CHE 573- Digital Signal Processing for Process Engineers

Solution to Assignment #4

Solution Provided by: Fariborz Kiasi

Q.1.

$x(t) = \sin(2\pi(100)t)$, using $t = nT$ we will have:

$x[n] = \sin(2\pi(100)nT) = \sin\left(2\pi(100)\frac{n}{400}\right) = \sin\left(\frac{n\pi}{2}\right)$

Q.2. Note that for periodicity $\omega N = 2\pi k \ \rightarrow \ N = (2\pi k)/\omega$ where N should be an integer.

a.
$\omega = 0.01\pi \ \rightarrow \ N = \frac{2\pi k}{0.01\pi} = 200k \ \ for\ k = 1: \ \boldsymbol{N = 200\ Samples}\ (fundamental\ period)$

b. $\omega = \frac{30}{105}\pi \ \rightarrow N = \frac{2\pi k}{\frac{30}{105}\pi} = \frac{21}{3}k \ \ for\ k = 1: \ \boldsymbol{N = 7\ Samples}\ \ (fundamental\ period)$

c. $\omega = 3\pi \ \rightarrow \ N = \frac{2\pi k}{3\pi} = \frac{2}{3}k \ \ fro\ k = 3: \ \boldsymbol{N = 2\ Samples}\ \ (fundamental\ period)$

d. $\omega = 3 \ \rightarrow N = \frac{2\pi k}{3} \neq integer \ \rightarrow \boldsymbol{it\ is\ not\ periodic}$

e.
$\omega = \frac{62}{10}\pi \ \rightarrow \ N = \frac{2\pi k}{\frac{62}{10}\pi} = \frac{20}{62}k = \frac{10}{31}k \ \ for\ k = 31: \ \boldsymbol{N = 10\ Samples}(fundamental\ period)$

Q.3.

Consider the following:

$$x(t) = Acos(2\pi Ft + \varphi) \xrightarrow{t=nT_s} x[n] = Acos\left(2\pi \frac{F}{F_s} n + \varphi\right) = Acos(2\pi fn + \varphi)$$

Note that $F/F_s \triangleq f$

$$x[n] = \cos\left(\frac{\pi}{4}n\right) \rightarrow \frac{2\pi F}{F_s} = \frac{\pi}{4} \rightarrow \frac{F}{F_s} = \frac{1}{8} \rightarrow F = \frac{F_s}{8} = \frac{1000}{8} = \frac{250}{2}$$

$$\rightarrow \Omega_0 = 2\pi F = 2\pi \times \frac{250}{2} = 250\pi$$

The other alternative solution can be $\Omega_0 = -250\pi$, due to the fact that cosine is an even function. However a more general solution is as follows:

$$t = nT_s = \frac{n}{F_s} \rightarrow n = F_s t$$

$$\text{x[n]} = \cos\left(\frac{\pi}{4}n\right) = \cos\left(\left(2\pi k \pm \frac{\pi}{4}\right)n\right) \xrightarrow{n=F_s t} x(t) = \cos\left(\left(2\pi k \pm \frac{\pi}{4}\right)F_s t\right)$$

$$\cos\left(\left(2\pi k \pm \frac{\pi}{4}\right)F_s t\right) = \cos(\Omega_0 t) \rightarrow \left(2\pi k \pm \frac{\pi}{4}\right)F_s = \Omega_0$$

$$For\ k = 0:\ \Omega_0 = \left(\pm\frac{\pi}{4}\right)F_s = \pm 250\pi$$

$$For\ k = 1:\ \Omega_0 = \left(2\pi \pm \frac{\pi}{4}\right)F_s \rightarrow \Omega_0 = 2250\pi\ \ or\ \ \Omega_0 = 1750\pi$$

Important Point:

"$\cos(2\pi Ft)$" is aliased with "$\cos(2\pi F_1 t)$" for all $F = F_1 + kF_s$ where k is an integer and F_s is the sampling frequency.

PROOF.

$$\mathrm{x(t)} = \cos(2\pi F_1 t + \varphi) \xrightarrow{t=\frac{n}{F_s}} x[n] = \cos\left(2\pi \frac{F_1}{F_s} n + \varphi\right) = \cos\left(2\pi \frac{F_1}{F_s} n + 2\pi k + \varphi\right)$$

$$\rightarrow x[n] = \cos\left(2\pi \frac{F_1 + kF_s}{F_s} n + \varphi\right) = \cos\,(2\pi \frac{F}{F_s} n + \varphi)$$

Q.4.

a.

$$x(t) = \cos(4000\pi t) \xrightarrow{t=nT} x[n] = \cos(4000\pi\, nT) \rightarrow \cos(4000Tn) = \cos\left(\frac{\pi}{3} n\right)$$

$$\rightarrow 4000\pi T = \frac{\pi}{3} \rightarrow T = \frac{1}{12000}$$

b. other choices for T:

