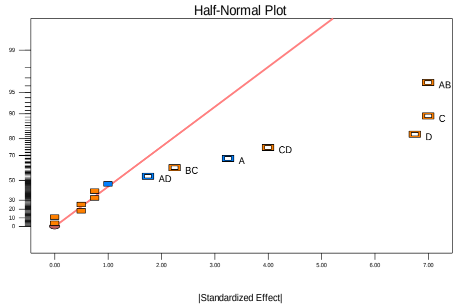
Solutions for HW 5

◊ Problem 7.18:

The runs for the experiment are shown below with the corresponding blocks.

Run	Block	Glucose (g dm^{-3})	NH_4NO_3 (g dm^{-3})	$FeSO_4$ (g dm^{-3} x 10^{-4})	$MnSO_4$ (g dm^{-3} x 10^{-2})	y $(CMC)^{-1}$
1	Block 2	20.00	2.00	6.00	4.00	23
2	Block 1	60.00	2.00	6.00	4.00	15
3	Block 1	20.00	6.00	6.00	4.00	16
4	Block 2	60.00	6.00	6.00	4.00	18
5	Block 1	20.00	2.00	30.00	4.00	25
6	Block 2	60.00	2.00	30.00	4.00	16
7	Block 2	20.00	6.00	30.00	4.00	17
8	Block 1	60.00	6.00	30.00	4.00	26
9	Block 1	20.00	2.00	6.00	20.00	28
10	Block 2	60.00	2.00	6.00	20.00	16
11	Block 2	20.00	6.00	6.00	20.00	18
12	Block 1	60.00	6.00	6.00	20.00	21
13	Block 2	20.00	2.00	30.00	20.00	36
14	Block 1	60.00	2.00	30.00	20.00	24
15	Block 1	20.00	6.00	30.00	20.00	33
16	Block 2	60.00	6.00	30.00	20.00	34

The analysis of the experiment shown below identifies the contribution of the blocks. By reducing the SS_E and MS_E, the *AD* and *CD* interactions now appear to be significant.



Response **1** **y**

ANOVA for selected factorial model

Analysis of variance table [Partial sum of squares - Type III]

Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**p-value Prob > F**	
Block	6.25	1	6.25			
Model	713.00	8	89.13	50.93	< 0.0001	significant
A-Glucose	*42.25*	*1*	*42.25*	*24.14*	*0.0027*	
B-NH4NO3	*0.000*	*1*	*0.000*	*0.000*	*1.0000*	
C-FeSO4	*196.00*	*1*	*196.00*	*112.00*	*< 0.0001*	
D-MnSO4	*182.25*	*1*	*182.25*	*104.14*	*< 0.0001*	
AB	*196.00*	*1*	*196.00*	*112.00*	*< 0.0001*	
AD	*12.25*	*1*	*12.25*	*7.00*	*0.0382*	
BC	*20.25*	*1*	*20.25*	*11.57*	*0.0145*	
CD	*64.00*	*1*	*64.00*	*36.57*	*0.0009*	
Residual	10.50	6	1.75			
Cor Total	729.75	15				

The Model F-value of 50.93 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, C, D, AB, AD, BC, CD are significant model terms.

◊ Problem 7.23

$$\text{Block Effect} = \bar{y}_{Block1} - \bar{y}_{Block2} = \frac{406}{8} - \frac{715}{8} = \frac{-309}{8} = -38.625$$

This is the block effect estimated in Example 7.2 plus the additional 20 units that were added to each observation in block 2. All other effects are the same.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A	1870.56	1	1870.56	89.93
C	390.06	1	390.06	18.75
D	855.56	1	855.56	41.13
AC	1314.06	1	1314.06	63.18
AD	1105.56	1	1105.56	53.15
Blocks	5967.56	1	5967.56	
Error	187.56	9	20.8	
Total	11690.93	15		

Design Expert Output

Response: Filtration in gal/hr

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**	
Block	5967.56	1	5967.56			
Model	5535.81	5	1107.16	53.13	< 0.0001	significant
A	*1870.56*	*1*	*1870.56*	*89.76*	*< 0.0001*	
C	*390.06*	*1*	*390.06*	*18.72*	*0.0019*	
D	*855.56*	*1*	*855.56*	*41.05*	*0.0001*	
AC	*1314.06*	*1*	*1314.06*	*63.05*	*< 0.0001*	
AD	*1105.56*	*1*	*1105.56*	*53.05*	*< 0.0001*	
Residual	187.56	9	20.84			
Cor Total	11690.94	15				

The Model F-value of 53.13 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, C, D, AC, AD are significant model terms.

