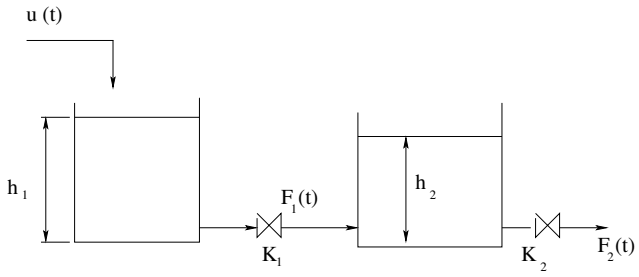
CHE 576 Assignment 3
due: February 10, 2012

Feb 3, 2012

Q1

(25 marks) Consider a model of a simple two tank system given in the Figure below:



Assuming that the flow rate, $u(t)$ is input and flow $F_2(t)$ is output, where K_1, K_2 (valve characteristics) and A_1 and A_2 (tank diameter) are constants:

- Find the continuous-time state space representation of the system. (hint: let $x_1 = h_1, x_2 = h_2$).
- Find discrete state space model representation for the following parameters $K_1 = 1$, $K_2 = 0.5$, $A_1 = 2$ $A_2 = 1$. and sampling time $\triangle t = 0.1$.
- Find the overall transfer function by using $\mathcal{Z}$-transform.
- Find the zero-input state response $x_{zero\ input}(n)$.
- Find the zero-state response $x_{zero\ state}(n)$ for the step input.
- Find the overall system response for the initial conditions $x(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Q2

(10 marks) Consider a process described by linear difference equation given as:

$$x(k+2) + 3x(k+1) + 2x(k) = u(k) \tag{1}$$

and initial conditions $x(0) = 1$ and $x(1) = -4$. Find the time response of the system ($x(n)$) to the unit step input.

Q3

(10 marks) For a given transfer function $G(z) = \frac{z(z+1)}{(z-1)^2}$ of the plant find an impulse response by the following methods:

- Power series method (use 5 terms)
- The partial fraction method

Confirm that both methods lead to the same result.

Q4

(25 marks) For the tank system model given in the Figure below by the following transfer function $G(z) = \frac{1}{z+0.1}$, find the response of the system to the step input signal. Comment what is response if $n \to \infty$ and graphically show the evolution of the state from $y(n = 0)$ to $y(n = 15)$.