CHE 576 ASSIGNMENT 4 SOLUTIONS

Q1. (8 marks)

- Poles: $z = 0.74 \pm 0.5i\sqrt{1.35}$, $\|z\| = 0.95 < 1 \Rightarrow$ stable.
- Poles: $z = \{2.192, 0.4 \pm 0.25i\}$, $2.192 > 1 \Rightarrow$ unstable.
- Poles: $z = \{0.35, 0.82 \pm 0.86i\}$, $\|0.82 + 0.86i\| = 1.92 > 1 \Rightarrow$ unstable.
- Poles: $z = \{0.7, 0.5 \pm 0.87i\}$, $\|0.5 + 0.87i\| = 1 \Rightarrow$ marginally stable.

Q2. (2 marks)

$$A = \begin{pmatrix} -0.5 & 1 \\ 0 & 0.5 \end{pmatrix}, \quad \text{eigenvalues}: \quad \lambda_1 = -0.5, \quad \lambda_2 = 0.5, \quad \|\lambda_i\| < 1 \Rightarrow \text{ stable}$$

$x(5) = Ax(4) = \cdots = A^5 x(0)$. Using Jordan form: $J = P^{-1}AQ$:

$$A^n = (PJP^{-1})(PJP^{-1})\cdots(PJP^{-1}) = PJ^nP^{-1}$$

Eigenvectors of A: $\phi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\phi_2 = \begin{pmatrix} 0.7071 \\ 0.7071 \end{pmatrix}$.

$$P = \begin{pmatrix} 1 & 0.7071 \\ 0 & 0.7071 \end{pmatrix}, \quad J = \begin{pmatrix} -0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \quad A^5 = PJ^5P^{-1} = \begin{pmatrix} -0.0312 & -0.0625 \\ 0 & 0.0312 \end{pmatrix}$$

Then:

$$x(5) = A^5 x(0) = \begin{pmatrix} -0.0312 & -0.0625 \\ 0 & 0.0312 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -0.0938 \\ -0.0312 \end{pmatrix}$$

Q3. (20 marks)

a) z-transfrom of signal sequence:

$$\mathcal{Z}\{0.21\}^n = \frac{z}{z - 0.21} \Longrightarrow \text{Let } G_f(z) = \frac{z - 0.21}{z}$$

Open loop transfer function:

$$G_{OL} = G_p G_f = \frac{1}{z^2 - 0.34z + 0.024} \frac{z - 0.21}{z} = \frac{z - 0.21}{z(z - 0.1)(z - 0.24)}$$

Poles of G_{OL} are $z_i = \{0, 0.1, 0.24\}$ so that $\|z_i\| < 1 \Rightarrow G_{OL}$ is stable.

b) Closed loop transfer function:

$$G_{CL} = \frac{G_p G_f}{1 + G_p G_f} = \frac{z - 0.21}{(z^2 - 0.1293z + 0.9968)(z - 0.2107)}$$

Poles of G_{CL} are $z_i = \{0.0647 \pm 0.9963i, 0.2107\}$, and $\|0.0647 \pm 0.9963i\| = 0.9984 \simeq 1$ (marginally stable). Look for new filter in the form of:

$$G_f^* = \frac{z - 0.21}{z + \alpha}$$

such that the closed loop system is stable and it's response is less oscillatory.

