$*$ show $\frac{1}{n} \le h_{ii} < 1$

$HH = H \qquad H = X(X'X)^{-1}X'$

$$(h_{i1}, h_{i2}, \cdots, h_{in}) \begin{pmatrix} h_{1i} \\ h_{2i} \\ \vdots \\ h_{ni} \end{pmatrix} = h_{ii} \iff \sum_{j=1}^{n} h_{ij} h_{ji} = \sum_{j=1}^{n} h_{ij}^2 \quad (\because \text{symmetric})$$

$\uparrow$

H의 i-th row　　H의 i-th column

$\therefore h_{ii}^2 \le h_{ii} \quad (h_{i1}^2 + h_{i2}^2 + \cdots + h_{ii}^2 + \cdots + h_{in}^2 = h_{ii})$

$\therefore h_{ii} \le 1$

$$\left.\begin{aligned} y_i &= \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + \varepsilon_i \\ &= \alpha + \beta_1 (x_{i1} - \bar{x}_1) + \cdots + \beta_k (x_{ik} - \bar{x}_k) + \varepsilon_i \end{aligned}\right\} \text{centering}$$

새로운 Design matrix

$$X = \begin{pmatrix} 1 & x_{11} - \bar{x}_1 & x_{12} - \bar{x}_2 & \cdots & x_{1k} - \bar{x}_k \\ 1 & x_{21} - \bar{x}_1 & x_{22} - \bar{x}_2 & \cdots & x_{2k} - \bar{x}_k \\ \vdots & \vdots & \vdots & \ddots & \\ 1 & x_{n1} - \bar{x}_1 & x_{n2} - \bar{x}_2 & & x_{nk} - \bar{x}_k \end{pmatrix}$$

$\uparrow$

상수항은 centering X

$*$ Idea

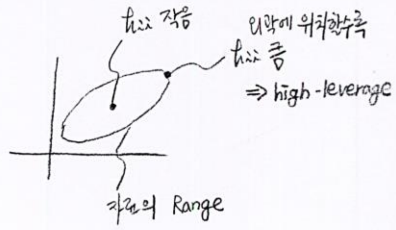
$$X'X = \begin{pmatrix} n & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & \boxed{X^{*\prime}X^{*}} & \\ 0 & & & \end{pmatrix} \qquad \therefore (X'X)^{-1} = \begin{pmatrix} \frac{1}{n} & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & \boxed{(X^{*\prime}X^{*})^{-1}} & \\ 0 & & & \end{pmatrix}$$

$$\therefore X(X'X)^{-1}X' = (\underset{\sim}{1}, \underset{\sim}{x_1} - \bar{x}_1 \underset{\sim}{1}, \underset{\sim}{x_2} - \bar{x}_2 \underset{\sim}{1}, \cdots, \underset{\sim}{x_k} - \bar{x}_k \underset{\sim}{1}) \begin{pmatrix} \frac{1}{n} & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & \boxed{(X^{*\prime}X^{*})^{-1}} & \\ 0 & & & \end{pmatrix} \begin{pmatrix} \underset{\sim}{1}' \\ \underset{\sim}{x_1}' - \bar{x}_1 \underset{\sim}{1}' \\ \underset{\sim}{x_2}' - \bar{x}_2 \underset{\sim}{1}' \\ \vdots \\ \underset{\sim}{x_k}' - \bar{x}_k \underset{\sim}{1}' \end{pmatrix}$$

$$h_{ii} = (1, \underbrace{x_{i1} - \bar{x}_1, \cdots, x_{ik} - \bar{x}_k}_{ⓐ}) \begin{pmatrix} \frac{1}{n} & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & \underbrace{\boxed{(X^{*\prime}X^{*})^{-1}}}_{ⓑ} & \\ 0 & & & \end{pmatrix} \underbrace{\begin{pmatrix} 1 \\ x_{i1} - \bar{x}_1 \\ \vdots \\ x_{ik} - \bar{x}_k \end{pmatrix}}_{ⓒ}$$

$$= \frac{1}{n} + \underline{ⓐ \times ⓑ \times ⓒ} \ge \frac{1}{n}$$

$x - \bar{x}$가 작을 수록 작아짐. / 자료의 중심에 있을 수록



$\therefore \frac{1}{n} \le h_{ii} \le 1$