# Review of section material

January 25, 2012

## Sampling of signals

- How to choose a sampling time
- Aliasing

## Discretization of continuous system: $\dot{x}(t) = Ax(t) + Bu(t)$

$$x(t_{k+1}) = e^{A(t_{k+1}-t_k)}x(t_k) + \int_{t_k}^{t_{k+1}} e^{A(t_{k+1}-s)}Bu(s)ds$$

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) \\ \Phi &= e^{A\triangle t} \\ \Gamma &= \int_0^{\triangle t} e^{As} ds B \end{aligned}$$

**For scalar systems:** $\dot{x}(t) = ax(t) + bu(t)$

$$\begin{aligned} \Phi &= e^{a\triangle t} \\ \Gamma &= \int_0^{\triangle t} e^{as} ds\ b = \frac{b}{a}(e^{a\triangle t} - 1) \end{aligned}$$

**For matrix systems:**

$$\begin{aligned} \Phi &= e^{a\triangle t} = \mathcal{L}^{-1}\left[(sI - A)^{-1}\right] \\ \Gamma &= \int_0^{\triangle t} e^{As} ds B \end{aligned}$$

**Discretization of continuous system by finite difference:**

$$
\begin{aligned}
x(k+1) &= \bar{\Phi}x(k) + \bar{\Gamma}u(k) \\
\bar{\Phi} &= 1 + A\triangle t \\
\bar{\Gamma} &= B\triangle t
\end{aligned}
$$

**Discretization of continuous system with time delay $\tau_d$: $\dot{x}(t) = Ax(t) + Bu(t-\tau_d)$**

- For $\triangle t > \tau_d$

$$
\begin{aligned}
x(k+1) &= \Phi x(k) + \Gamma_1 u(k-1) + \Gamma_0 u(k) \\
\Phi &= e^{A\triangle t} \\
\Gamma_0 &= \int_0^{\triangle t - \tau_d} e^{As} ds B \\
\Gamma_1 &= e^{A(\triangle t - \tau_d)} \int_0^{\tau_d} e^{As} ds B
\end{aligned}
$$

- For $\triangle t < \tau_d$ find the $\tau_d'$ and $d$ such that $\tau_d = (d-1)\triangle t + \tau_d'$

$$
\begin{aligned}
x(k+1) &= \Phi x(k) + \Gamma_1 u(k-d) + \Gamma_0 u(k-(d-1)) \\
\Phi &= e^{A\triangle t} \\
\Gamma_0 &= \int_0^{\triangle t - \tau_d'} e^{As} ds B \\
\Gamma_1 &= e^{A(\triangle t - \tau_d')} \int_0^{\tau_d'} e^{As} ds B
\end{aligned}
$$