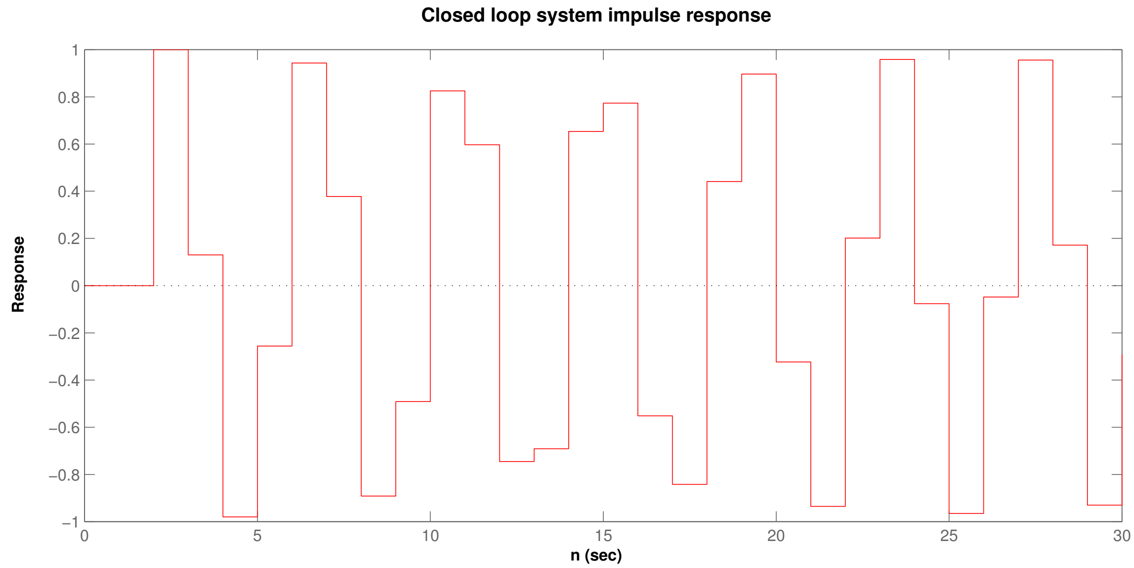

FIGURE 1. System response of G_{CL} to unit impulse

Closed loop transfer function:

$$G^*_{CL} = \frac{G_p G^*_f}{1 + G_p G^*_f} = \frac{z - 0.21}{z^3 + (\alpha - 0.34)z^2 + (1.024 - 0.34\alpha)z + (0.024\alpha - 0.21)}$$

Stability: Poles of G^*_{CL} satisfy $\|z_i\| < 1$ if $0 \le \alpha \le 1.8$, i.e. Let $\alpha = 1.5$:

$$z_i = \{0.2149, -0.6875 \pm 0.5804i\}, \quad \| - 0.6875 \pm 0.5804\| = 0.8997 < 1 \Rightarrow G^*_{CL} \text{ is stable}$$

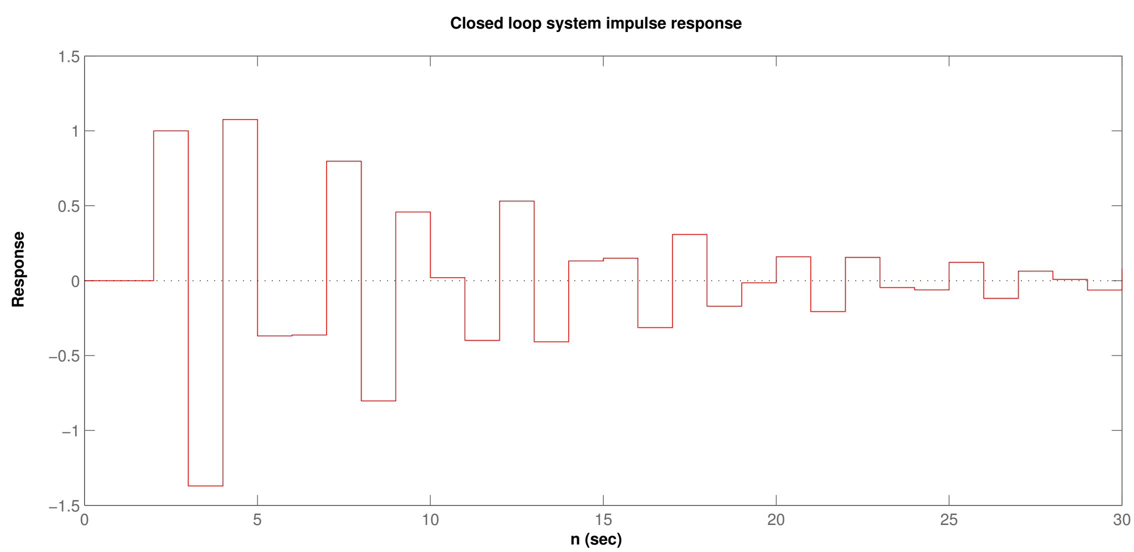

FIGURE 2. System response of G^*_{CL} with $\alpha = 1.5$ to unit impulse