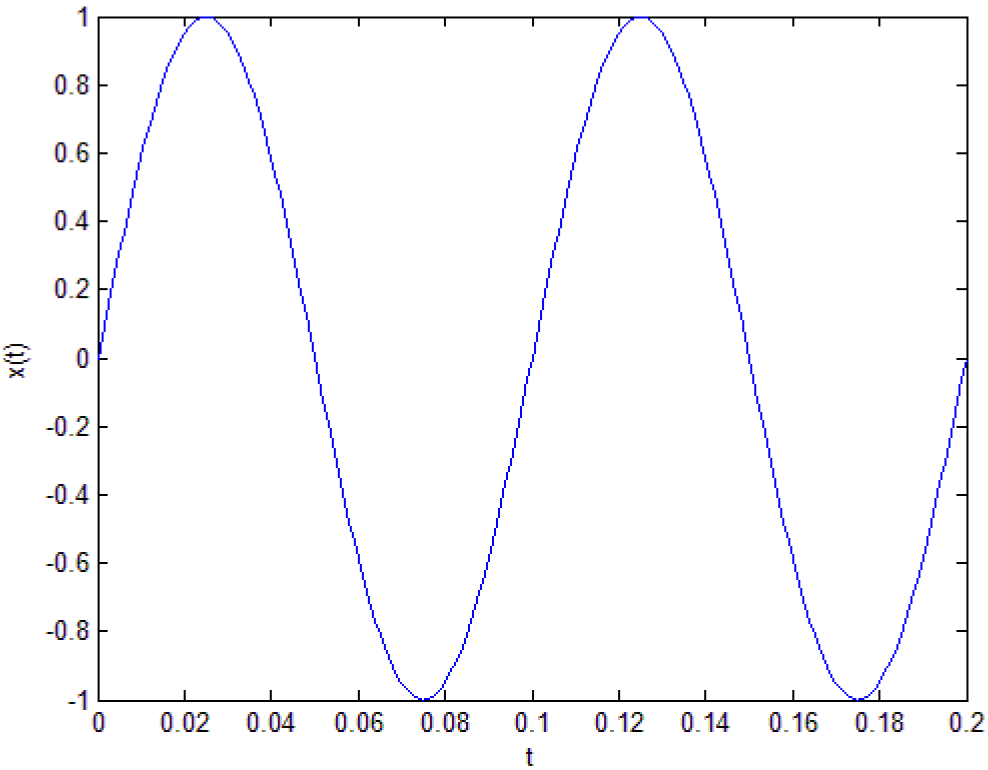
$$\cos\left(\frac{\pi}{3} n\right) = \cos\left(2\pi k \pm \frac{\pi}{3}\right)$$

$$for\ k = 1: \quad \frac{7\pi}{3} = 4000\pi T \rightarrow T = \frac{7}{12000} \quad or \quad \frac{5\pi}{3} = 4000\pi T \rightarrow T = \frac{5}{12000}$$

Q.5.

a.

```
t=0:0.001:.2
x=sin(2*pi*10*t);
plot(t,x);
xlabel('t');ylabel('x(t)')
```



b.

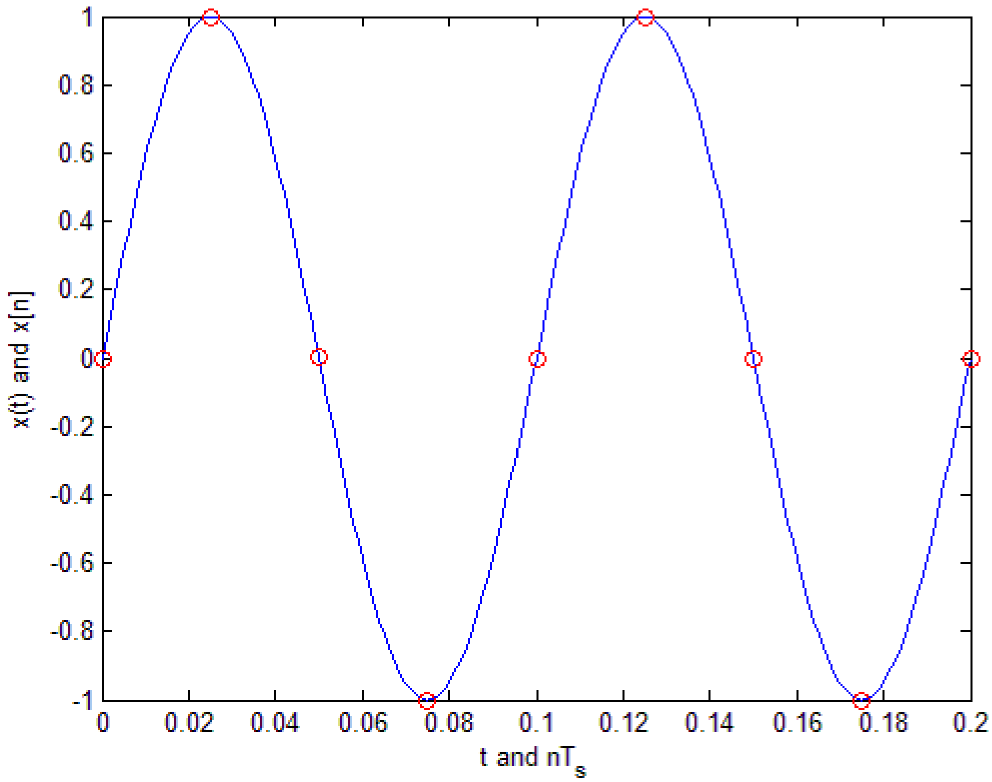
$$x(t) = \sin(2\pi(10)t) \xrightarrow{t=n/F_s} x[n] = \sin\left(\frac{2\pi(10)n}{F_s}\right) = \sin\left(\frac{2\pi(10)n}{40}\right) = \sin\left(\frac{\pi}{2}n\right)$$