◊ Problem 8.4

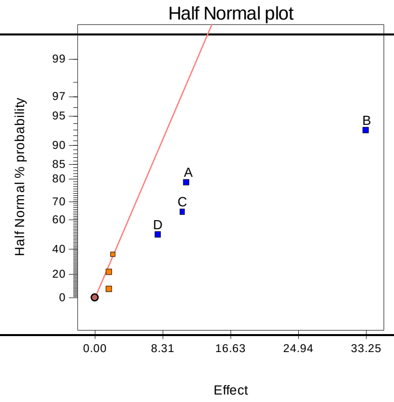
I = ABD = ACE = BCDE

A	*(ABD)*	*=BD*	*A*	*(ACE)*	*=CE*	*A*	*(BCDE)*	*=ABCDE*	*A=BD=CE=ABCDE*
B	*(ABD)*	*=AD*	*B*	*(ACE)*	*=ABCE*	*B*	*(BCDE)*	*=CDE*	*B=AD=ABCE=CDE*
C	*(ABD)*	*=ABCD*	*C*	*(ACE)*	*=AE*	*C*	*(BCDE)*	*=BDE*	*C=ABCD=AE=BDE*
D	*(ABD)*	*=AB*	*D*	*(ACE)*	*=ACDE*	*D*	*(BCDE)*	*=BCE*	*D=AB=ACDE=BCE*
E	*(ABD)*	*=ABDE*	*E*	*(ACE)*	*=AC*	*E*	*(BCDE)*	*=BCD*	*E=ABDE=AC=BCD*
BC	*(ABD)*	*=ACD*	*BC*	*(ACE)*	*=ABE*	*BC*	*(BCDE)*	*=DE*	*BC=ACD=ABE=DE*
BE	*(ABD)*	*=ADE*	*BE*	*(ACE)*	*=ABC*	*BE*	*(BCDE)*	*=CD*	*BE=ADE=ABC=CD*

A	*B*	*C*	*D=AB*	*E=AC*		
-	-	-	+	+	*de*	6
+	-	-	-	-	*a*	9
-	+	-	-	+	*be*	35
+	+	-	+	-	*abd*	50
-	-	+	+	-	*cd*	18
+	-	+	-	+	*ace*	22
-	+	+	-	-	*bc*	40
+	+	+	+	+	*abcde*	63

Design Expert Output

	Term	Effect	SumSqr	% Contribtn
Model	Intercept			
Model	A	11.25	253.125	8.91953
Model	B	33.25	2211.13	77.9148
Model	C	10.75	231.125	8.1443
Model	D	7.75	120.125	4.23292
Error	E	2.25	10.125	0.356781
Error	BC	-1.75	6.125	0.215831
Error	BE	1.75	6.125	0.215831
	Lenth's ME	28.232		
	Lenth's SME	67.5646		



The main *A*, *B*, *C*, and *D* are large. However, recall that we are really estimating *A+BD+CE*, *B+AD*, *C+DE* and *D+AD*. There are other possible interpretations of the experiment because of the aliasing.

Design Expert Output

Response: Yield

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**	
Model	2815.50	4	703.88	94.37	0.0017	significant
A	*253.13*	*1*	*253.13*	*33.94*	*0.0101*	
B	*2211.12*	*1*	*2211.12*	*296.46*	*0.0004*	
C	*231.13*	*1*	*231.13*	*30.99*	*0.0114*	
D	*120.13*	*1*	*120.13*	*16.11*	*0.0278*	
Residual	22.38	3	7.46			
Cor Total	2837.88	7				

The Model F-value of 94.37 implies the model is significant. There is only a 0.17% chance that a "Model F-Value" this large could occur due to noise.

Std. Dev.	2.73	R-Squared	0.9921
Mean	30.38	Adj R-Squared	0.9816
C.V.	8.99	Pred R-Squared	0.9439
PRESS	159.11	Adeq Precision	25.590

Factor	**Coefficient Estimate**	**DF**	**Standard Error**	**95% CI Low**	**95% CI High**	**VIF**
Intercept	30.38	1	0.97	27.30	33.45	
A-Aperture	5.63	1	0.97	2.55	8.70	1.00
B-Exposure Time	16.63	1	0.97	13.55	19.70	1.00
C-Develop Time	5.37	1	0.97	2.30	8.45	1.00
D-Mask Dimension	3.87	1	0.97	0.80	6.95	1.00

Final Equation in Terms of Coded Factors:

Yield =
+30.38
+5.63 * A
+16.63 * B
+5.37 * C
+3.87 * D

Final Equation in Terms of Actual Factors:

