Other Model Types: ARMA, Impulse response and Step-response models

$$y(k) = \underbrace{\frac{B(z^{-1})}{A(z^{-1})} \cdot u(k)}_{\text{ARMA model}} = \underbrace{\sum_{i=1}^{\infty} h_i \cdot u(k-i)}_{\text{Impulse response}} = \frac{B(z^{-1})}{A(z^{-1}) \cdot \Delta} \cdot \Delta u(k) = \underbrace{\sum_{i=1}^{\infty} S_i \cdot \Delta u(k-i)}_{\text{Step response}}$$

- All models are equivalent.
- Step response and Impulse response models are easy to understand and relate to by plant operating personnel.
- Empirical identification of a step model is relatively simple. In the simplest case it involves a step test.
- Step and impulse response models can account for the past history of the process via past input values.

Correlation Analysis

Consider a sampled data system with the impulse response coefficients $\{ h_k \}$:

$$y(t) = \sum_{k=0}^{\infty} h_k u(t-k) + v(t)$$

Let $\{ u(t) \}$ be a signal that is a realization of a stochastic process with zero mean value and covariance function $\nu_{uu}(\tau)$:

$$\nu_{uu}(\tau) = Eu(t)u(t-\tau)$$

and assume that $\{u(t)\}$ and $\{v(t)\}$ are uncorrelated. The cross variance function between u and y is then:

$$\nu_{yu}(\tau) = Ey(t)u(t-\tau) = \sum_{k=0}^{\infty} h_k Eu(t-k)u(t-\tau) + Ev(t)u(t-\tau) = \sum_{k=0}^{\infty} h_k \nu_{uu}(\tau - k)$$

If the input is white noise,

$$\nu_{uu}(\tau) = \begin{cases} \sigma_u^{\,2}, \; \tau = k \\ 0, \; \tau \neq k \end{cases}$$

we obtain

$$\nu_{yu}(\tau) = \sigma_u^{\,2} h_\tau$$

The cross covariance function $\nu_{yu}(\tau)$ *will thus be proportional to the impulse response coefficient at* $k=\tau$. *This function can be estimated via the Matlab function* `'cra'`.

Algorithm CRA

1. Collect data *y(k), u(k),* $k=1,\ldots,N$
2. Subtract sample means from each signal: $\tilde{y}(k) = y(k) - \bar{y}$ and $\tilde{u}(k) = u(k) - \bar{u}$
3. Form the signals: $y_f(t) = L(q)\tilde{y}(t)$ and $u_f(t) = L(q)\tilde{u}(t)$
4. Form the estimates