For periodicity $= 2\pi k$:

$$N = \frac{2\pi k}{\omega} = \frac{2\pi k}{\frac{\pi}{2}} = 4k$$

$$for\ k = 1{:}\ \boldsymbol{N = 4\ Samples}\ \ (fundamental\ period)$$

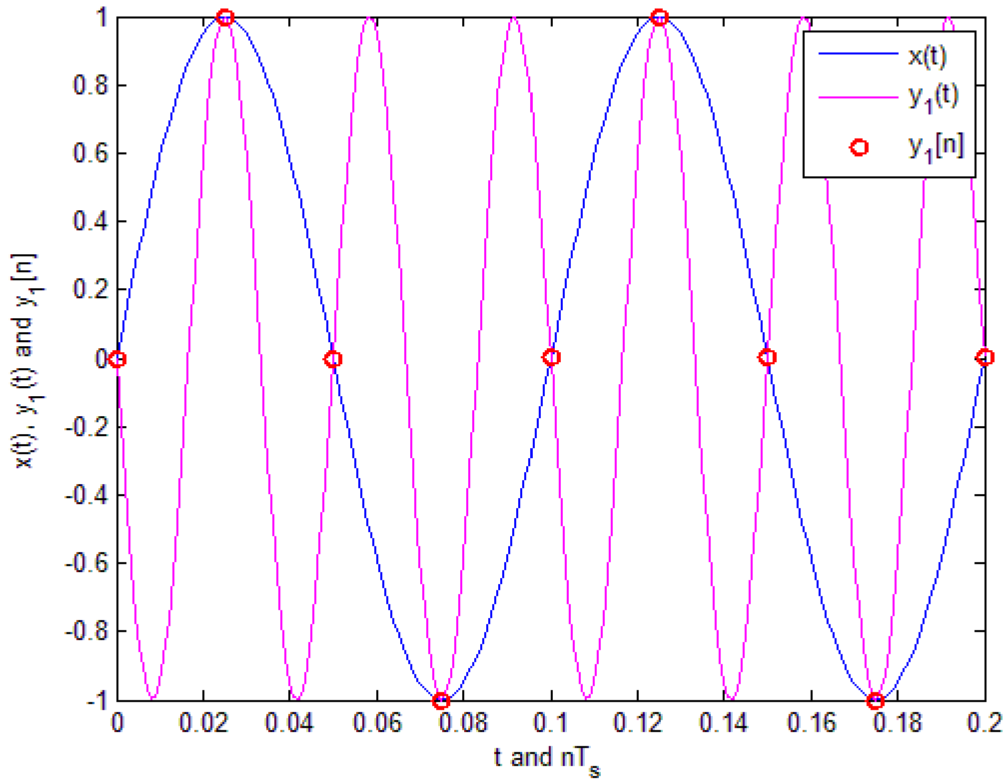
```
t=0:0.001:.2;
x=sin(2*pi*10*t);
plot(t,x);
xlabel('t');ylabel('x(t)')
hold on
n=0:1/40:0.2;
xn=sin(2*pi*10*n);
plot(n,xn,'ro')
xlabel('t and nT_s');ylabel('x(t) and x[n]')
```



c.

