CHE 576 Midterm Examination

February 27, 2010

Basic instructions

The midterm is to be completed in the allotted amount of time. Non-programmable calculators may be used. The one side page hand written sheet is allowed and no external formula sheets are allowed. Complete all parts of each question. You must provide brief explanations where required and show all your work to receive full marks. Good luck!

Question 1

Consider the following continuous signal $y(t)$ given in the Fig.1 below:

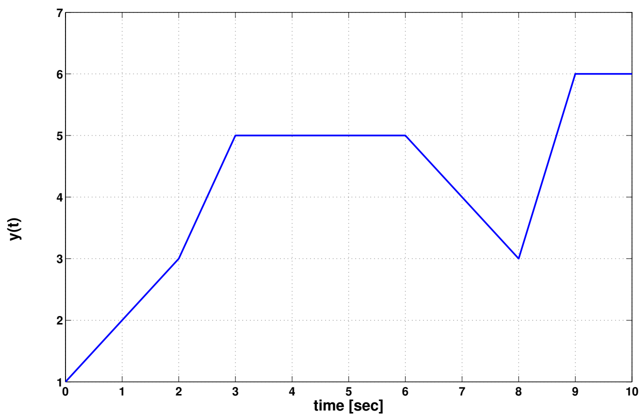


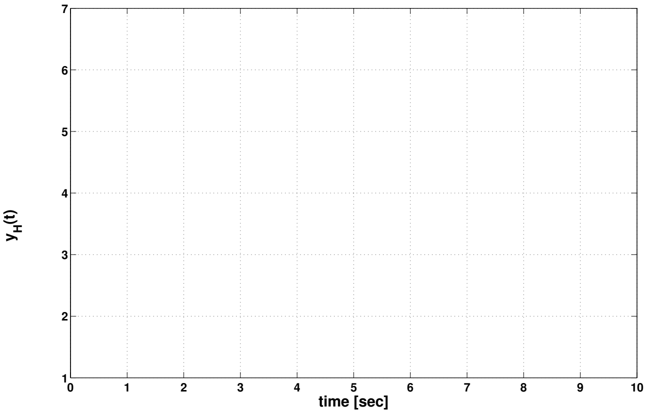
Figure 1: Graph of signal $y(t)$.

1.) (5 marks) What are the values of the sampled signal for 10 signal sampled instances if sampling time is $\triangle t = 1 sec$?

$$y(k) = \{ \qquad \qquad \qquad \qquad \}$$

$y(k) = \{1, 2, 3, 5, 5, 5, 5, 4, 3, 6\}$

2.) (5 marks) Reconstruct the continuous time signal y(t) given a discrete input signal y(k) = 2; 3; 4; 4; 2 to zero-order-hold (ZOH) digital to analog converter with hold time $\triangle t = 2sec$:



3.) (5 marks) Find the sampling time for the following two processes:

$$a) \quad G(s) = \frac{e^{-0.1s}}{s+0.01}$$
$$b) \quad G(s) = \frac{e^{-1.75s}}{s+0.5}$$

$G(s) = 100\frac{e^{-0.1s}}{100s+1}$, $\tau = 100$, $\tau_d = 0.1$, $\triangle t = 0.1 * 100 = 10$
$G(s) = 2\frac{e^{-1.75s}}{2s+1}$, $\tau = 2$, $\tau_d = 1.75$, $\triangle t = 0.1 * 1.75 = 0.175$

Question 2

The process of interest for control and dynamics response analysis is given as two tanks system in series which is given in the picture below, Fig.2. Assuming that the flow rate, $u(t)$ is input

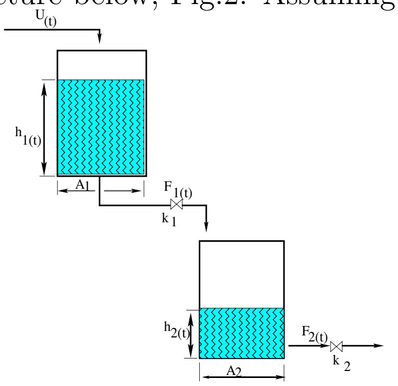


Figure 2: Two tanks model.

and flow $F_2(t)$ is output, and K_1, K_2 (valve characteristics) and A_1 and A_2 are constants:

1.) (7 marks) Find the continuous-time state space representation of the system. (hint: let $x_1 = h_1, x_2 = h_2$).

