CH 6: Fixed, Random, and Mixed Models

6.1 Introduction

- Level of Factor: Fixed or Random (Decided before running the experiments)

Fixed: the results to be judged for these levels only
← Temperature, Time, or Pressure (near Extremes and some Intermediate Points)

Random: the results may be extended to more levels
← Operators, Days, or Batches (small Sample of All Possible Population)
Factors: Temperature (25 → 55), Altitude (0 K → 3 K)

6.2 Single-Factor Models

- Model: $Y_{ij} = \mu + \tau_j + \varepsilon_{ij}$, with $\varepsilon_{ij} \sim NID(0,\ \sigma_\varepsilon^2)$

- Comparison between Fixed and Random Models

	Fixed Models	Random Models
Assumptions	$\sum_{j=1}^{k} \tau_j = \sum_{j=1}^{k} (\mu_j - \mu) = 0$	$\tau_j \sim NID(0,\ \sigma_\tau^2)$
Analysis	Shown in CH 3	Same as Fixed models
EMS	**Source** / **df** / **EMS**: τ_j / $k-1$ / $\sigma_\varepsilon^2 + n\phi_\tau$ ε_{ij} / $k(n-1)$ / σ_ε^2	**Source** / **df** / **EMS**: τ_j / $k-1$ / $\sigma_\varepsilon^2 + n\sigma_\tau^2$ ε_{ij} / $k(n-1)$ / σ_ε^2
Hypothesis	$H_0 : \tau_j = 0;\ j = 1, \cdots, k$ $F = MS_{treatment} / MS_{error}$	$H_0 : \sigma_\tau^2 = 0$ $F = MS_{treatment} / MS_{error}$

Where $\phi_\tau = \dfrac{\sum_j \tau_j^2}{n-1}$

6.3 Two-Factor Models

- **Model:** $Y_{ijk} = \mu + A_i + B_j + AB_{ij} + \varepsilon_{k(ij)}$

  With $i = 1, \cdots, a \quad j = 1, \cdots, b \quad k = 1, \cdots, n$

- **EMS**

Fixed	Random	Mixed (A fixed, B Random)

1. Assumptions

Fixed	Random	Mixed (A fixed, B Random)
A_i's are **Fixed** constants		A_i's are **Fixed** constants
$\sum_{i=1}^{a} A_i = 0$	$A_i \sim NID(0,\ \sigma_A^2)$	$\sum_{i=1}^{a} A_i = 0$
B_j's are **Fixed** constants		B_j's are **Random**
$\sum_{j=1}^{b} B_j = 0$	$B_j \sim NID(0,\ \sigma_B^2)$	$B_j \sim NID(0,\ \sigma_B^2)$
AB_{ij}'s are **Fixed** constants		$AB_{ij} \sim NID(0,\ \sigma_{AB}^2)$ but
$\sum_{i=1}^{a} AB_{ij} = 0$	$AB_{ij} \sim NID(0,\ \sigma_{AB}^2)$	$\sum_{i=1}^{a} AB_{ij} = 0$
$\sum_{j=1}^{b} AB_{ij} = 0$		$(\sum_{j=1}^{b} AB_{ij} \neq 0)$

2. Analysis Procedures (CH5 for Sum of Squares)

Fixed	Random	Mixed
Same	**Same**	**Same**

3. EMS

Source	d.f.	EMS		
		Fixed	**Random**	**Mixed**
A_i	$a-1$	$\sigma_\varepsilon^2 + nb\phi_A$	$\sigma_\varepsilon^2 + n\sigma_{AB}^2 + nb\sigma_A^2$	$\sigma_\varepsilon^2 + n\sigma_{AB}^2 + nb\phi_A$
B_j	$b-1$	$\sigma_\varepsilon^2 + na\phi_B$	$\sigma_\varepsilon^2 + n\sigma_{AB}^2 + na\sigma_B^2$	$\sigma_\varepsilon^2 + na\sigma_B^2$
AB_{ij}	$(a-1)(b-1)$	$\sigma_\varepsilon^2 + n\phi_{AB}$	$\sigma_\varepsilon^2 + n\sigma_{AB}^2$	$\sigma_\varepsilon^2 + n\sigma_{AB}^2$
$\varepsilon_{k(ij)}$	$ab(n-1)$	σ_ε^2	σ_ε^2	σ_ε^2