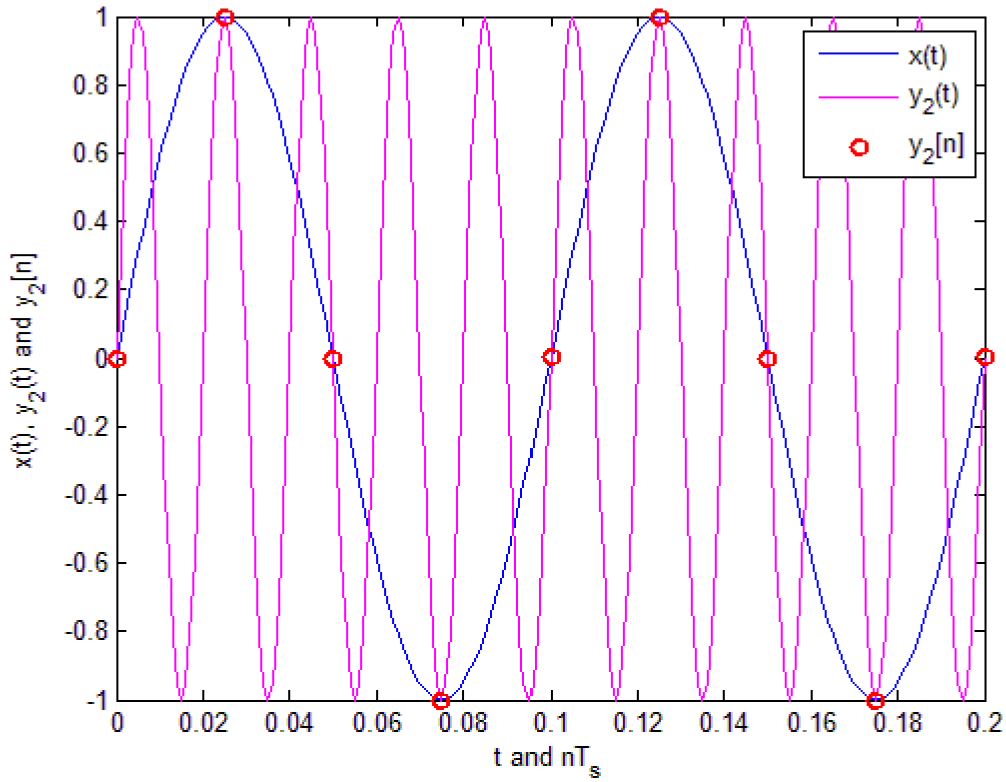
1. $y_1[n] = \sin(-2\pi 30 n T_s)$

```
t=0:0.001:.2;
x=sin(2*pi*10*t);
plot(t,x);
xlabel('t');ylabel('x(t)')
hold on
y1=sin(-2*pi*30*t);
plot(t,y1,'m')
n=0:1/40:0.2;
xn=sin(-2*pi*30*n);
plot(n,xn,'ro','LineWidth',2)
xlabel('t and nT_s');ylabel('x(t), y_1(t) and y_1[n]')
legend('x(t)','y_1(t)','y_1[n]')
```



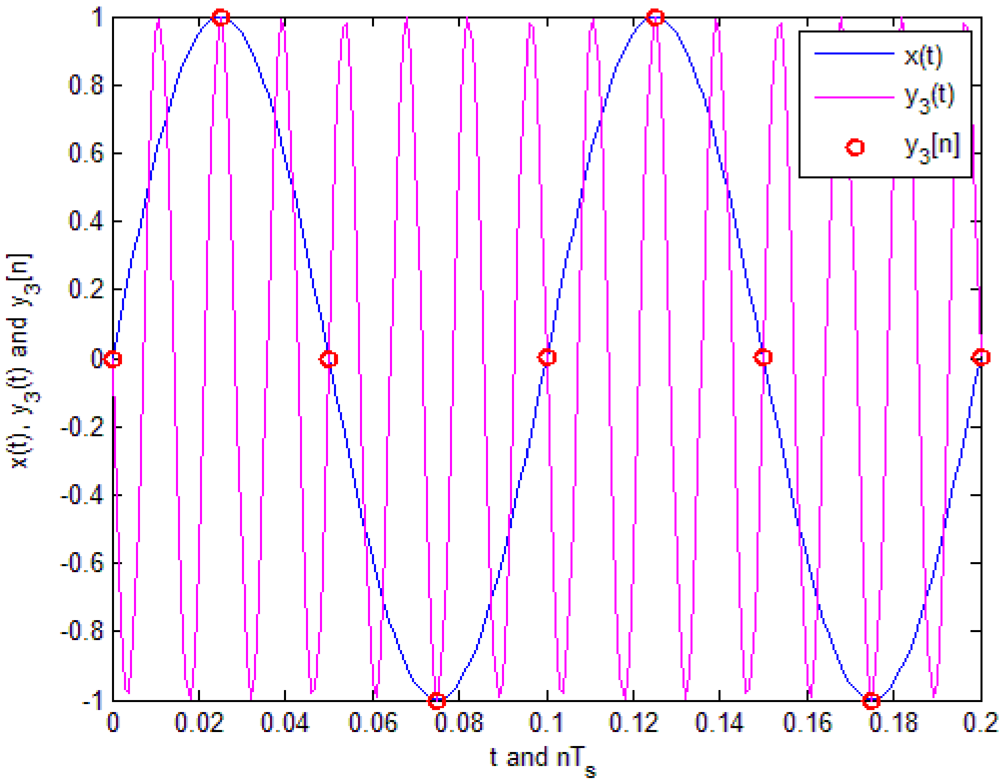
2. $y_2[n] = \sin{(2\pi 50 n T_s)}$

```
t=0:0.001:.2;
x=sin(2*pi*10*t);
plot(t,x);
xlabel('t');ylabel('x(t)')
hold on
y2=sin(2*pi*50*t);
plot(t,y2,'m')
n=0:1/40:0.2;
xn=sin(2*pi*50*n);
plot(n,xn,'ro','LineWidth',2)
xlabel('t and nT_s');ylabel('x(t), y_2(t) and y_2[n]')
legend('x(t)','y_2(t)','y_2[n]')
```