$$\begin{aligned} \frac{dx}{dt} &= \begin{bmatrix} -\frac{1}{K_1A_1} & 0 \\ \frac{1}{K_1A_2} & -\frac{1}{K_2A_2} \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & \frac{1}{K_2} \end{bmatrix} x(t) \end{aligned}$$

2.) (7 marks) Find the discrete state space model representation for the following parameters $K_1 = 0.5,\ K_2 = 1,\ A_1 = 1\ A_2 = 2$ and sampling time of $\triangle t = 1$.

$$x(k+1) = exp(\begin{bmatrix} -\frac{1}{K_1A_1} & 0 \\ \frac{1}{K_1A_2} & -\frac{1}{K_2A_2} \end{bmatrix} 1)x(k) + \Gamma u(k) \tag{1}$$

$A = \begin{bmatrix} -\frac{1}{K_1A_1} & 0 \\ \frac{1}{K_1A_2} & -\frac{1}{K_2A_2} \end{bmatrix}$ $B = \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & \frac{1}{K_2} \end{bmatrix}$

$A = \begin{bmatrix} -\frac{1}{0.5*1} & 0 \\ \frac{1}{0.5*2} & -\frac{1}{1*2} \end{bmatrix}$ $B = \begin{bmatrix} \frac{1}{1} \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & \frac{1}{1} \end{bmatrix}$

$\Phi = exp(A * \triangle t) = \begin{bmatrix} 0.1353 & 0 \\ 0.3141 & 0.6065 \end{bmatrix}$ is obtained as $\Phi = \mathcal{L}^{-1}\left\{ [sI - \begin{bmatrix} -2 & 0 \\ 1 & -0.5 \end{bmatrix}]^{-1} \right\}$

$\Phi = \mathcal{L}^{-1}\left\{ \begin{bmatrix} s+2 & 0 \\ -1 & s+0.5 \end{bmatrix}^{-1} \right\}$

$\Phi = \mathcal{L}^{-1}\left\{ \begin{bmatrix} \frac{1}{s+2} & 0 \\ \frac{A}{s+0.5} + \frac{B}{s+2} & \frac{1}{s+0.5} \end{bmatrix} \right\} = \begin{bmatrix} e^{-2t} & 0 \\ Ae^{-2t} + Be^{-0.5t} & e^{-0.5t} \end{bmatrix}$, $A = -1/1.5,\ B = 1/1.5$.

$\Gamma = \int_0^1 \begin{bmatrix} e^{-2\tau} & 0 \\ Ae^{-2\tau} + Be^{-0.5\tau} & e^{-0.5\tau} \end{bmatrix} \begin{bmatrix} \frac{1}{1} \\ 0 \end{bmatrix} d\tau = \int_0^1 \begin{bmatrix} e^{-2\tau} \\ Ae^{-2\tau} + Be^{-0.5\tau} \end{bmatrix} d\tau = \begin{bmatrix} 0.4323 \\ 0.2364 \end{bmatrix}$

3.) (7 marks) Find the overall transfer function which relates $y(z)$, $x(0)$ and $u(z)$ by using $\mathcal{Z}$-transform.

$y(z) = Cz(zI - \Phi)^{-1}x(0) + C(zI - \Phi)^{-1}\Gamma u(z)$

$y(z) = z\left(-\frac{0.62826}{(z-0.6065)\,(z-0.13533)} \quad \frac{1.0}{z-0.6065} \right) x(0) + \frac{0.47280}{z-0.6065} - \frac{0.2716}{(z-0.6065)\,(z-0.13533)} u(z)$

4.) (7 marks) Find the expression for the output response $y(n)$, if the input is zero and initial conditions are $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$y(z) = z\left(-\frac{0.62826}{(z-0.6065)\,(z-0.13533)} \quad \frac{1.0}{z-0.6065} \right) x(0) = \frac{1}{z-0.6065} \rightarrow y(n) = (0.6065)^n$

Another way is to observe that the $u(k) = 0$, and initial tank $x_1(0) = 0$ (empty tank 1), so one can find that $x_2(k+1) = -0.5x_2(k)$, $y(k) = x_2(k)$, and easily find that $y(z) = \frac{z}{z-exp(-0.5)}$, which leads to $y(n) = (0.6065)^n$

5.) (7 marks) Find the output response $y(n)$ if the initial conditions are zero and input is given as impulse function.

