Solutions for Homework 4

6.1

(a) Analyze the data from this experiment.

Design Expert Output

Response: Vibration

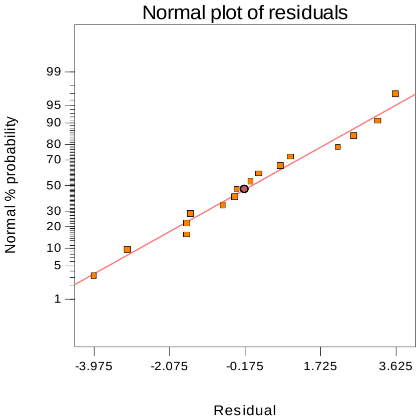
ANOVA for Selected Factorial Model

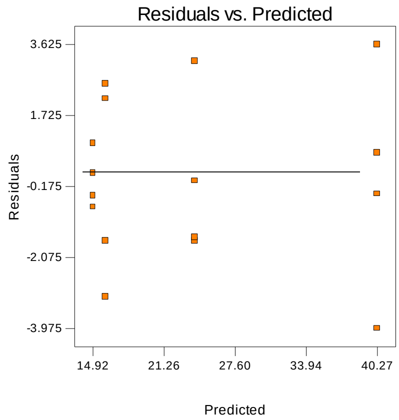
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model	1638.11	3	546.04	91.36	< 0.0001	significant
A	*1107.23*	*1*	*1107.23*	*185.25*	*< 0.0001*	
B	*227.26*	*1*	*227.26*	*38.02*	*< 0.0001*	
AB	*303.63*	*1*	*303.63*	*50.80*	*< 0.0001*	
Residual	71.72	12	5.98			
Lack of Fit	*0.000*	*0*				
Pure Error	*71.72*	*12*	*5.98*			
Cor Total	1709.83	15				

The Model F-value of 91.36 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

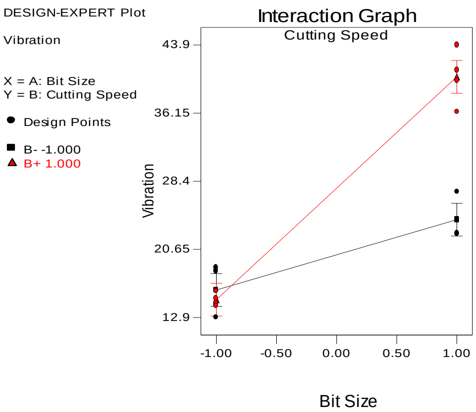
(b) Construct a normal probability plot of the residuals, and plot the residuals versus the predicted vibration level. Interpret these plots.





➔ There is nothing unusual about the residual plots.

(c) Draw the *AB* interaction plot. Interpret this plot. What levels of bit size and speed would you recommend for routine operation?



To reduce the vibration, use the smaller bit. Once the small bit is specified, either speed will work equally well, because the slope of the curve relating vibration to speed for the small tip is approximately zero.
The process is robust to speed changes if the small bit is used.

6.12

(a) Estimate the factor effects.

Design Expert Output

	Term	Effect	SumSqr	% Contribtn
Model	Intercept			
Error	A	-0.31725	0.40259	6.79865
Error	B	0.586	1.37358	23.1961
Error	AB	0.2815	0.316969	5.35274
Error	Lack Of Fit		0	0
Error	Pure Error		3.82848	64.6525

(b) Conduct an analysis of variance. Which factors are important?

From the analysis of variance shown below, no factors appear to be important. Factor *B* is only marginally interesting with an *F*-value of 4.31.

Design Expert Output

Response: Thickness

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**	
Model	2.09	3	0.70	2.19	0.1425	not significant
A	*0.40*	*1*	*0.40*	*1.26*	*0.2833*	
B	*1.37*	*1*	*1.37*	*4.31*	*0.0602*	
AB	*0.32*	*1*	*0.32*	*0.99*	*0.3386*	
Residual	3.83	12	0.32			
Lack of Fit	*0.000*	*0*				
Pure Error	*3.83*	*12*	*0.32*			
Cor Total	5.92	15				

The "Model F-value" of 2.19 implies the model is not significant relative to the noise. There is a 14.25 % chance that a "Model F-value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.
In this case there are no significant model terms.