3. $y_3[n] = \sin(-2\pi 70 n T_s)$

```
t=0:0.001:.2;
x=sin(2*pi*10*t);
plot(t,x);
xlabel('t');ylabel('x(t)')
hold on
y3=sin(-2*pi*70*t);
plot(t,y3,'m')
n=0:1/40:0.2;
xn=sin(-2*pi*70*n);
plot(n,xn,'ro','LineWidth',2)
xlabel('t and nT_s');ylabel('x(t), y_3(t) and y_3[n]')
legend('x(t)','y_3(t)','y_3[n]')
```



Recall that: "$\sin(2\pi F t)$" is aliased with "$\sin(2\pi F_1 t)$" for all $F = F_1 + kF_s$ where k is an integer and F_s is the sampling frequency.

<u>*Note that all signals here are aliases of the 10Hz signal. But, we do not say that they are aliases of each other. Alias refers to the signal with **the lowest frequency** for which a high frequency signal could be mistaken for.*</u>

Q.6.

1.

$$y_1(t) = \sin(2\pi 40t) \xrightarrow{t=\frac{n}{F_s}} y_1[n] = \sin\left(2\pi \frac{40}{F_s} n\right) = \sin\left(\left(2\pi \frac{40}{40} + 2\pi k\right) n\right)$$
$$= \sin\left[\left(\frac{40}{40} + k\right) 2\pi n\right]$$

$$for\ k = 1:\ y_1[n] = \sin\left(\frac{80}{40} 2\pi n\right) \rightarrow 80Hz\ is\ an\ alias\ of\ 40Hz\ when\ F_s = 40\ Hz$$

$$for\ k = -1:\ y_1[n] = \sin\left(\frac{40-40}{40} 2\pi n\right) \rightarrow 40Hz\ is\ an\ alias\ of\ 0Hz\ when\ F_s = 40\ Hz$$

$$for\ k = 2:\ y_1[n] = \sin\left(\frac{120}{40} 2\pi n\right) \rightarrow 120Hz\ is\ an\ alias\ of\ 40Hz\ when\ F_s = 40\ Hz$$

2.

$$y_1(t) = \sin(2\pi 80t) \xrightarrow{t=\frac{n}{F_s}} y_1[n] = \sin\left(2\pi \frac{80}{F_s} n\right) = \sin\left(\left(2\pi \frac{80}{40} + 2\pi k\right) n\right)$$
$$= \sin\left[\left(\frac{80}{40} + k\right) 2\pi n\right]$$

$$for\ k = 1:\ y_1[n] = \sin\left(\frac{120}{40} 2\pi n\right) \rightarrow 120Hz\ is\ an\ alias\ of\ 40Hz\ when\ F_s = 40\ Hz$$

$$for\ k = -1:\ y_1[n] = \sin\left(\frac{40}{40} 2\pi n\right) \rightarrow 80Hz\ is\ an\ alias\ of\ 40Hz\ when\ F_s = 40\ Hz$$