4. Hypothesis

Fixed	Random	Mixed
$H_0: A_i = 0$ for all i	$H_0: \sigma_A^2 = 0$	$H_0: A_i = 0$ for all i
$H_0: B_j = 0$ for all j	$H_0: \sigma_B^2 = 0$	$H_0: \sigma_B^2 = 0$
$H_0: AB_{ij} = 0$ for all i, j	$H_0: \sigma_{AB}^2 = 0$	$H_0: \sigma_{AB}^2 = 0$

- **Determining the Test Statistics (Based on EMS)**

Source	*d.f.*	EMS		
		Fixed	Random	Mixed
A_i	$a-1$	$\sigma_\varepsilon^2 + nb\phi_A$	$\sigma_\varepsilon^2 + n\sigma_{AB}^2 + nb\sigma_A^2$	$\sigma_\varepsilon^2 + n\sigma_{AB}^2 + nb\phi_A$
B_j	$b-1$	$\sigma_\varepsilon^2 + na\phi_B$	$\sigma_\varepsilon^2 + n\sigma_{AB}^2 + na\sigma_B^2$	$\sigma_\varepsilon^2 + na\sigma_B^2$
AB_{ij}	$(a-1)(b-1)$	$\sigma_\varepsilon^2 + n\phi_{AB}$	$\sigma_\varepsilon^2 + n\sigma_{AB}^2$	$\sigma_\varepsilon^2 + n\sigma_{AB}^2$
$\varepsilon_{k(ij)}$	$ab(n-1)$	σ_ε^2	σ_ε^2	σ_ε^2

Source	Test Statistics		
	Fixed	Random	Mixed
A_i	MS_A / MS_E	MS_A / MS_{AB}	MS_A / MS_{AB}
B_j	MS_B / MS_E	MS_B / MS_{AB}	MS_B / MS_E
AB_{ij}	MS_{AB} / MS_E	MS_{AB} / MS_E	MS_{AB} / MS_E

6.4 EMS Rules

- A Simple Method of determining EMS for the Balanced Case
- **Determination of EMS:**

Example) $Y_{ijk} = \mu + A_i + B_j + AB_{ij} + \varepsilon_{k(ij)}$ (Fixed A, Random B)

Step 0: 모형의 Factor들을 표의 행에 나열. 표의 열에는 첨자를 기준으로 해당 Factor의 **(1) 수준 수 (2) 종류(F or R), (3) 첨자(i, j, k 등)를 기록** **(단, 반복은 random으로 간주)**

Factor	*a* F *i*	*b* R *j*	*n* R *k*
A_i			
B_j			
AB_{ij}			
$\varepsilon_{k(ij)}$			

Step 1: 각 행에서, 행의 첨자와 다른 첨자의 열과 만나는 **Cell**에 열의 수준(또는 반복) 수를 적는다.

Step 2: 행에 있는 첨자 중에서 괄호 속에 있는 것이 있으면, 그 행에서 괄호 속과 같은 첨자의 열을 찾아 만나는 곳에 **1**을 적는다.

Step 3: 나머지 빈 **Cell**에 대해, 해당 열의 첨자가 고정 요인이면 **0**, 랜덤 요인이면 **1**을 기록

Factor	a F i	b R j	n R k
A_i	③ **0**	① **b**	① **n**
B_j	① **a**	③ **1**	① **n**
AB_{ij}	③ **0**	③ **1**	① **n**
$\varepsilon_{k(ij)}$	② **1**	② **1**	③ **1**

Step 4: 작성된 표를 이용하여 각 요인의 E(MS) 계산

i) 각 행에서, 괄호 밖에 있는 첨자와 같은 첨자의 열을 가리고 ii)의 방식으로 EMS를 계산. (예로, A_i 요인의 EMS 계산 시 i 첨자 열을 제외하고, $\varepsilon_{k(ij)}$ 의 EMS 계산 시 k 첨자 열을 제외)

ii) 각 요인의 EMS는 그 행의 첨자가 포함된 행을 대상으로, 남은 열의 숫자 (문자)와 그 행의 산포 측도 (ϕ_τ 또는 σ_τ^2)의 곱들을 합한다.

