Solutions for HW 3

4.16. Assuming that chemical types and bolts are fixed, estimate the model parameters τ_i and β_j in Problem 4.4.
Using Equations 4.18, applying the constraints, we obtain:

$$\hat{\mu}=\frac{35}{20},\ \hat{\tau}_1=\frac{-23}{20},\ \hat{\tau}_2=\frac{-7}{20},\ \hat{\tau}_3=\frac{13}{20},\hat{\tau}_4=\frac{17}{20},\ \hat{\beta}_1=\frac{35}{20},\ \hat{\beta}_2=\frac{-65}{20},\ \hat{\beta}_3=\frac{75}{20},\ \hat{\beta}_4=\frac{20}{20},\hat{\beta}_5=\frac{-65}{20}$$

4.18. Suppose that the observation for chemical type 2 and bolt 3 is missing in Problem 4.4. Analyze the problem by estimating the missing value. Perform the exact analysis and compare the results.

y_{23} is missing. $\hat{y}_{23}=\dfrac{ay'_{2.}+by'_{.3}-y'_{..}}{(a-1)(b-1)}=\dfrac{4(282)+5(227)-1360}{(3)(4)}=75.25$

Therefore, $y_{2.}$=357.25, $y_{.3}$=302.25, and $y_{..}$=1435.25

Source	*SS*	*DF*	*MS*	F_0
Chemicals	12.7844	3	4.2615	2.154
Bolts	158.8875	4		
Error	21.7625	11	1.9784	
Total	193.4344	18		

$F_{0.05,3,11}$=3.59, Chemicals are not significant. This is the same result as found in Problem 4.3.

4.22. An industrial engineer is investigating the effect of four assembly methods (*A, B, C, D*) on the assembly time for a color television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time. To account for this source of variability, the engineer uses the Latin square design shown below. Analyze the data from this experiment (α = 0.05) draw appropriate conclusions.

Order of Assembly	Operator 1	2	3	4
1	*C*=10	*D*=14	*A*=7	*B*=8
2	*B*=7	*C*=18	*D*=11	*A*=8
3	*A*=5	*B*=10	*C*=11	*D*=9
4	*D*=10	*A*=10	*B*=12	*C*=14

The Minitab output below identifies assembly method as having a significant effect on assembly time. **Minitab Output**

```
                      General Linear Model
Factor     Type Levels Values
Order    random      4  1  2  3  4
Operator random      4  1  2  3  4
Method    fixed      4  A  B  C  D

Analysis of Variance for Time, using Adjusted SS for Tests
Source     DF     Seq SS     Adj SS     Adj MS       F      P
Method      3     72.500     72.500     24.167   13.81  0.004
Order       3     18.500     18.500      6.167    3.52  0.089
Operator    3     51.500     51.500     17.167    9.81  0.010
Error       6     10.500     10.500      1.750
Total      15    153.000
```

4.28. Consider the gene expression experiment in Problem 4.11. Assume that the subjects used in this experiment are random. Estimate the block variance component
The block variance component is:

$$\hat{\sigma}_\beta^2 = \frac{\left[MS_{\text{Blocks}} - MS_E\right]}{a} = \frac{\left[102300 - 73130.15\right]}{3} = 9723.28$$

5.1. The following output was obtained from a computer program that performed a two-factor ANOVA on a factorial experiment.

(a) Fill in the blanks in the ANOVA table. You can use bounds on the *P*-values.

Two-way ANOVA: y versus A, B

Source	DF	SS	MS	F	P
A	1	**(0.0002)**	0.0002	**(0.00001)**	**(0.998)**
B	**(3)**	180.378	**(60.1260)**	**(3.02907)**	**(0.093)**
Interaction	3	8.479	**(2.8263)**	**(0.14239)**	0.932
Error	8	158.797	**(19.8496)**		
Total	15	347.653			

(a) How many levels were used for factor *B*? 4 levels.

(b) How many replicates of the experiment were performed? 2 replicates.

(c) What conclusions would you draw about this experiment?