(c) Write down a regression equation that could be used to predict epitaxial layer thickness over the region of arsenic flow rate and deposition time used in this experiment.

Design Expert Output

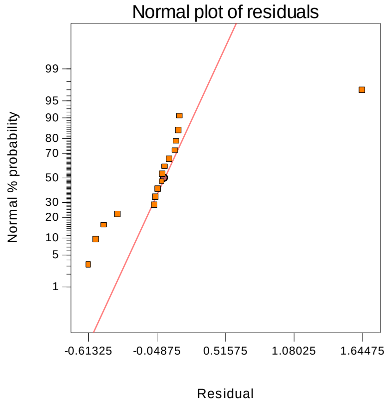
Final Equation in Terms of Coded Factors:

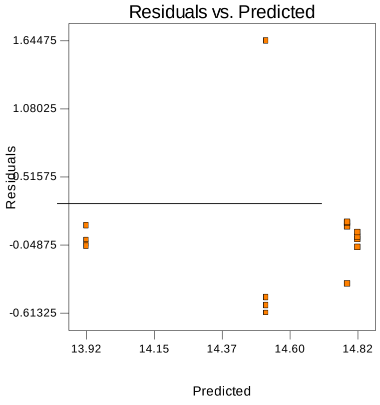
Thickness	=
+14.51	
-0.16	* A
+0.29	* B
+0.14	* A * B

Final Equation in Terms of Actual Factors:

Thickness	=
+37.62656	
-0.43119	* Flow Rate
-1.48735	* Dep Time
+0.028150	* Flow Rate * Dep Time

(d) Analyze the residuals. Are there any residuals that should cause concern? Observation #2 falls outside the groupings in the normal probability plot and the plot of residual versus predicted.





(e) Discuss how you might deal with the potential outlier found in part (d).

One approach would be to replace the observation with the average of the observations from that experimental cell. Another approach would be to identify if there was a recording issue in the original data. The first analysis below replaces the data point with the average of the other three. The second analysis assumes that the reading was incorrectly recorded and should have been 14.165.

The analysis with the run associated with standard order 2 replaced with the average of the remaining three runs in the cell, 13.972, is shown below.

Response: Thickness

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**	
Model	2.97	3	0.99	53.57	< 0.0001	significant
A	*7.439E-003*	*1*	*7.439E-003*	*0.40*	*0.5375*	
B	*2.96*	*1*	*2.96*	*160.29*	*< 0.0001*	
AB	*2.176E-004*	*1*	*2.176E-004*	*0.012*	*0.9153*	
Pure Error	*0.22*	*12*	*0.018*			
Cor Total	3.19	15				

The Model F-value of 53.57 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

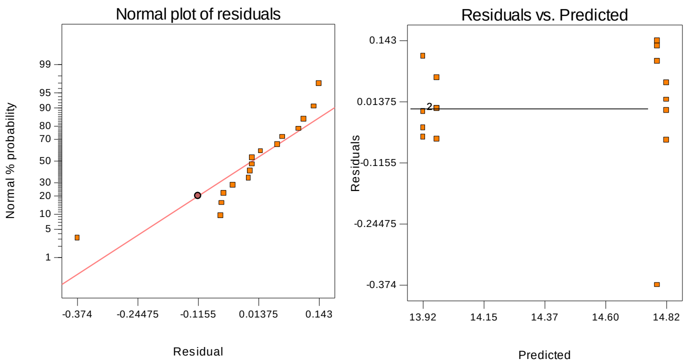
Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case B are significant model terms.

Final Equation in Terms of Coded Factors:

Thickness	=	
+14.38		
-0.022		* A
+0.43		* B
+3.688E-003		* A * B

Final Equation in Terms of Actual Factors:

Thickness	=	
+13.36650		
-0.020000		* Flow Rate
+0.12999		* Dep Time
+7.37500E-004		* Flow Rate * Dep Time



A new outlier is present and should be investigated.