Aperture small
Mask Dimension Small
Yield =
-6.00000
+0.83125 * Exposure Time
+0.71667 * Develop Time

Aperture large

Mask Dimension	Small
Yield	=
+5.25000	
+0.83125	* Exposure Time
+0.71667	* Develop Time
Aperture	small
Mask Dimension	Large
Yield	=
+1.75000	
+0.83125	* Exposure Time
+0.71667	* Develop Time
Aperture	large
Mask Dimension	Large
Yield	=
+13.00000	
+0.83125	* Exposure Time
+0.71667	* Develop Time

◊ Problem 8.5

We could fold over the original design by changing the signs on the generators $D = AB$ and $E = AC$ to produce the following new experiment.

A	*B*	*C*	*D=-AB*	*E=-AC*		
-	-	-	-	-	(1)	7
+	-	-	+	+	*ade*	10
-	+	-	+	-	*bd*	32
+	+	-	-	+	*abe*	52
-	-	+	-	+	*ce*	15
+	-	+	+	-	*acd*	21
-	+	+	+	+	*bcde*	41
+	+	+	-	-	*abc*	60

A	(-ABD)	=-BD	A	(-ACE)	=-CE	A	(BCDE)	=ABCDE	A=-BD=-CE=ABCDE
B	(-ABD)	=-AD	B	(-ACE)	=-ABCE	B	(BCDE)	=CDE	B=-AD=-ABCE=CDE
C	(-ABD)	=-ABCD	C	(-ACE)	=-AE	C	(BCDE)	=BDE	C=-ABCD=-AE=BDE
D	(-ABD)	=-AB	D	(-ACE)	=-ACDE	D	(BCDE)	=BCE	D=-AB=-ACDE=BCE
E	(-ABD)	=-ABDE	E	(-ACE)	=-AC	E	(BCDE)	=BCD	E=-ABDE=-AC=BCD
BC	(-ABD)	=-ACD	BC	(-ACE)	=-ABE	BC	(BCDE)	=DE	BC=-ACD=-ABE=DE
BE	(-ABD)	=-ADE	BE	(-ACE)	=-ABC	BE	(BCDE)	=CD	BE=-ADE=-ABC=CD

Assuming all three factor and higher interactions to be negligible, all main effects can be separated from their two-factor interaction aliases in the combined design.

◊ Problem 8.13

I=*CDEF=ABCG=ABDEFG*, Resolution IV

	A	B	C	D	E	F=CDE	G=ABC	
1	-	-	-	-	-	-	-	(1)
2	+	-	-	-	-	-	+	*ag*
3	-	+	-	-	-	-	+	*bg*
4	+	+	-	-	-	-	-	*ab*
5	-	-	+	-	-	+	+	*cfg*
6	+	-	+	-	-	+	-	*acf*
7	-	+	+	-	-	+	-	*bcf*
8	+	+	+	-	-	+	+	*abcfg*
9	-	-	-	+	-	+	-	*df*
10	+	-	-	+	-	+	+	*adfg*
11	-	+	-	+	-	+	+	*bdfg*
12	+	+	-	+	-	+	-	*abdf*
13	-	-	+	+	-	-	+	*cdg*
14	+	-	+	+	-	-	-	*acd*
15	-	+	+	+	-	-	-	*bcd*
16	+	+	+	+	-	-	+	*abcdg*

17	-	-	-	-	+	+	-	*ef*
18	+	-	-	-	+	+	+	*aefg*
19	-	+	-	-	+	+	+	*befg*
20	+	+	-	-	+	+	-	*abef*
21	-	-	+	-	+	-	+	*ceg*
22	+	-	+	-	+	-	-	*ace*
23	-	+	+	-	+	-	-	*bce*
24	+	+	+	-	+	-	+	*abceg*
25	-	-	-	+	+	-	-	*de*
26	+	-	-	+	+	-	+	*adeg*
27	-	+	-	+	+	-	+	*bdeg*
28	+	+	-	+	+	-	-	*abde*
29	-	-	+	+	+	+	+	*cdefg*
30	+	-	+	+	+	+	-	*acdef*
31	-	+	+	+	+	+	-	*bcdef*
32	+	+	+	+	+	+	+	*abcdefg*