$$\hat{\nu}_{y_f u_f}(\tau) = \frac{1}{N}\sum_{t=1}^{N} y_f(t) u_f(t-\tau)$$

$$\hat{\sigma}_{u(N)}^{\;\;2} = \frac{1}{N}\sum_{t=1}^{N} u_f^2(t)$$

5. The impulse response estimate is now: $\hat{h}_\tau^N = \dfrac{\nu_{y_f u_f}^N(\tau)}{\hat{\sigma}_{u(N)}^{\;\;2}}$

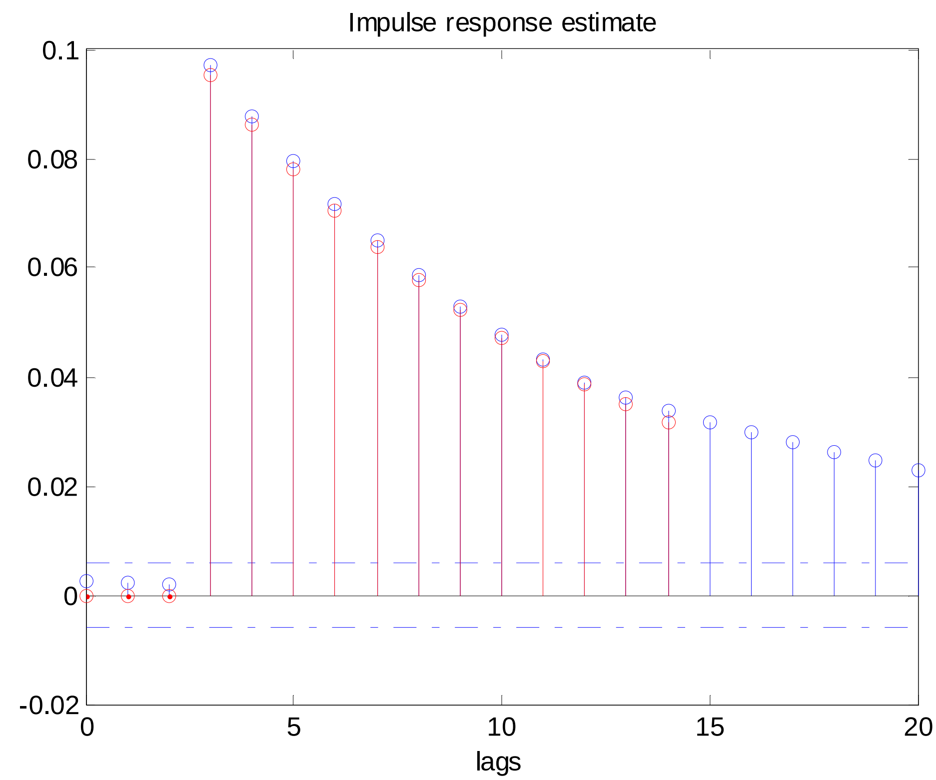
```
>> sysc=tf(1, [10 1], 'iodelay', 2.0);
>> sysc

Transfer function:
              1
exp(-2*s) * --------
            10 s + 1
```

```
>> sysd=c2d(sysc, 1.0);

>> sysd

Transfer function:
          0.09516
z^(-2) * ----------
         z - 0.9048

Sampling time: 1
```

```
>> help deconv
 DECONV Deconvolution and polynomial division.
    [Q,R] = DECONV(B,A) deconvolves vector A out of vector B.  The result
    is returned in vector Q and the remainder in vector R such that
    B = conv(A,Q) + R.

    If A and B are vectors of polynomial coefficients, deconvolution
    is equivalent to polynomial division.  The result of dividing B by
    A is quotient Q and remainder R.

    Class support for inputs B,A:
       float: double, single

    See also conv, residue.
    Reference page in Help browser
       doc deconv

>> [q,r]=deconv([0 0 0 .09516 0 0 0 0 0 0 0 0 0 0 0], [ 1  -0.9048]);
>> q

q=
  Columns 1 through 11
 0       0       0    0.0952    0.0861    0.0779    0.0705    0.0638    0.0577    0.0522   .0472

 Columns 12 through 15
    0.0427    0.0387    0.0350    0.0317
```

```
>> y=filter([0 0 0 .09516],[1  -.9048],u);
>> z=[y u];
>> cra(z, 20)
```

```
ans =
    0.0026
    0.0023
    0.0021
    0.0970
    0.0878
    0.0794
    0.0718
    0.0649
    0.0586
    0.0529
    0.0478
    0.0431
    0.0388
    0.0363
    0.0339
    0.0318
    0.0298
    0.0280
    0.0263
    0.0248
    0.0230
```




```
>> hold;
Current plot held
>> size(q)
ans =      1    15
>> stem([0:14],q, 'r') % See figure above.
```

Spectral Analysis

Similarly for spectral analysis, if a system is represented by:

$$y(t) = G(q)u(t) + H(q)e(t)$$

and if u(t) and e(t) are independent, then we have

$$\phi_y(\omega) = \left|G(e^{jw})\right|^2 \phi_u(\omega) + \left|H(e^{jw})\right|^2 \lambda^2$$
$$\phi_{yu}(\omega) = G(e^{jw})\phi_u(\omega)$$

Note that in using both of these non-parametric type estimation techniques, based on correlations, the main assumption is that *u* and *e* are independent. This is only true for open loop systems. For closed loop systems *u* is dependent on *e* because of feedback.

An empirical transfer functions estimate (function ETFE) or the spectral estimate (function SPA with default or user specified window) can then be obtained from:

$$\hat{G}_N(iw) = \frac{\hat{\phi}_{yu}^N(\omega)}{\hat{\phi}_u^N(\omega)}$$

Furthermore, the disturbance spectrum can be estimated as follows:

$$\phi_y(\omega) = \left|G(i\omega)\right|^2 \phi_u(\omega) + \phi_v(\omega)$$

$$\phi_v(\omega) = \phi_y(\omega) - \left|G(i\omega)\right|^2 \phi_u(w) \quad \Rightarrow \quad \hat{\phi}_v^N(\omega) = \hat{\phi}_y^N(\omega) - \frac{\left|\hat{\phi}_{yu}^N(\omega)\right|^2}{(\hat{\phi}_u^N(\omega))^2}\ \hat{\phi}_u^N(\omega)$$

or

$$\hat{\phi}_v^N(\omega) = \hat{\phi}_y^N(\omega) - \frac{\left|\hat{\phi}_{yu}^N(\omega)\right|^2}{\hat{\phi}_u^N(\omega)}$$