Factor	a F i	b R j	n R k	산포	EMS
A_i	**0**	**b**	**n**	ϕ_A	$\sigma_\varepsilon^2 + n\sigma_{AB}^2 + nb\phi_A$
B_j	**a**	**1**	**n**	σ_B^2	$\sigma_\varepsilon^2 + na\sigma_B^2$
AB_{ij}	**0**	**1**	**n**	σ_{AB}^2	$\sigma_\varepsilon^2 + n\sigma_{AB}^2$
$\varepsilon_{k(ij)}$	**1**	**1**	**1**	σ_ε^2	σ_ε^2

□ Example (Table 6.3: page 175)

Response: Time required to position and assemble mating parts

Treatment: Operators (6; **Random**): O_i

Angles (4 levels; **Fixed**) → (0°, 30°, 60°, 90°): A_j
Clearances (5 levels; **Fixed**): C_k
Locations (2 levels; **Fixed**): L_m

Replications: *q*

i) Step 1:

Source		DF	6 R i	4 F j	5 F k	2 F m	6 R q	EMS
O_i	σ_O^2	5	1	**4**	**5**	**2**	**6**	
A_j	ϕ_A	3	**6**	0	**5**	**2**	**6**	
OA_{ij}	σ_{OA}^2	15	1	0	**5**	**2**	**6**	
C_k	ϕ_C	4	**6**	**4**	0	**2**	**6**	
OC_{ik}	σ_{OC}^2	20	1	**4**	0	**2**	**6**	
AC_{jk}	ϕ_{AC}	12	**6**	0	0	**2**	**6**	
OAC_{ijk}	σ_{OAC}^2	60	1	0	0	**2**	**6**	
L_m	ϕ_L	1	**6**	**4**	**5**	0	**6**	
OL_{im}	σ_{OL}^2	5	1	**4**	**5**	0	**6**	
AL_{jm}	ϕ_{AL}	3	**6**	0	**5**	0	**6**	
OAL_{ijm}	σ_{OAL}^2	15	1	0	**5**	0	**6**	
CL_{km}	ϕ_{CL}	4	**6**	**4**	0	0	**6**	
OCL_{ikm}	σ_{OCL}^2	20	1	**4**	0	0	**6**	
ACL_{jkm}	ϕ_{ACL}	12	**6**	0	0	0	**6**	
$OACL_{ijkm}$	σ_{OACL}^2	60	1	0	0	0	**6**	
$\varepsilon_{q(ijkm)}$	σ_ε^2	1200	1	1	1	1	1	

ii) Step 2 and Step 3:

Source		6 R i	4 F j	5 F k	2 F m	6 R q	EMS
O_i	σ_O^2	**1**	**4**	**5**	**2**	**6**	$\sigma_\varepsilon^2 + 240\sigma_O^2$
A_j	ϕ_A	**6**	**0**	**5**	**2**	**6**	
OA_{ij}	σ_{OA}^2	**1**	**0**	**5**	**2**	**6**	
C_k	ϕ_C	**6**	**4**	**0**	**2**	**6**	
OC_{ik}	σ_{OC}^2	**1**	**4**	**0**	**2**	**6**	
AC_{jk}	ϕ_{AC}	**6**	**0**	**0**	**2**	**6**	
OAC_{ijk}	σ_{OAC}^2	**1**	**0**	**0**	**2**	**6**	
L_m	ϕ_L	**6**	**4**	**5**	**0**	**6**	
OL_{im}	σ_{OL}^2	**1**	**4**	**5**	**0**	**6**	
AL_{jm}	ϕ_{AL}	**6**	**0**	**5**	**0**	**6**	
OAL_{ijm}	σ_{OAL}^2	**1**	**0**	**5**	**0**	**6**	
CL_{km}	ϕ_{CL}	**6**	**4**	**0**	**0**	**6**	
OCL_{ikm}	σ_{OCL}^2	**1**	**4**	**0**	**0**	**6**	
ACL_{jkm}	ϕ_{ACL}	**6**	**0**	**0**	**0**	**6**	
$OACL_{ijkm}$	σ_{OACL}^2	**1**	**0**	**0**	**0**	**6**	
$\varepsilon_{q(ijkm)}$	σ_ε^2	**1**	**1**	**1**	**1**	**1**	