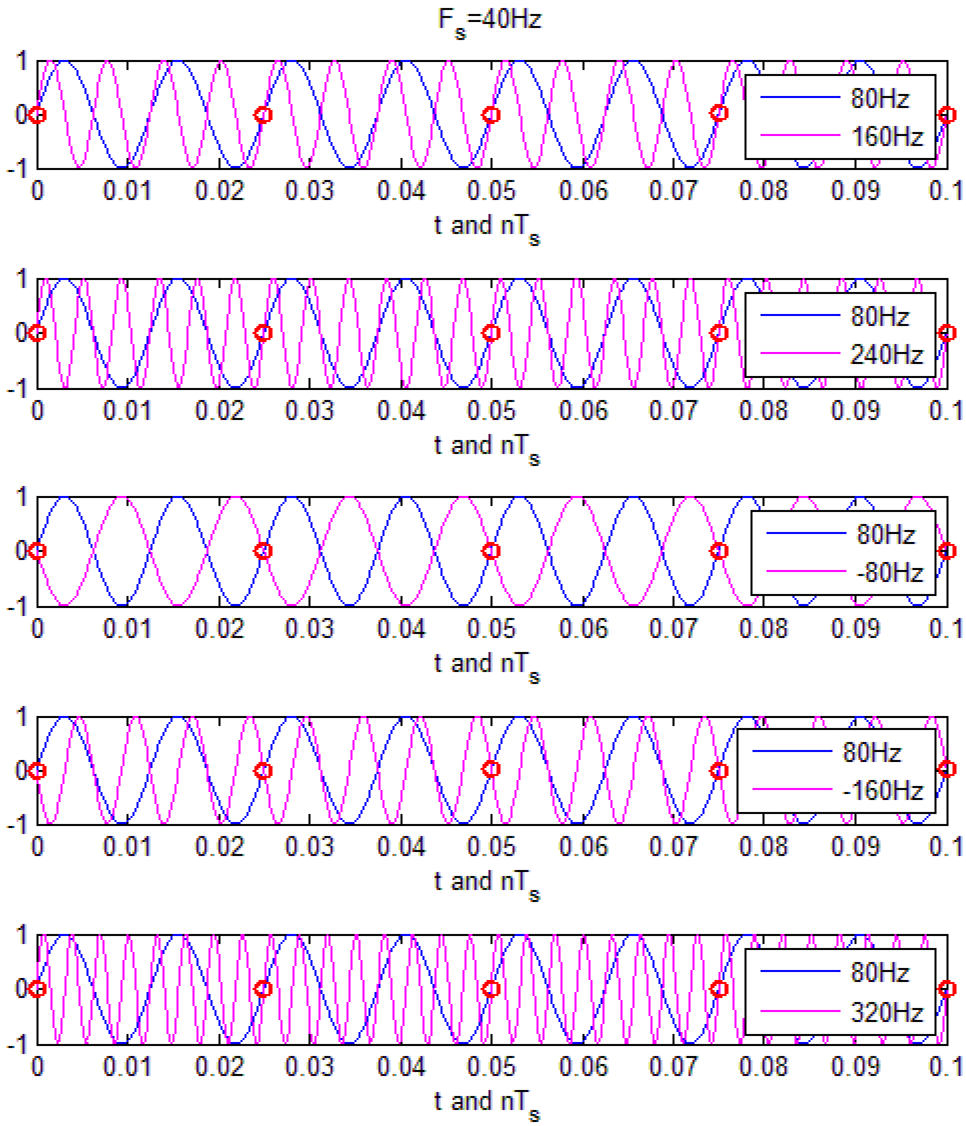
$$for\ k = -2:\ y_1[n] = \sin\left(\frac{80-80}{40} 2\pi n\right) \rightarrow 80Hz\ is\ an\ alias\ of\ 0Hz\ when\ F_s = 40\ Hz$$

Note that both 40Hz and 80Hz signals are aliases of 0Hz when the sampling frequency is 40Hz.

MATLAB code for 40Hz signal and aliases:

```
clc
clear all
close all
t=0:0.0001:0.2;
n=0:1/40:0.2;
```

