(x) : c \leq 2$

$T(x,y) = \Theta(x) + S(x, \frac{y}{2})$

$S(c,y) = \Theta(y) : c \leq 2$

$S(x,y) = \Theta(y) + T(x/2, y)$

$\rightarrow T(x,y) = \Theta(x) + \Theta(y/2) + T(x/2, y/2)$

$\rightarrow T(x,y) = cx + c\,y/2 + T(x/2, y/2)$

$\rightarrow T(x,y) = cx + c\,y/2 + c\,x/2 + c\,y/4 + c\,x/4 + c\,y/8 + \cdots$

$\rightarrow T(x,y) = c\frac{(2x+y)}{2} + c\frac{(2x+y)}{4} + \cdots$

bound $\hookrightarrow T = 2c(2x+y) \rightarrow T(n,n) = 2c(3n) \rightarrow \Theta(n)$