iii) Step 4: Computation of EMS

e.g.) O_i 의 EMS 계산

Source		6 R i	4 F j	5 F k	2 F m	6 R q	EMS
O_i	σ_O^2	1	4	5	2	6	$\sigma_\varepsilon^2 + 240\sigma_0^2$
A_j	ϕ_A	6	0	5	2	6	
OA_{ij}	σ_{OA}^2	1	0	5	2	6	
C_k	ϕ_C	6	4	0	2	6	
OC_{ik}	σ_{OC}^2	1	4	0	2	6	
AC_{jk}	ϕ_{AC}	6	0	0	2	6	
OAC_{ijk}	σ_{OAC}^2	1	0	0	2	6	
L_m	ϕ_L	6	4	5	0	6	
OL_{im}	σ_{OL}^2	1	4	5	0	6	
AL_{jm}	ϕ_{AL}	6	0	5	0	6	
OAL_{ijm}	σ_{OAL}^2	1	0	5	0	6	
CL_{km}	ϕ_{CL}	6	4	0	0	6	
OCL_{ikm}	σ_{OCL}^2	1	4	0	0	6	
ACL_{jkm}	ϕ_{ACL}	6	0	0	0	6	
$OACL_{ijkm}$	σ_{OACL}^2	1	0	0	0	6	
$\varepsilon_{q(ijkm)}$	σ_ε^2	1	1	1	1	1	

$$\begin{aligned} E[MS_O] &= 4\cdot5\cdot2\cdot6\sigma_O^2 + 0\cdot5\cdot2\cdot6\sigma_{OA}^2 + 4\cdot0\cdot2\cdot6\sigma_{OC}^2 + 0\cdot0\cdot2\cdot6\sigma_{OAC}^2 \\ \Rightarrow \quad & +4\cdot5\cdot0\cdot6\sigma_{OL}^2 + 0\cdot5\cdot0\cdot6\sigma_{OAL}^2 + 4\cdot0\cdot0\cdot6\sigma_{OCL}^2 + 0\cdot0\cdot0\cdot6\sigma_{OACL}^2 + \sigma_\varepsilon^2 \\ &= \sigma_\varepsilon^2 + 240\sigma_O^2 \end{aligned}$$

e.g.) AC_{jk} 의 EMS 계산

Source		6 R i	4 F j	5 F k	2 F m	6 R q	EMS
O_i	σ_O^2	1	4	5	2	6	
A_j	ϕ_A	6	0	5	2	6	
OA_{ij}	σ_{OA}^2	1	0	5	2	6	
C_k	ϕ_C	6	4	0	2	6	
OC_{ik}	σ_{OC}^2	1	4	0	2	6	
AC_{jk}	ϕ_{AC}	6	0	0	2	6	$72\phi_{AC} + 12\sigma_{OAC}^2 + \sigma_\varepsilon^2$
OAC_{ijk}	σ_{OAC}^2	1	0	0	2	6	
L_m	ϕ_L	6	4	5	0	6	
OL_{im}	σ_{OL}^2	1	4	5	0	6	
AL_{jm}	ϕ_{AL}	6	0	5	0	6	
OAL_{ijm}	σ_{OAL}^2	1	0	5	0	6	
CL_{km}	ϕ_{CL}	6	4	0	0	6	
OCL_{ikm}	σ_{OCL}^2	1	4	0	0	6	
ACL_{jkm}	ϕ_{ACL}	6	0	0	0	6	
$OACL_{ijkm}$	σ_{OACL}^2	1	0	0	0	6	
$\varepsilon_{q(ijkm)}$	σ_ε^2	1	1	1	1	1	

➔ $E[MS_O] = 6\cdot 2\cdot 6\phi_{AC} + 1\cdot 2\cdot 6\sigma_{OAC}^2 + 6\cdot 0\cdot 6\phi_{ACL} + 1\cdot 0\cdot 6\sigma_{OALC}^2 + \sigma_\varepsilon^2$

6.5 EMS Derivations

☐ Two Factor Experiment

Model: $Y_{ijk} = \mu + A_i + B_j + AB_{ij} + \varepsilon_{k(ij)}$ **(Example: Fixed A, Random B)**

→ $SS_A = \sum_{i=1}^{k} nb(\bar{Y}_{i..} - \bar{Y}_{...})^2$

i)
$$\bar{Y}_{i..} = \sum_j^b \sum_k^n Y_{ijk} / nb = \sum_j^b \sum_k^n (\mu + A_i + B_j + AB_{ij} + \varepsilon_{k(ij)}) / nb$$
$$= \mu + A_i + \sum_j^b B_j / b + \sum_j^b AB_{ij} / b + \sum_j^b \sum_k^n \varepsilon_{k(ij)} / nb$$

ii)
$$\bar{Y}_{...} = \sum_i^a \sum_j^b \sum_k^n Y_{ijk} / nb = \sum_i^a \sum_j^b \sum_k^n (\mu + A_i + B_j + AB_{ij} + \varepsilon_{k(ij)}) / anb$$
$$= \mu + \sum_i^a A_i / a + \sum_j^b B_j / b + \sum_i^a \sum_j^b AB_{ij} / ab + \sum_i^a \sum_j^b \sum_k^n \varepsilon_{k(ij)} / anb$$