Analysis with the run associated with standard order 2 replaced with the value 14.165:

Response: Thickness

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**	
Model	2.82	3	0.94	45.18	< 0.0001	significant
A	*0.018*	*1*	*0.018*	*0.87*	*0.3693*	
B	*2.80*	*1*	*2.80*	*134.47*	*< 0.0001*	
AB	*3.969E-003*	*1*	*3.969E-003*	*0.19*	*0.6699*	
Pure Error	*0.25*	*12*	*0.021*			
Cor Total	3.07	15				

The Model F-value of 45.18 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

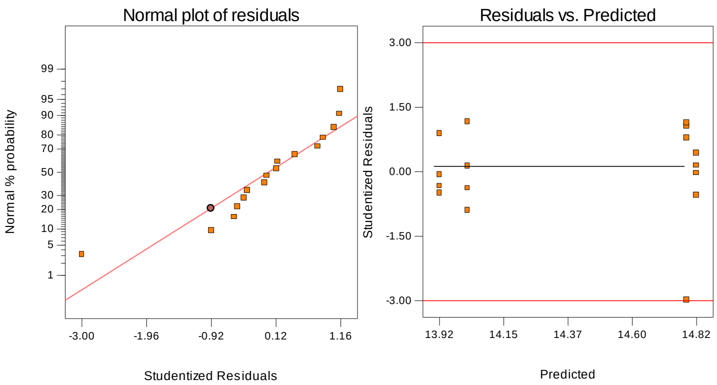
Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case B are significant model terms.

Final Equation in Terms of Coded Factors:

Thickness =
+14.39
-0.034 * A
+0.42 * B
+0.016 * A * B

Final Equation in Terms of Actual Factors:

Thickness =
+15.50156
-0.056188 * Flow Rate
-0.012350 * Dep Time
+3.15000E-003 * Flow Rate * Dep Time



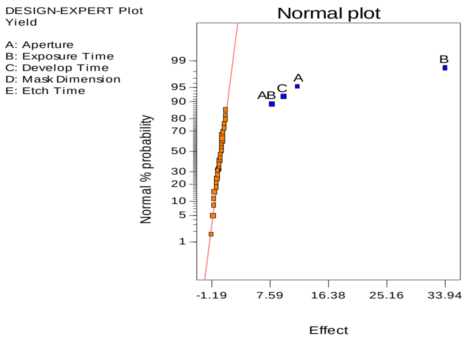
→ Another outlier is present and should be investigated.

6.25

(1) =	7	d =	8	e =	8	de =	6
a =	9	ad =	10	ae =	12	ade =	10
b =	34	bd =	32	be =	35	bde =	30
ab =	55	abd =	50	abe =	52	$abde$ =	53
c =	16	cd =	18	ce =	15	cde =	15
ac =	20	acd =	21	ace =	22	$acde$ =	20
bc =	40	bcd =	44	bce =	45	$bcde$ =	41
abc =	60	$abcd$ =	61	$abce$ =	65	$abcde$ =	63

(a) Construct a normal probability plot of the effect estimates. Which effects appear to be large?

From the normal probability plot of effects shown below, effects *A*, *B*, *C*, and the *AB* interaction appear to be large.



(b) Conduct an analysis of variance to confirm your findings for part (a).

Response: Yield

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model	11585.13	4	2896.28	991.83	< 0.0001	significant
A	*1116.28*	*1*	*1116.28*	*382.27*	*< 0.0001*	
B	*9214.03*	*1*	*9214.03*	*3155.34*	*< 0.0001*	
C	*750.78*	*1*	*750.78*	*257.10*	*< 0.0001*	
AB	*504.03*	*1*	*504.03*	*172.61*	*< 0.0001*	
Residual	78.84	27	2.92			
Cor Total	11663.97	31				

The Model F-value of 991.83 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, B, C, AB are significant model terms.

(c) Write down the regression model relating yield to the significant process variables.

Final Equation in Terms of Actual Factors:

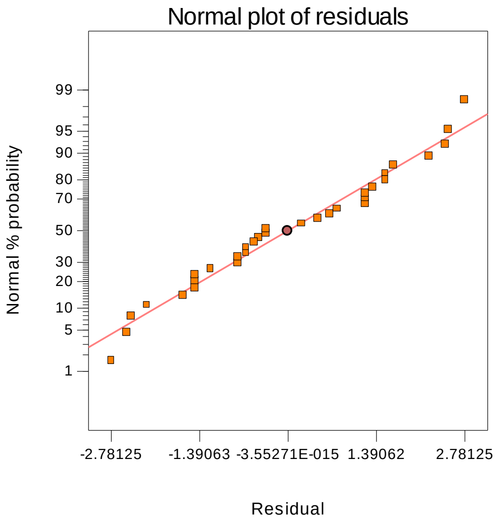
Aperture small

Yield =
+0.40625
+0.65000 * Exposure Time
+0.64583 * Develop Time

Aperture large

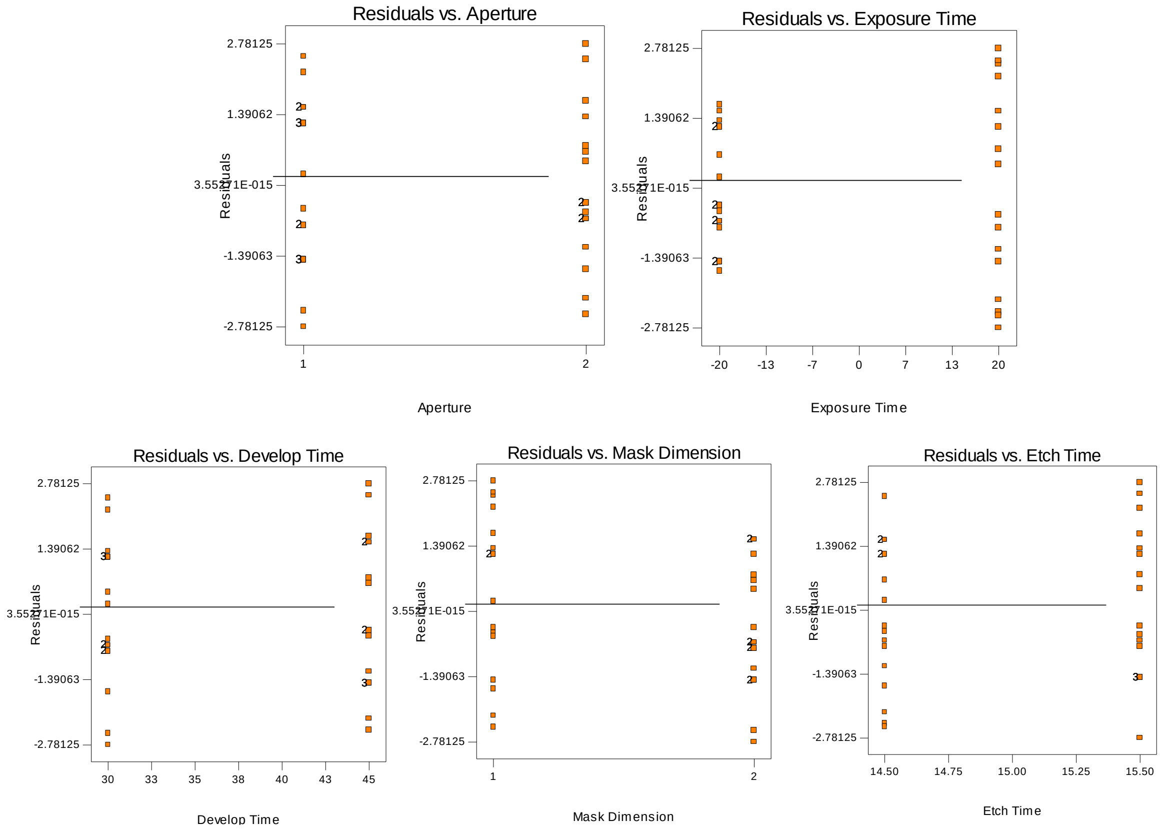
Yield =
+12.21875
+1.04688 * Exposure Time
+0.64583 * Develop Time

(d) Plot the residuals on normal probability paper. Is the plot satisfactory?



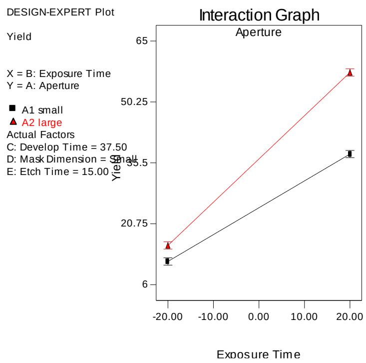
→ There is nothing unusual about this plot.

