CHE 576 - Assignment 1

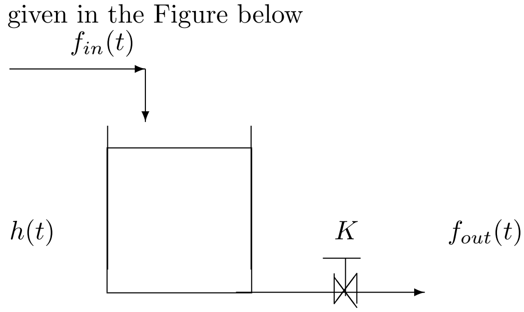
Winter 2011

Due 11:00 a.m., January 25, Wednesday

Q.1: Select the sampling period for the following transfer functions:

1. $\frac{y(s}{u(s)} = \frac{e^{-2s}}{s+0.01}$
2. $\frac{y(s}{u(s)} = \frac{e^{-2s}}{s+0.1}$
3. $\frac{y(s}{u(s)} = \frac{e^{-0.1s}}{(0.3s+1)(s+0.1)}$
4. $\frac{y(s}{u(s)} = \frac{e^{-3s}}{(s+2)(s+0.25)}$
5. $\frac{y(s}{u(s)} = \frac{e^{-2s}}{s+0.5s+0.5}$

Q.2: For the physical processes given in the Figure below



1.) Provide the model description in the form of the physical law that govern the process (Hint: Mass Balance)

2.) Find the state space representation $\Sigma(A, B, C, D)$

3.) Input is flow in the tank $f_{in}(t)$, output is height $h(t)$

4.) Find the corresponding discrete system state space representation $\Sigma(\Phi, \Gamma, \Theta, D)$ when the sample time is $\triangle t = 0.1$

5.) Find a sampling time if $K = 10$, $\bar{A} = 0.1$ (tank base), $\rho = 1$.

Q.3: Consider the first-order differential equation

$$\frac{dy(t)}{dt} - 0.5y(t) = 1.5u(t), \quad y(0) = 3 \tag{1}$$

1.) Find an integral form solution for Eq.1.

2.) Find the difference equation when sample time is $\triangle t = 0.1$ by exact discretization.

3.) Find the difference equation with time derivative approximated at $\triangle t = 0.1$ by finite difference method.

4.) Find a difference equation using MATLAB

5.) Plot the sequence of numbers obtained by the exact difference equation and by finite difference method and applied piecewise constant input given as follows

$$\begin{aligned} u(0) &= -2 \\ u(1\triangle t) &= -2e^{-2*1\triangle t} \\ u(2\triangle t) &= -2e^{-2*2\triangle t} \\ \cdots &= \cdots \\ u(k\triangle t) &= -2e^{-2*k\triangle t} \end{aligned}$$

with k=20.

Q.4: Find a state space representation (that is $\sum_c(A, B, C, D)$) of the system described by the following differential equations:

$$\begin{aligned} &M_1\ddot{x}_1 + C\dot{x}_1 + k_1x_1 - k_2x_2 = f(t) \\ &M_2\ddot{x}_2 + k_2x_2 = 0 \end{aligned} \tag{2}$$

$x_1(0) = x_0$, $\dot{x}_1(0) = x_{10}$, $x_2(0) = x_{20}$, $\dot{x}_2(0) = x_{30}$.
Obtain the discrete state space representations by exact discretization method (that is $\sum_d = (\Phi, \Gamma, \Theta, D)$), and $\triangle t = 0.1$, $M_1 = 1$, $M_2 = 2$, $k_1 = 0.5$, $k_2 = 0.2$, $c = 0.1$. (Hint: Example 1.3 and 2.1 in notes).