Solution Set for HW 1

2.20. The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

Days	
108	138
124	163
124	159
106	134
115	139

(a) We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.

H_0: $\mu = 120$ $\qquad\qquad$ H_1: $\mu > 120$

(b) Test these hypotheses using $\alpha = 0.01$. What are your conclusions?

$\bar{y} = 131$

$S^2 = 3438 / 9 = 382$ $\qquad \rightarrow \quad t_0 = \dfrac{\bar{y} - \mu_0}{S/\sqrt{n}} = \dfrac{131 - 120}{19.54/\sqrt{10}} = 1.78$

$S = \sqrt{382} = 19.54$ $\qquad$ (since $t_{0.01,9} = 2.821$; do not reject H_0)

Minitab Output

```
T-Test of the Mean

Test of mu = 120.00 vs mu > 120.00

Variable     N      Mean    StDev   SE Mean        T         P
Shelf Life  10    131.00    19.54      6.18     1.78     0.054

T Confidence Intervals

Variable     N      Mean    StDev  SE Mean       99.0 % CI
Shelf Life  10    131.00    19.54     6.18   (  110.91,  151.09)
```

(c) Find the P-value for the test in part (b). P=0.054

(d) Construct a 99 percent confidence interval on the mean shelf life.

The 99% confidence interval is $\bar{y} - t_{\alpha/2, n-1}\dfrac{S}{\sqrt{n}} \le \mu \le \bar{y} + t_{\alpha/2, n-1}\dfrac{S}{\sqrt{n}}$ with $\alpha = 0.01$.

$$131 - (3.250)\left(\frac{19.54}{\sqrt{10}}\right) \le \mu \le 131 + (3.250)\left(\frac{19.54}{\sqrt{10}}\right)$$

$$110.91 \le \mu \le 151.08$$

2.27. An article in *Solid State Technology*, "Orthogonal Design of Process Optimization and Its Application to Plasma Etching" by G.Z. Yin and D.W. Jillie (May, 1987) describes an experiment to determine the effect of C_2F_6 flow rate on the uniformity of the etch on a silicon wafer used in integrated circuit manufacturing. Data for two flow rates are as follows:

C_2F_6	Uniformity Observation					
(SCCM)	1	2	3	4	5	6
125	2.7	4.6	2.6	3.0	3.2	3.8
200	4.6	3.4	2.9	3.5	4.1	5.1

(a)Does the C_2F_6 flow rate affect average etch uniformity? Use α = 0.05.

No, C_2F_6 flow rate does not affect average etch uniformity.

Minitab Output

Two Sample T-Test and Confidence Interval

Two sample T for Uniformity

Flow Rat	N	Mean	StDev	SE Mean
125	6	3.317	0.760	0.31
200	6	3.933	0.821	0.34

95% CI for mu (125) - mu (200): (-1.63, 0.40)
T-Test mu (125) = mu (200) (vs not =): T = -1.35 P = 0.21 DF = 10
Both use Pooled StDev = 0.791

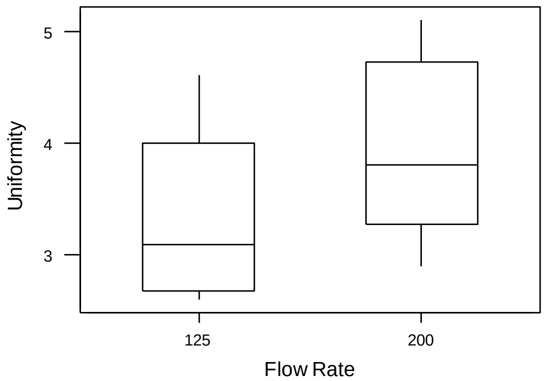
(b) What is the *P*-value for the test in part (a)? From the *Minitab* output, *P*=0.21

(c) Does the C_2F_6 flow rate affect the wafer-to-wafer variability in etch uniformity? Use α = 0.05.

$$H_0: \sigma_1^2 = \sigma_2^2$$
$$H_1: \sigma_1^2 \neq \sigma_2^2$$
$$F_{0.025,5,5} = 7.15$$
$$F_{0.975,5,5} = 0.14$$
$$F_0 = \frac{0.5776}{0.6724} = 0.86$$

➔ Do not reject; C_2F_6 flow rate does not affect wafer-to-wafer variability.

(d) Draw box plots to assist in the interpretation of the data from this experiment.
The box plots shown below indicate that there is little difference in uniformity at the two gas flow rates. Any observed difference is not statistically significant. See the *t*-test in part (a).



2.38. Two popular pain medications are being compared on the basis of the speed of absorption by the body. Specifically, tablet 1 is claimed to be absorbed twice as fast as tablet 2. Assume that σ_1^2 and σ_2^2 are known. Develop a test statistic for

H_0: $2\mu_1 = \mu_2$

H_1: $2\mu_1 \neq \mu_2$

$2\bar{y}_1 - \bar{y}_2 \sim N\left(2\mu_1 - \mu_2, \frac{4\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$, assuming that the data is normally distributed.

