

CHE 576 Assignment 5

due: March 9, 2010

March 2, 2012

Q1

Given the difference equation:

$$y(k+3) + 3y(k+2) + 2y(k+1) - y(k) = u(k)$$

- Find a state space realization (Hint: use $y(k) = x_1(k), \dots$)?
- Is system controllable
- Find a block diagram by using back-shift operator?
- Is the system observable?
- Find a system transfer function $G(z)$ assuming zero initial conditions.

Q2

Consider the system given by:

$$\mathbf{x}(k+1) = \begin{bmatrix} 2 & 3 & 6 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(k) \quad (1)$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}(k) \quad (2)$$

- Is the system controllable?
- Is the system observable?
- Find a transformation matrix T_c such that (A,B,C,D) is in controllable state space form.
- Find a transformation matrix T_o such that (A,B,C,D) is in observable state space form.

Q3

Consider the discrete-time state space system given by:

$$x(k+1) = \begin{bmatrix} 1 & 1 & -2 \\ -2 & 1 & 1 \\ -7 & 2 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(k) \quad (3)$$

$$y(k) = \begin{bmatrix} 4 & 0 & 0 \end{bmatrix} x(k) \quad (4)$$

- Determine if the system is controllable/observable.
- Find in how many steps one can reach the state $x^* = [a \ 2a \ 3a]$ and find the input sequence $\{u(0), u(1), \dots\}$ in terms of a , assuming that initial condition is $x(0) = [0 \ 0 \ 0]'$.
- If measurements $[y(0) \ y(1) \ y(2)] = [3a \ 2a \ a]$ are given, use the input sequence determined from above and measurement $[y(0) \ y(1) \ y(2)]$ to reconstruct the $x(0)$.