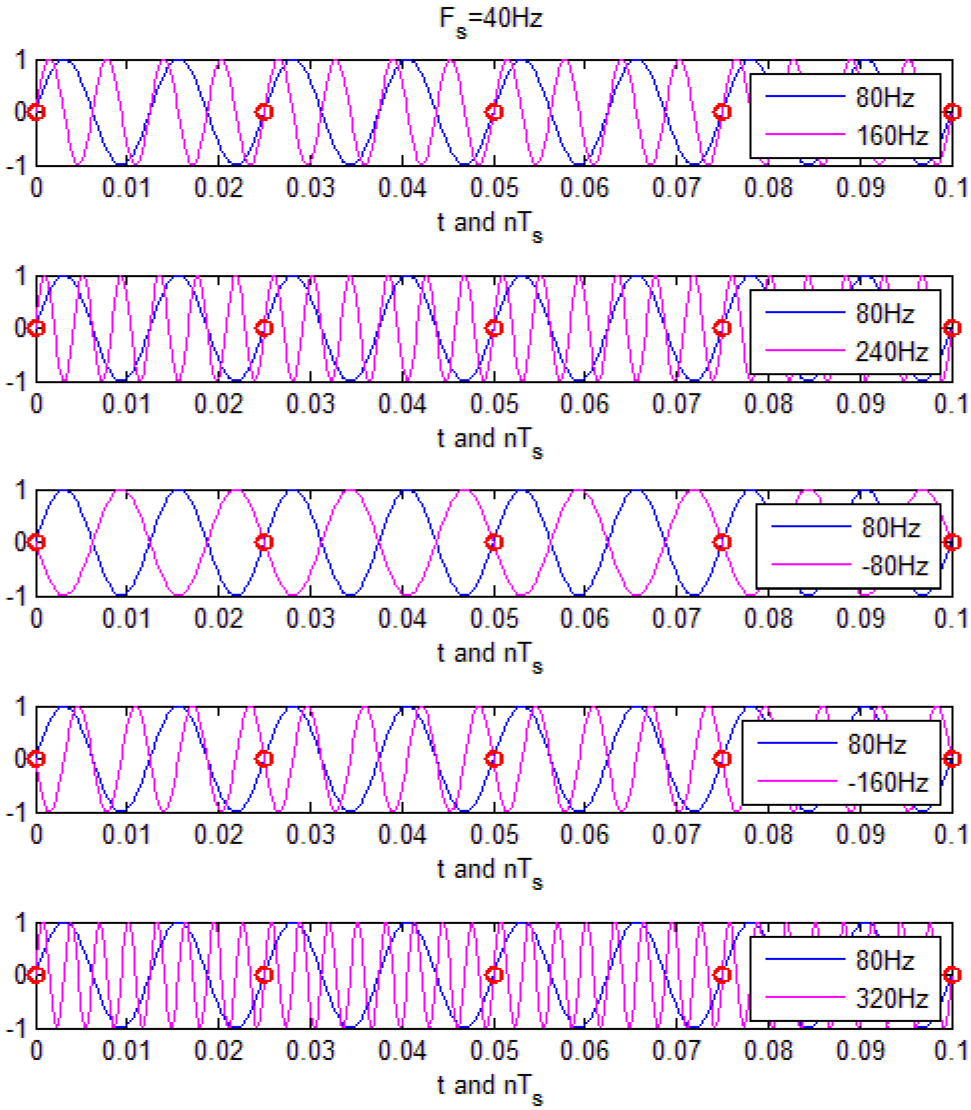
```
y1=sin(2*pi*40*t);
y11=sin(2*pi*80*t);
y12=sin(2*pi*120*t);
y13=sin(2*pi*-40*t);
y14=sin(2*pi*-80*t);
y15=sin(2*pi*160*t);

y11d=sin(2*pi*80*n);
y12d=sin(2*pi*120*n);
y13d=sin(2*pi*-40*n);
y14d=sin(2*pi*-80*n);
y15d=sin(2*pi*160*n);

subplot(5,1,1)
plot(t,y1);
hold on
plot(t,y11,'m')
plot(n,y11d,'ro','LineWidth',2)
xlabel('t and T_s');
% legend('40Hz','80Hz','Sampled Signal')
legend('40Hz','80Hz')
title('F_s=40Hz')
subplot(5,1,2)
plot(t,y1);
hold on
plot(t,y12,'m')
plot(n,y12d,'ro','LineWidth',2)
xlabel('t and nT_s');
% legend('40Hz','120Hz','Sampled Signal')
legend('40Hz','120Hz')
subplot(5,1,3)
plot(t,y1);
hold on
plot(t,y13,'m')
plot(n,y13d,'ro','LineWidth',2)
xlabel('t and nT_s');
% legend('40Hz','-40Hz','Sampled Signal')
legend('40Hz','-40Hz')
subplot(5,1,4)
plot(t,y1);
hold on
plot(t,y14,'m')
plot(n,y14d,'ro','LineWidth',2)
xlabel('t and nT_s');
% legend('40Hz','-80Hz','Sampled Signal')
legend('40Hz','-80Hz')
subplot(5,1,5)
plot(t,y1);
hold on
plot(t,y15,'m')
plot(n,y15d,'ro','LineWidth',2)
xlabel('t and nT_s');
% legend('40Hz','160Hz','Sampled Signal')
legend('40Hz','160Hz')
```



F_s=40Hz
80Hz
160Hz
240Hz
-80Hz
-160Hz
320Hz
t and nT_s

MATLAB code for 80Hz signal and aliases:

