CHE 576 ASSIGNMENT 2 SOLUTIONS

Q1.

Part 1. Model description:

$$\frac{dh}{dt} = -3h + f_{in}(t - 4.1430), \qquad f_{out} = 3h$$

Part 2. Discretization:

$$\Delta t < \tau_d = 4.1430, \quad \tau_d = (d-1)\Delta t + \tau_d^* \Rightarrow d = 5 \Rightarrow \tau_d^* = 0.143$$

$$x(k+1) = e^{-3*1}x(k) - \int_0^{1-0.1430} e^{-3s} ds\; u(k-4) - \int_1^{0.857} e^{-3s} ds\; u(k-5)$$

$$x(k+1) = 0.0498x(k) + 0.308u(k-4) + 0.0089u(k-5)$$

Q2. Continuous system:

$$\frac{dx(t)}{dt} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t - 0.2)$$

Using the sampling interval $\Delta t = 0.3$.

$$x(k+1) = \begin{pmatrix} 1.35 & 0 \\ 0.405 & 1.35 \end{pmatrix} x(k) + \begin{pmatrix} 0.11 \\ 0.0054 \end{pmatrix} u(k) + \begin{pmatrix} 0.25 \\ 0.05 \end{pmatrix}$$

Q3.

- By using the forward shift operator we obtain:

$$\begin{aligned} x(k+2) - 0.5x(k+1) + 0.3x(k) &= u(k+1) \\ q^2x(k) - 0.5qx(k) + 0.3x(q) &= u(k)q \\ x(k) &= \frac{q}{q^2 - 0.5q + 0.3}u(k) \end{aligned}$$

  Then:

$$A(q) = q^2 - 0.5q + 0.3$$

$$B(q) = q$$

- By using the back shift operator we obtain:

$$\begin{aligned} x(k) - 0.5x(k-1) &= u(k-3) + 0.4u(k-5) \\ x(k) - 0.5q^{-1}x(k) &= u(k)q^{-3} + 0.4u(k)q^{-5} \\ x(k) &= \frac{q^{-3} + 0.4q^{-5}}{1 - 0.5q^{-1}}u(k) \\ x(k) &= \frac{1 + 0.4q^{-2}}{1 - 0.5q^{-1}}u(k-3) \end{aligned}$$

(1)

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Then:

$$A^*(q^{-1}) = 1 - 0.5q^{-1}$$

$$B^*(q^{-1}) = 1 + 0.4q^{-1}$$

Q4. See notes Page 61.