Factor *B* is moderately significant with a *P*-value of 0.93. Factor *A* and the two-factor interaction are not significant.

5.6. An article in *Industrial Quality Control* (1956, pp. 5-8) describes an experiment to investigate the effect of the type of glass and the type of phosphor on the brightness of a television tube. The response variable is the current necessary (in microamps) to obtain a specified brightness level. The data are as follows:

(a) Is there any indication that either factor influences brightness? Use α = 0.05.

Both factors, phosphor type (*A*) and Glass type (*B*) influence brightness.

Design Expert Output

Response: Current in microamps
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model	15516.67	5	3103.33	58.80	< 0.0001	significant
A	*933.33*	*2*	*466.67*	*8.84*	*0.0044*	
B	*14450.00*	*1*	*14450.00*	*273.79*	*< 0.0001*	
AB	*133.33*	*2*	*66.67*	*1.26*	*0.3178*	
Residual	633.33	12	52.78			
Lack of Fit	*0.000*	*0*				
Pure Error	*633.33*	*12*	*52.78*			
Cor Total	16150.00	17				

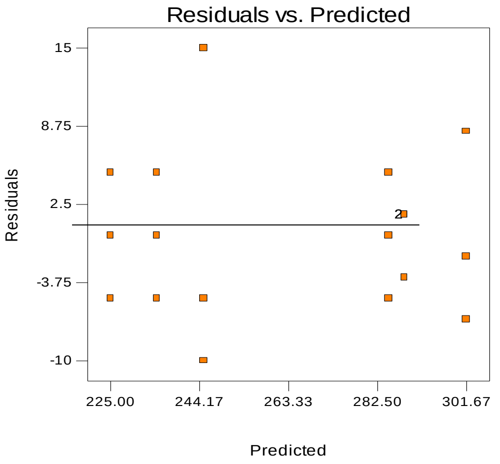
The Model F-value of 58.80 implies the model is significant. There is only
a 0.01% chance that a "Model F-Value" this large could occur due to noise.

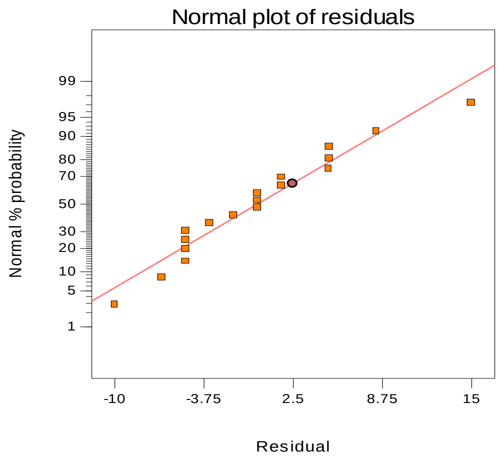
Values of "Prob > F" less than 0.0500 indicate model terms are significant.
In this case A, B are significant model terms.

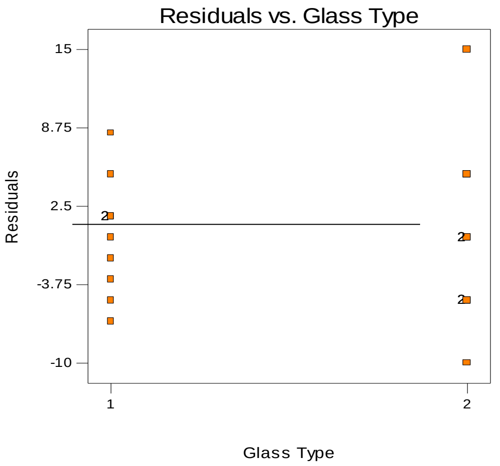
(b) Do the two factors interact? Use α = 0.05. **There is no interaction effect**.

