

One – factor Experiments & Regression model using dummy variables

$y_{ij} = \mu + \tau_j + \varepsilon_{ij}$, $\varepsilon_{ij} \sim \text{NID}(0, \sigma_\varepsilon^2)$, τ_j : j – th treatment effect . For example, $j = 1, 2, 3, 4$

• NID : Normally and Identically Distributed

$$y_{ij} = \mu + \tau_j + \varepsilon_{ij} \rightarrow \begin{aligned} y_{i1} &= \mu + \tau_1 + \varepsilon_{i1}, \text{ where } \#(y_{i1}) = n_1 \\ y_{i2} &= \mu + \tau_2 + \varepsilon_{i2}, \text{ where } \#(y_{i2}) = n_2 \\ y_{i3} &= \mu + \tau_3 + \varepsilon_{i3}, \text{ where } \#(y_{i3}) = n_3 \\ y_{i4} &= \mu + \tau_4 + \varepsilon_{i4}, \text{ where } \#(y_{i4}) = n_4 \end{aligned}$$

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$$y_i = (\beta_0 + \beta_1 x_{i1}) + \delta_1 z_{i1} + \delta_2 z_{i2} + \delta_3 z_{i3} + \varepsilon_i \rightarrow \begin{aligned} y_i &= \beta_0 + \beta_1 x_{i1} + \delta_1 + \varepsilon_i, \text{ where } y_i \in z_{i1} (0 \text{ or } 1) \\ y_i &= \beta_0 + \beta_1 x_{i1} + \delta_2 + \varepsilon_i, \text{ where } y_i \in z_{i2} (0 \text{ or } 1) \\ y_i &= \beta_0 + \beta_1 x_{i1} + \delta_3 + \varepsilon_i, \text{ where } y_i \in z_{i3} (0 \text{ or } 1) \\ y_i &= \beta_0 + \beta_1 x_{i1} \varepsilon_i, \text{ where } y_i \notin \{z_{i1}, z_{i2}, z_{i3}\} \end{aligned}$$

Two types of One – factor Experiments

① Fixed effect model

$$y_{ij} = \mu + \tau_j + \varepsilon_{ij} \rightarrow \begin{aligned} y_{i1} &= \mu + \tau_1 + \varepsilon_{i1} \\ y_{i2} &= \mu + \tau_2 + \varepsilon_{i2} \\ y_{i3} &= \mu + \tau_3 + \varepsilon_{i3} \\ y_{i4} &= \mu + \tau_4 + \varepsilon_{i4} \end{aligned} \text{ where } \tau_1, \tau_2, \tau_3, \tau_4 \text{ are fixed constants}$$

$$H_0 : \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0 \text{ vs } H_1 : \text{at least one } \tau_j \text{ is different.}$$

② Random effect model : $y_{ij} = \mu + \tau_j + \varepsilon_{ij}$

$$y_{ij} = \mu + \tau_j + \varepsilon_{ij} \rightarrow \begin{aligned} y_{i1} &= \mu + \tau_1 + \varepsilon_{i1} \\ y_{i2} &= \mu + \tau_2 + \varepsilon_{i2} \\ y_{i3} &= \mu + \tau_3 + \varepsilon_{i3} \\ y_{i4} &= \mu + \tau_4 + \varepsilon_{i4} \end{aligned} \text{ where } \tau_j \sim \text{NID}(0, \sigma_\tau^2)$$

$$H_0 : \sigma_\tau^2 = 0 \text{ vs } H_1 : \sigma_\tau^2 \neq 0 \therefore \tau(\text{random effect}) \text{ is significant.}$$