➔
$$\bar{Y}_{i..} - \bar{Y}_{...} = (A_i - \sum_i^a A_i / a) + (\sum_j^b AB_{ij} / b - \sum_i^a \sum_j^b AB_{ij} / ab)$$
$$+ (\sum_j^b \sum_k^n \varepsilon_{k(ij)} / nb - \sum_i^a \sum_j^b \sum_k^n \varepsilon_{k(ij)} / anb)$$

➔
$$SS_A = \sum_i^a (\bar{Y}_{i..} - \bar{Y}_{...})^2 = nb \sum_i^a (A_i - \sum_i^a A_i / a)^2$$
$$+ nb \sum_i^a (\sum_j^b AB_{ij} / b - \sum_i^a \sum_j^b AB_{ij} / ab)^2$$
$$+ nb \sum_i^a (\sum_j^b \sum_k^n \varepsilon_{k(ij)} / nb - \sum_i^a \sum_j^b \sum_k^n \varepsilon_{k(ij)} / anb)^2$$

Since $\sum_i^a A_i = 0$ and $\sum_i^a AB_{ij} = 0$ but $\sum_j^b AB_{ij} \neq 0$ with

$\sum_j^b AB_{ij} / b = \overline{AB}_{i.} \sim N(0, \sigma_{AB}^2 / b)$, $\sum_i^a \sum_j^b AB_{ij} / ab = \overline{AB}_{..} \sim N(0, \sigma_{AB}^2 / ab)$,

$\sum_j^b \sum_k^n \varepsilon_{k(ij)} / nb = \bar{\varepsilon}_{i..} \sim NID(0, \sigma_\varepsilon^2 / nb)$, $\sum_i^a \sum_j^b \sum_k^n \varepsilon_{k(ij)} / anb = \bar{\varepsilon}_{...} \sim N(0, \sigma_\varepsilon^2 / anb)$,

$$(E[\sum_i^a (\overline{AB}_{i.} - \overline{AB}_{..})^2] = (a-1) Var[\overline{AB}_{i.}] = (a-1)\sigma_{AB}^2 / b \text{ and}$$

$$E[\sum_i^a (\bar{\varepsilon}_{i..} - \bar{\varepsilon}_{...})^2] = (a-1)\sigma_\varepsilon^2 / bn)$$

$$E[SS_A] = nb \sum_i^a A_i^2 + nbE[\sum_i^a (\overline{AB}_{i.} - \overline{AB}_{..})^2] + nbE[\sum_i^a (\bar{\varepsilon}_{i..} - \bar{\varepsilon}_{...})^2]$$
$$= nb \sum_i^a A_i^2 + nb(a-1)\sigma_{AB}^2 / b + nb(a-1)\sigma_\varepsilon^2 / bn$$
$$= nb \sum_i^a A_i^2 + n(a-1)\sigma_{AB}^2 + (a-1)\sigma_\varepsilon^2$$

➔ $E[MS_A] = E[SS_A]/(a-1) = nb \sum_i^a A_i^2 / (a-1) + n\sigma_{AB}^2 + \sigma_\varepsilon^2$

6.6 The Pseudo-F Test

☐ Cases of No Exact F-Test (Example 6.3)

- **Response:** Thickness of Dry-Film

  Treatments: Day (Random) – 2 levels
  Operators (Random) – 3 levels
  Gate Settings (Fixed) – 3 levels