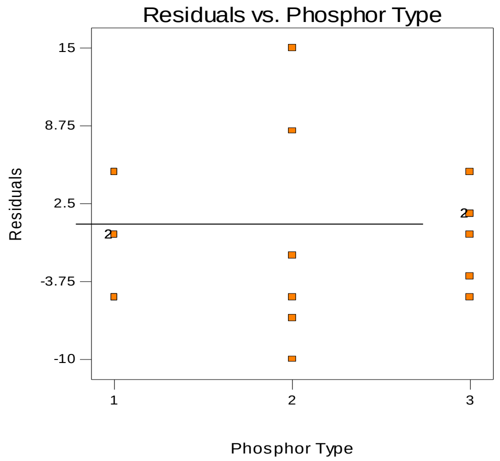
(c) Analyze the residuals from this experiment.

The residual plot of residuals versus phosphor content indicates a very slight inequality of variance. It is not serious enough to be of concern, however.









5.7. Johnson and Leone (*Statistics and Experimental Design in Engineering and the Physical Sciences*, Wiley 1977) describe an experiment to investigate the warping of copper plates. The two factors studied were the temperature and the copper content of the plates. The response variable was a measure of the amount of warping. The data were as follows:

(a) Is there any indication that either factor affects the amount of warping? Is there any interaction between the factors? Use α = 0.05.

Both factors, copper content (*A*) and temperature (*B*) affect warping, the interaction does not.

Design Expert Output

Response: Warping

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

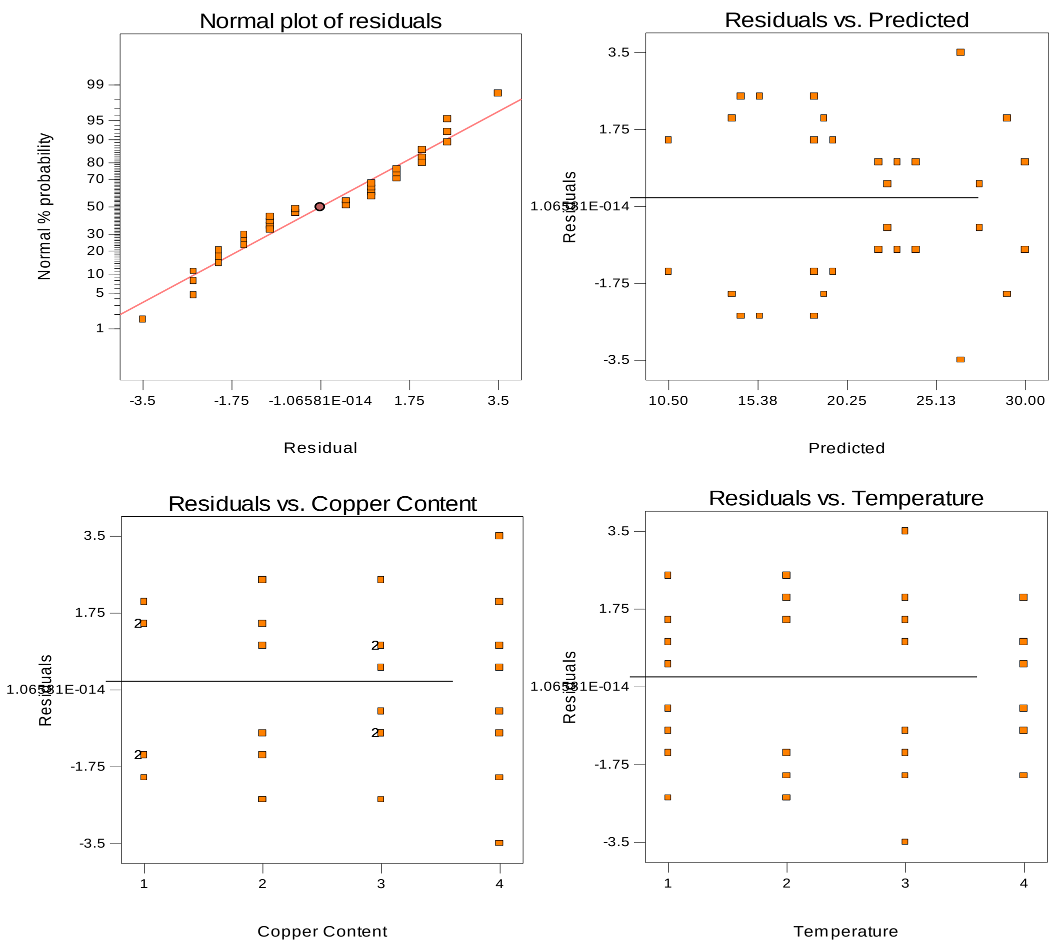
Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**	
Model	968.22	15	64.55	9.52	< 0.0001	significant
A	*698.34*	*3*	*232.78*	*34.33*	*< 0.0001*	
B	*156.09*	*3*	*52.03*	*7.67*	*0.0021*	
AB	*113.78*	*9*	*12.64*	*1.86*	*0.1327*	
Residual	108.50	16	6.78			
Lack of Fit	*0.000*	*0*				
Pure Error	*108.50*	*16*	*6.78*			
Cor Total	1076.72	31				