(e) Plot the residuals versus the predicted yields and versus each of the five factors. Comment on the plots.



→ The plot of residual versus exposure time shows some very slight inequality of variance. There is no strong evidence of a potential problem.

(f) Interpret any significant interactions.

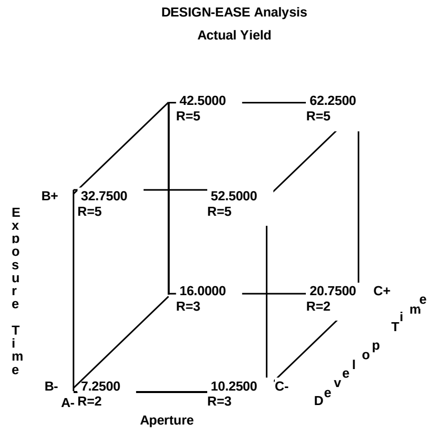


→ Factor A does not have as large an effect when B is at its low level as it does when B is at its high level.

(g) What are your recommendations regarding process operating conditions?

→ To achieve the highest yield, run B at the high level, A at the high level, and C at the high level.

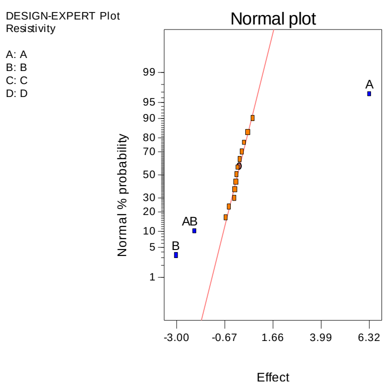
(h) Project the 2^5 design in this problem into a 2^k design in the important factors. Sketch the design and show the average and range of yields at each run. Does this sketch aid in interpreting the results of this experiment?



→ This cube plot aids in interpretation. The strong AB interaction and the large positive effect of C are clearly evident.

6.36

(a) Estimate the factor effects. Plot the effect estimates on a normal probability plot and select a tentative model.



(b) Fit the model identified in part (a) and analyze the residuals. Is there any indication of model inadequacy?

→ The normal probability plot of residuals is not satisfactory. The plots of residual versus predicted, residual versus factor *A*, and the residual versus factor *B* are funnel shaped indicating non-constant variance.

Response: Resistivity

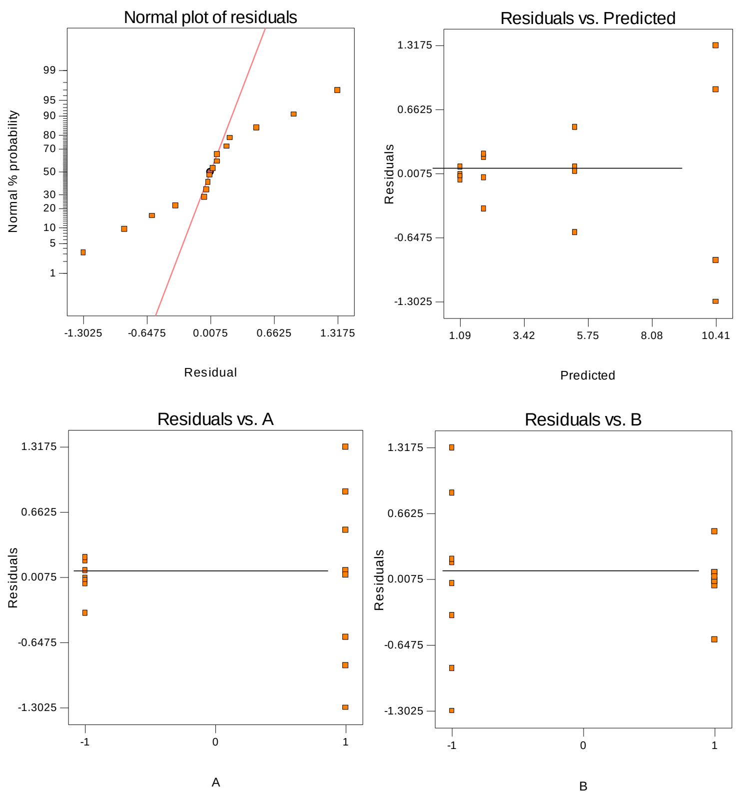
ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