- **ANOVA Table**

Source	DF	SS	MS	EMS
D	1	.0010	.0010	$\sigma_\varepsilon^2 + 6\sigma_{DO}^2 + 18\sigma_D^2$
O	2	.1121	.0560	$\sigma_\varepsilon^2 + 6\sigma_{DO}^2 + 12\sigma_O^2$
D*O	2	.0060	.0030	$\sigma_\varepsilon^2 + 6\sigma_{DO}^2$
G	2	1.573	.7866	$\sigma_\varepsilon^2 + 2\sigma_{DOG}^2 + 4\sigma_{OG}^2 + 6\sigma_{DG}^2 + 12\phi_G$
D*G	2	.0113	.0056	$\sigma_\varepsilon^2 + 2\sigma_{DOG}^2 + 6\sigma_{DG}^2$
O*G	4	.0428	.0107	$\sigma_\varepsilon^2 + 2\sigma_{DOG}^2 + 4\sigma_{OG}^2$
D*O*G	4	.0099	.0025	$\sigma_\varepsilon^2 + 2\sigma_{DOG}^2$
Error	18	.0059	.0003	σ_ε^2
Total	35	1.7622		

- **Question: How to Test Gate Main Effect?**

  - If D*G Interaction Effects is assumed to be Zero, O*G can be applied
  - If O*G Interaction Effects is assumed to be Zero, D*G can be applied
  - If neither of these two interaction is significant: **(by Satterthwaite)**

  → Consider constructing a Mean Square as a Linear Combination of other MS in the Experiment:

  $\sigma_\varepsilon^2 + 2\sigma_{DOG}^2 + 4\sigma_{OG}^2 + 6\sigma_{DG}^2$ can be expressed by the Linear Combination,

  $$MS = MS_{DG} + MS_{OG} - MS_{DOG} \quad (= 0.0138)$$

  $$\leftarrow \quad E[MS] = (\sigma_\varepsilon^2 + 2\sigma_{DOG}^2 + 4\sigma_{OG}^2) + (\sigma_\varepsilon^2 + 2\sigma_{DOG}^2 + 6\sigma_{DG}^2) - (\sigma_\varepsilon^2 + 2\sigma_{DOG}^2)$$

- **Degree of Freedom for MS**

  $$MS = a_1 MS_1 + a_2 MS_2 + \cdots \rightarrow \nu = \frac{(MS)^2}{a_1^2 (MS_1)^2 / \nu_1 + a_2^2 (MS_2)^2 / \nu_2 + \cdots}$$

- **Example Data:** $a_1 = 1,\ a_2 = 1,\ a_3 = -1$ with $\nu_1 = 4,\ \nu_2 = 2,\ \nu_3 = 4$

  $$\rightarrow \quad \nu = \frac{(0.0138)^2}{1^2 (0.0107)^2 / 4 + 1^2 (0.0056)^2 / 2 + (-1)^2 (0.0025)^2 / 4} = 4.2$$

  $$\rightarrow \quad F' = MS_G / MS = 0.7866 / 0.0138 = 57.0 : \text{Significant for } F_{2,\ 4.2}$$

6.8 Remarks (Single Observation per Cell)

- **Model:** $Y_{ijk} = \mu + A_i + B_j + AB_{ij} + \varepsilon_{ij}$

- **ANOVA Table**

Source	EMS **Fixed**	EMS **Random**	EMS **Mixed**
A_i	$\sigma_\varepsilon^2 + b\phi_A$	$\sigma_\varepsilon^2 + \sigma_{AB}^2 + b\sigma_A^2$	$\sigma_\varepsilon^2 + \sigma_{AB}^2 + b\phi_A$
B_j	$\sigma_\varepsilon^2 + a\phi_B$	$\sigma_\varepsilon^2 + \sigma_{AB}^2 + a\sigma_B^2$	$\sigma_\varepsilon^2 + a\sigma_B^2$
AB_{ij} (or ε_{ij})	$\sigma_\varepsilon^2 + \phi_{AB}$	$\sigma_\varepsilon^2 + \sigma_{AB}^2$	$\sigma_\varepsilon^2 + \sigma_{AB}^2$

- **For the Fixed Model, There is No TEST for the main effects *A* and *B***
  ← If there is no interaction ($\phi_{AB} = 0$), the main effects can be tested

- For the Random Effect Model, both main effects *A* and *B* can be tested

- **For the Mixed Model, the random main effects *B* can not be tested**

☐ EMS For Non-Equal Replications

- For the Single-Factor Experiment

- EMS: Employing $n_0 = \dfrac{N^2 - \sum_{j=1}^{k} n_j^2}{(k-1)N}$, where $N = \sum_{j=1}^{k} n_j$

  Fixed Effect: $\sigma_\varepsilon^2 + n_0\phi_\tau$

  Random Effect: $\sigma_\varepsilon^2 + n_0\sigma_\tau^2$