The test statistic is: $z_o = \dfrac{2\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{4\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$, reject if $|z_o| > z_{\alpha/2}$

2.39. Continuation of Problem 2.38. An article in *Nature* (1972, pp.225-226) reported on the levels of monoamine oxidase in blood platelets for a sample of 43 schizophrenic patients resulting in $\bar{y}_1$ = 2.69 and s_1 = 2.30 while for a sample of 45 normal patients the results were $\bar{y}_2$ = 6.35 and s_2 = 4.03. The units are nm/mg protein/h. Use the results of the previous problem to test the claim that the mean monoamine oxidase level for normal patients is at least twice the mean level for schizophrenic patients. Assume that the sample sizes are large enough to use the sample standard deviations as the true parameter values.

$$z_o = \frac{2\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{4\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \frac{2(2.69) - 6.35}{\sqrt{\dfrac{4(2.30)^2}{43} + \dfrac{4.03^2}{45}}} = \frac{-0.97}{.92357} = -1.05$$

z_0 = -1.05; using α=0.05, $z_{\alpha/2}$ = 1.96, do not reject.

3.22. The response time in milliseconds was determined for three different types of circuits that could be used in an automatic valve shutoff mechanism. The results are shown in the following table.

Circuit Type	Response Time				
1	9	12	10	8	15
2	20	21	23	17	30
3	6	5	8	16	7

(a) Test the hypothesis that the three circuit types have the same response time. Use α = 0.01.
From the computer printout, F=16.08, so there is at least one circuit type that is different.

Design Expert Output

Response:Response Time in ms
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**	
Model	543.60	2	271.80	16.08	0.0004	significant
A	*543.60*	*2*	*271.80*	*16.08*	*0.0004*	
Residual	202.80	12	16.90			
Lack of Fit	*0.000*	*0*				
Pure Error	*202.80*	*12*	*16.90*			
Cor Total	746.40	14				

The Model F-value of 16.08 implies the model is significant. There is only a 0.04% chance that a "Model F-Value" this large could occur due to noise.

Treatment Means (Adjusted, If Necessary)

	Estimated Mean	**Standard Error**
1-1	10.80	1.84
2-2	22.20	1.84
3-3	8.40	1.84

Treatment	**Mean Difference**	**DF**	**Standard Error**	**t for H0 Coeff=0**	**Prob > |t|**
1 vs 2	-11.40	1	2.60	-4.38	0.0009
1 vs 3	2.40	1	2.60	0.92	0.3742
2 vs 3	13.80	1	2.60	5.31	0.0002

(b) Use Tukey's test to compare pairs of treatment means. Use $\alpha = 0.01$.

$$S_{\bar{y}_{i.}} = \sqrt{\frac{MS_E}{n}} = \sqrt{\frac{16.90}{5}} = 1.8385$$

$$q_{0.01,(3,12)} = 5.04$$

$$t_0 = 1.8385(5.04) = 9.266$$

1 vs. 2: $|10.8-22.2| = 11.4 > 9.266$

1 vs. 3: $|10.8-8.4| = 2.4 < 9.266$

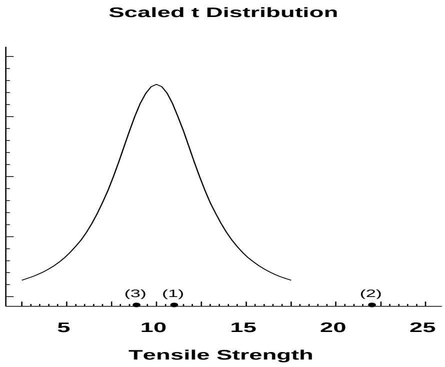
2 vs. 3: $|22.2-8.4| = 13.8 > 9.266$

1 and 2 are different. 2 and 3 are different.

Notice that the results indicate that the mean of treatment 2 differs from the means of both treatments 1 and 3, and that the means for treatments 1 and 3 are the same. Notice also that the Fisher LSD procedure (see the computer output) gives the same results.

(c) Use the graphical procedure in Section 3.5.3 to compare the treatment means. What conclusions can you draw? How do they compare with the conclusions from part (a).

The scaled-*t* plot agrees with part (b). In this case, the large difference between the mean of treatment 2 and the other two treatments is very obvious.



(d) Construct a set of orthogonal contrasts, assuming that at the outset of the experiment you suspected the response time of circuit type 2 to be different from the other two.

$$H_0 = \mu_1 - 2\mu_2 + \mu_3 = 0 \qquad C_1 = y_{1.} - 2y_{2.} + y_{3.}$$
$$H_1 = \mu_1 - 2\mu_2 + \mu_3 \neq 0 \qquad C_1 = 54 - 2(111) + 42 = -126$$

$$SS_{C1} = \frac{(-126)^2}{5(6)} = 529.2 \qquad F_{C1} = \frac{529.2}{16.9} = 31.31$$

→ Type 2 differs from the average of type 1 and type 3.

(e) If you were a design engineer and you wished to minimize the response time, which circuit type would you select?

→ Either type 1 or type 3 as they are not different from each other and have the lowest response time.

(f) Analyze the residuals from this experiment. Are the basic analysis of variance assumptions satisfied?

→ The normal probability plot has some points that do not lie along the line in the upper region. This may indicate potential outliers in the data.