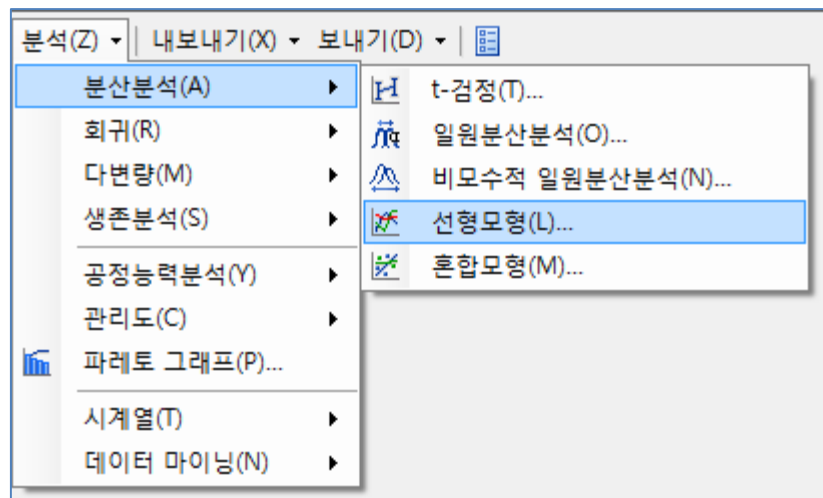


실험계획법 실습 - 7주차

중간고사 대비



```
proc glm data=ex;  
class treat;  
model y=treat;  
random treat;  
run;
```

```
proc varcomp data=ex method=type1;  
class treat;  
model y=treat;  
run;
```

Fixed Effect vs Random Effect Model

● 고정모형 vs 변량모형

3.6 Components of Variance

- Model: Single-Factor Experiment with No Restriction on Randomization

$$Y_{ij} = \mu + \tau_j + \varepsilon_{ij}$$

	Fixed Effect Model	Random Effect Model
Levels	Intentionally Selected	Randomly Selected
Assumption	$\sum_{j=1}^k \tau_j = 0$	$\tau_j \sim NID(0, \sigma_\tau^2)$
Hypothesis	$H_0 : \tau_1 = \dots = \tau_k = 0$	$H_0 : \sigma_\tau^2 = 0$
Decision	Restricted to selected levels	All levels of population
Interest	Mean Difference	Component of Variance

i) **Fixed Model:** Testing hypothesis/ Confidence Intervals/ Contrast in Means

ii) **Random Model:** Estimating Components of Variance.

→ Variance attributed from the true differences in treatment means, and
Variance due to random error about these means

Fixed Effect vs Random Effect Model

- 주요관심

- 고정의 경우 주 관심은 선택된 소수의 특정 수준들의 비교!!
- 변량의 경우 주 관심은 전체 수준들에서 반응에 차이가 있는지와 전체 수준들에서 분산(산포)의 크기가 어느 정도인가에 있다.
이들을 알기 위한 방법으로 EMS(expected mean square)
- 강의노트 ch03 42~47 참고

The linear statistical model is

$$y_{ij} = \mu + \tau_i + \epsilon_{ij} \quad \begin{cases} i = 1, 2, \dots, a \\ j = 1, 2, \dots, n \end{cases} \quad (3.47)$$

where both the treatment effects τ_i and ϵ_{ij} are random variables. We will assume that the treatment effects τ_i are NID $(0, \sigma_\tau^2)$ random variables¹ and that the errors are NID $(0, \sigma^2)$, random variables, and that the τ_i and ϵ_{ij} are independent. Because τ_i is independent of ϵ_{ij} , the variance of any observation is

$$V(y_{ij}) = \sigma_\tau^2 + \sigma^2$$



Variance components

$$MS_{\text{Treatments}} = \sigma^2 + n\sigma_\tau^2$$

$$\hat{\sigma}^2 = MS_E$$



$$MS_E = \sigma^2$$

$$\hat{\sigma}_\tau^2 = \frac{MS_{\text{Treatments}} - MS_E}{n}$$

EXAMPLE 3.11

A textile company weaves a fabric on a large number of looms. It would like the looms to be homogeneous so that it obtains a fabric of uniform strength. The process engineer suspects that, in addition to the usual variation in strength within samples of fabric from the same loom, there may also

be significant variations in strength between looms. To investigate this, she selects four looms at random and makes four strength determinations on the fabric manufactured on each loom. This experiment is run in random order, and the data obtained are shown in Table 3.17. The ANOVA is con-

■ **TABLE 3.17**
Strength Data for Example 3.11

Looms	Observations				$y_{\bar{i}}$
	1	2	3	4	
1	98	97	99	96	390
2	91	90	93	92	366
3	96	95	97	95	383
4	95	96	99	98	388

$$1527 = y_{..}$$

ducted and is shown in Table 3.18. From the ANOVA, we conclude that the looms in the plant differ significantly.

The variance components are estimated by $\hat{\sigma}^2 = 1.90$ and

$$\hat{\sigma}_{\tau}^2 = \frac{29.73 - 1.90}{4} = 6.96$$

Therefore, the variance of any observation on strength is estimated by

$$\hat{\sigma}_y^2 = \hat{\sigma}^2 + \hat{\sigma}_{\tau}^2 = 1.90 + 6.96 = 8.86.$$

Most of this variability is attributable to differences *between* looms.

■ **TABLE 3.18**
Analysis of Variance for the Strength Data

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P -Value
Looms	89.19	3	29.73	15.68	<0.001
Error	22.75	12	1.90		
Total	111.94	15			