The Model F-value of 9.52 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, B are significant model terms.

(b) Analyze the residuals from this experiment.

→ There is nothing unusual about the residual plots.



(c) Plot the average warping at each level of copper content and compare them to an appropriately scaled *t* distribution. Describe the differences in the effects of the different levels of copper content on warping. If low warping is desirable, what level of copper content would you specify?

Design Expert Output

Factor	**Name**	**Level**	**Low Level**	**High Level**			
A	Copper Content	40	40	100			
B	Temperature	Average	50	125			
	Prediction	**SE Mean**	**95% CI low**	**95% CI high**	**SE Pred**	**95% PI low**	**95% PI high**
Warping		15.5	0.92	13.55	17.45	2.76	9.64
21.36							

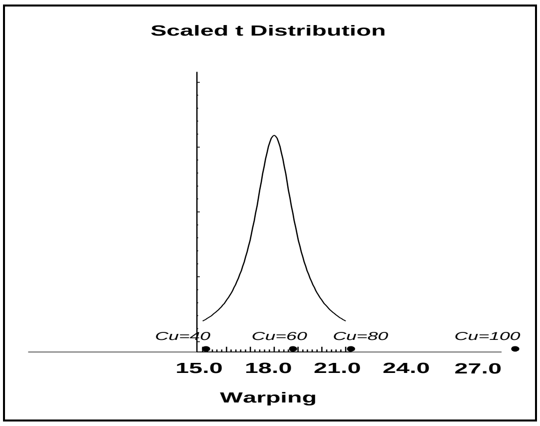
Factor	**Name**	**Level**	**Low Level**	**High Level**			
A	Copper Content	60	40	100			
B	Temperature	Average	50	125			
	Prediction	**SE Mean**	**95% CI low**	**95% CI high**	**SE Pred**	**95% PI low**	**95% PI high**
Warping		18.875	0.92	16.92	20.83	2.76	13.02
24.73							

Factor	**Name**	**Level**	**Low Level**	**High Level**			
A	**Copper Content**	**80**	**40**	**100**			
B	**Temperature**	**Average**	**50**	**125**			
	Prediction	**SE Mean**	**95% CI low**	**95% CI high**	**SE Pred**	**95% PI low**	**95% PI high**
Warping		21	0.92	19.05	22.95	2.76	15.14
26.86							

Factor	**Name**	**Level**	**Low Level**	**High Level**			
A	**Copper Content**	**100**	**40**	**100**			
B	**Temperature**	**Average**	**50**	**125**			
	Prediction	**SE Mean**	**95% CI low**	**95% CI high**	**SE Pred**	**95% PI low**	**95% PI high**
Warping		28.25	0.92	26.30	30.20	2.76	22.39
34.11							

Use a copper content of 40 for the lowest warping.

$$S = \sqrt{\frac{MS_E}{b}} = \sqrt{\frac{6.78125}{8}} = 0.92$$



(d) Suppose that temperature cannot be easily controlled in the environment in which the copper plates are to be used. Does this change your answer for part (c)?

Use a copper of content of 40. This is the same as for part (c).