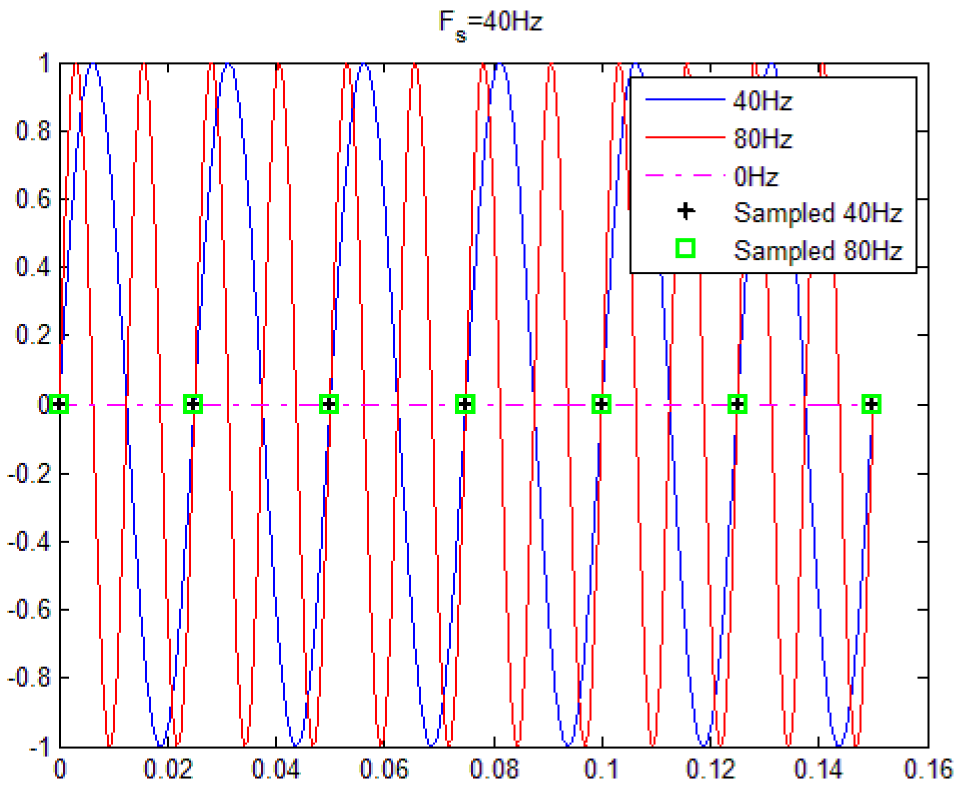
```
clc
clear all
close all
t=0:0.0001:0.1;
n=0:1/40:0.1;
y1=sin(2*pi*80*t);
y11=sin(2*pi*160*t);
y12=sin(2*pi*240*t);
y13=sin(2*pi*-80*t);
y14=sin(2*pi*-160*t);
y15=sin(2*pi*320*t);

y11d=sin(2*pi*160*n);
y12d=sin(2*pi*240*n);
y13d=sin(2*pi*-80*n);
y14d=sin(2*pi*-160*n);
y15d=sin(2*pi*320*n);

subplot(5,1,1)
plot(t,y1);
hold on
plot(t,y11,'m')
plot(n,y11d,'ro','LineWidth',2)
xlabel('t and nT_s');
legend('80Hz','160Hz')
title('F_s=40Hz')
subplot(5,1,2)
plot(t,y1);
hold on
plot(t,y12,'m')
plot(n,y12d,'ro','LineWidth',2)
xlabel('t and nT_s');
legend('80Hz','240Hz')
subplot(5,1,3)
plot(t,y1);
hold on
plot(t,y13,'m')
plot(n,y13d,'ro','LineWidth',2)
xlabel('t and nT_s');
legend('80Hz','-80Hz')
subplot(5,1,4)
plot(t,y1);
hold on
plot(t,y14,'m')
plot(n,y14d,'ro','LineWidth',2)
xlabel('t and nT_s');
legend('80Hz','-160Hz')
subplot(5,1,5)
plot(t,y1);
hold on
plot(t,y15,'m')
plot(n,y15d,'ro','LineWidth',2)
xlabel('t and nT_s');
legend('80Hz','320Hz')
```



Note that both 40Hz and 40Hz signals are aliases of 0Hz signal when $F_s = 40Hz$.

```
clc
clear all
close all
t=0:0.0001:0.15;
n=0:1/40:0.15;
y1=sin(2*pi*40*t);
y2=sin(2*pi*80*t);
y3=zeros(size(y1));
y1d=sin(2*pi*40*n);
y2d=sin(2*pi*80*n);
```

```
plot(t,y1,t,y2,'r',t,y3,'-.m')
hold on
plot(n,y1d,'k+','LineWidth',2)
plot(n,y2d,'gs','LineWidth',2)
legend('40Hz','80Hz','0Hz','Sampled 40Hz','Sampled 80Hz')
title('F_s=40Hz')
```



As it can be seen from the folding frequency plot, both 40Hz and 80Hz sinusoids are aliases of the 0Hz one.