Alias Structure

A(CDEF)= ACDEF	*A(ABCG)= BCG*	*A(ABDEFG)= BDEFG*	*A=ACDEF=BCG=BDEFG*
B(CDEF)= BCDEF	*B(ABCG)= ACG*	*B(ABDEFG)= ADEFG*	*B=BCDEF=ACG=ADEFG*
C(CDEF)= DEF	*C(ABCG)= ABG*	*C(ABDEFG)= ABCDEFG*	*C=DEF=ABG=ABCDEFG*
D(CDEF)= CEF	*D(ABCG)= ABCDG*	*D(ABDEFG)= ABEFG*	*D=CEF=ABCDG=ABEFG*
E(CDEF)= CDF	*E(ABCG)= ABCEG*	*E(ABDEFG)= ABDFG*	*E=CDF=ABCEG=ABDFG*
F(CDEF)= CDE	*F(ABCG)= ABCFG*	*F(ABDEFG)= ABDEG*	*F=CDE=ABCFG=ABDEG*
G(CDEF)= CDEFG	*G(ABCG)= ABC*	*G(ABDEFG)= ABDEF*	*G=CDEFG=ABC=ABDEF*
AB(CDEF)= ABCDEF	*AB(ABCG)= CG*	*AB(ABDEFG)= DEFG*	*AB=ABCDEF=CG=DEFG*
AC(CDEF)= ADEF	*AC(ABCG)= BG*	*AC(ABDEFG)= BCDEFG*	*AC=ADEF=BG=BCDEFG*
AD(CDEF)= ACEF	*AD(ABCG)= BCDG*	*AD(ABDEFG)= BEFG*	*AD=ACEF=BCDG=BEFG*
AE(CDEF)= ACDF	*AE(ABCG)= BCEG*	*AE(ABDEFG)= BDFG*	*AE=ACDF=BCEG=BDFG*
AF(CDEF)= ACDE	*AF(ABCG)= BCFG*	*AF(ABDEFG)= BDEG*	*AF=ACDE=BCFG=BDEG*
AG(CDEF)= ACDEFG	*AG(ABCG)= BC*	*AG(ABDEFG)= BDEF*	*AG=ACDEFG=BC=BDEF*
BD(CDEF)= BCEF	*BD(ABCG)= ACDG*	*BD(ABDEFG)= AEFG*	*BD=BCEF=ACDG=AEFG*
BE(CDEF)= BCDF	*BE(ABCG)= ACEG*	*BE(ABDEFG)= ADFG*	*BE=BCDF=ACEG=ADFG*
BF(CDEF)= BCDE	*BF(ABCG)= ACFG*	*BF(ABDEFG)= ADEG*	*BF=BCDE=ACFG=ADEG*
CD(CDEF)= EF	*CD(ABCG)= ABDG*	*CD(ABDEFG)= ABCEFG*	*CD=EF=ABDG=ABCEFG*
CE(CDEF)= DF	*CE(ABCG)= ABEG*	*CE(ABDEFG)= ABCDFG*	*CE=DF=ABEG=ABCDFG*
CF(CDEF)= DE	*CF(ABCG)= ABFG*	*CF(ABDEFG)= ABCDEG*	*CF=DE=ABFG=ABCDEG*
DG(CDEF)= CEFG	*DG(ABCG)= ABCD*	*DG(ABDEFG)= ABEF*	*DG=CEFG=ABCD=ABEF*
EG(CDEF)= CDFG	*EG(ABCG)= ABCE*	*EG(ABDEFG)= ABDF*	*EG=CDFG=ABCE=ABDF*
FG(CDEF)= CDEG	*FG(ABCG)= ABCF*	*FG(ABDEFG)= ABDE*	*FG=CDEG=ABCF=ABDE*

Analysis of Variance Table

Source	Degrees of Freedom
A	1
B	1
C	1
D	1
E	1
F	1
G	1
AB=CG	1
AC=BG	1
AD	1
AE	1
AF	1
AG=BC	1
BD	1
BE	1
BF	1
CD=EF	1
CE=DF	1
CF=DE	1
DG	1
EG	1
FG	1

Error	9
Total	31

◊ Problem 8.40

Run	Treatment Combination		A	B	C	D=ABC	y
1	(1)		-1	-1	-1	-1	8
2	ad		1	-1	-1	1	10
3	bd		-1	1	-1	1	12
4	ab		1	1	-1	-1	7
5	cd		-1	-1	1	1	13
6	ac		1	-1	1	-1	6
7	bc		-1	1	1	-1	5
8	abcd		1	1	1	1	11
		Avg (+)	8.5	4.5	8.75	11.5	
		Avg (-)	9.5	9.25	9.25	26	
		Effect	-0.5	-2.375	-0.25	-7.25	

(a) How many factors did this experiment investigate? 4
(b) What is the resolution of this design? IV
(c) Calculate the estimates of the effects. See table above.
(d) What is the complete defining relation? I = ABCD