$y(z) = \frac{0.4728z+0.2076}{z^2-0.7419z+0.08208}u(z) = \frac{A}{z-0.6065} + B\frac{B}{z-0.1353}$

$y(n) = 1.0486(0.6065)^n - 0.5763(0.1353)^n$

Question 3

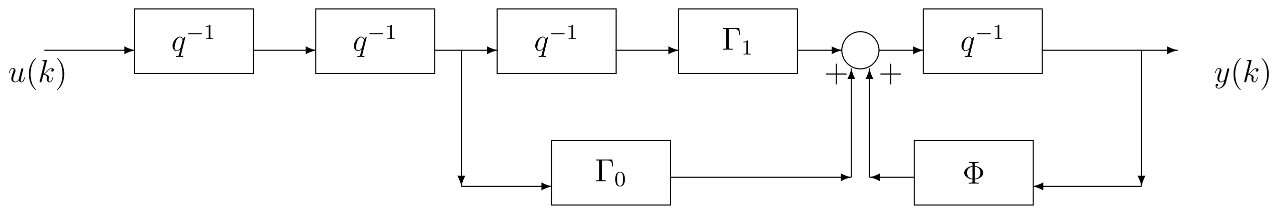
The continuous time system model is given as follows:

$$\frac{dx(t)}{dt} = -x(t) + u(t - 2.1) \tag{2}$$

and the appropriate sampling time is $\triangle t = 1$.

1.) (10 marks) Find the discrete state space model representation?
Therefore we have, $\tau = 2.1 = (3 - 1) * 1 + 0.1$, so that $d = 3$ and $\tau' = 0.1$. We calculate $\Phi = e^{-1}$, $\Gamma_0 = \int_0^{h-\tau'} e^{-s} ds B = \int_0^{1-0.1} e^{-s} ds B = [1 - e^{-0.9}]$, $\Gamma_1 = e^{1(1-0.1)} \int_0^{\tau'} e^{-s} ds$.
$x(k+1) = \Phi x(k) + \Gamma_0 u(k-2) + \Gamma_1 u(k-3) = 0.36x(k) + 0.5934u(k-2) + 0.0387u(k-3)$.

2.) (10 marks) Construct a block-diagram by using the appropriately chosen shift operator and determine if the system is stable?



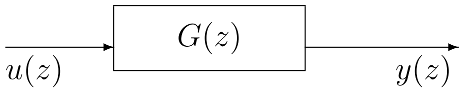
Question 4

The discrete time model of a chemical process of interest is given by:

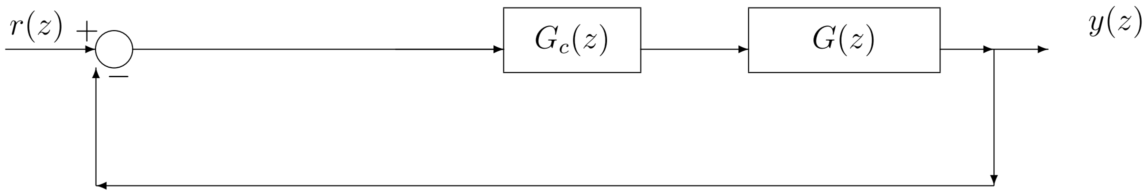
$$\begin{aligned} x(k+1) &= ax(k) + bu(k), \quad x(0) = 0 \\ y(k) &= cx(k) \end{aligned}$$

1.) (5 marks) Construct a block-diagram of the state-space model by using appropriate shift operator?

2.) (5 marks) Find the overall transfer function $G(z)$ that relates the input $u(z)$ and the output $y(z)$?
$y(z) = \frac{cb}{z-a} u(z)$



3.) (10 marks) Find the range of the controller gain $G_c(z) = K$, with $a = 1.5$, $b = 2$, $c = 0.5$, such that the closed loop system is stable?
$G_{cl}(z) = \frac{K\frac{bc}{z-a}}{1+K\frac{bc}{z-a}}$, $\mathcal{P}(z) = z-a+Kbc = 0$, $|-a+Kbc| < 1$, solve for $|-1.5+K*2*0.5| < 1$ which is $(-1.5+K*2*0.5) < 1$ and for $-(-1.5+K*2*0.5) < 1$, finally $0.5 < K < 2.5$.



4.) (10 marks) Choose a value of K within the range found in Part 3) and determine the system response to an impulse disturbance?
$y(z) = \frac{1}{1+K\frac{bc}{z-a}}d(z) = \frac{z-a}{z-a+Kbc}$, take $K = 1$, $y(z) = \frac{z-1.5}{z-0.5} = \frac{z}{z-0.5} - \frac{1.5}{z-0.5}$, $y(n) = 0.5^n - 1.5(0.5)^{n-1}$