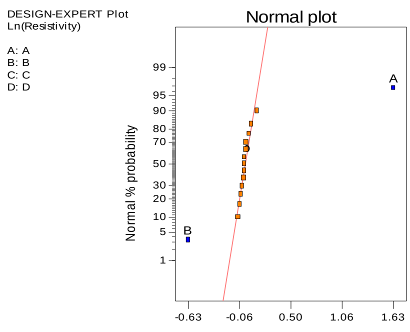
Source	**Sum of Squares**	**DF**	**Mean Square**	**F Value**	**Prob > F**	
Model	214.22	3	71.41	148.81	< 0.0001	significant
A	*159.83*	*1*	*159.83*	*333.09*	*< 0.0001*	
B	*36.09*	*1*	*36.09*	*75.21*	*< 0.0001*	
AB	*18.30*	*1*	*18.30*	*38.13*	*< 0.0001*	
Residual	5.76	12	0.48			
Cor Total	219.98	15				

The Model F-value of 148.81 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, B, AB are significant model terms.



(c) Repeat the analysis from parts (a) and (b) using ln(*y*) as the response variable. Is there any indication that the transformation has been useful?



Response: Resistivity Transform: Natural log Constant: 0.000

ANOVA for Selected Factorial Model

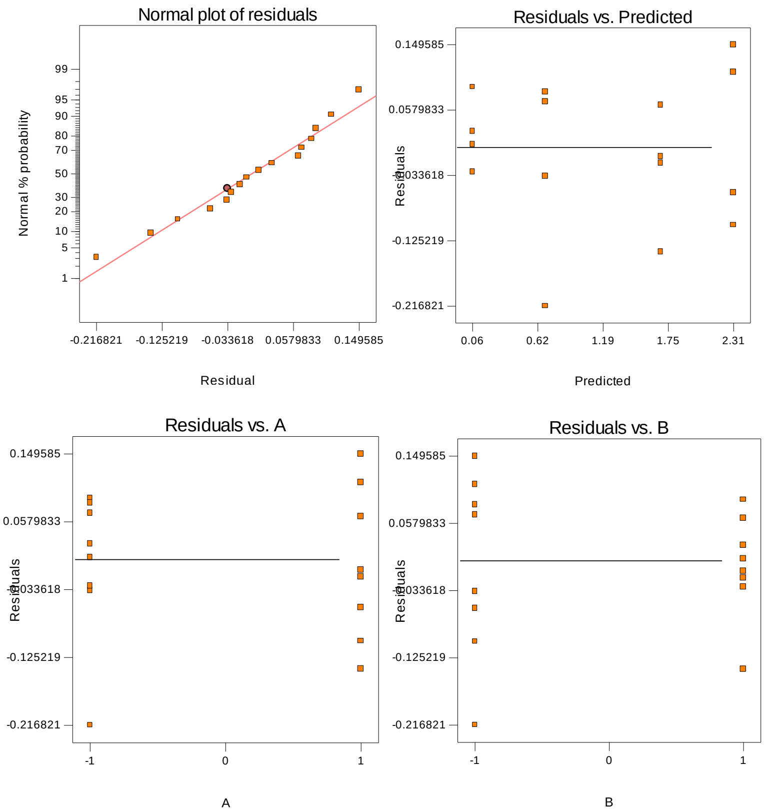
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model	12.15	2	6.08	553.44	< 0.0001	significant
A	*10.57*	*1*	*10.57*	*962.95*	*< 0.0001*	
B	*1.58*	*1*	*1.58*	*143.94*	*< 0.0001*	
Residual	0.14	13	0.011			
Cor Total	12.30	15				

The Model F-value of 553.44 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, B are significant model terms.

→ The transformed data no longer indicates that the *AB* interaction is significant. A simpler model has resulted from the log transformation.



→ The residual plots are much improved.

(d) Fit a model in terms of the coded variables that can be used to predict the resistivity.

Final Equation in Terms of Coded Factors:

$$\text{Ln(Resistivity)} = +1.19 + 0.81 * A - 0.31 * B$$

7.2 Consider the experiment described in Problem 6.1. Analyze this experiment assuming that each one of the four replicates represents a block.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Bit Size (*A*)	1107.23	1	1107.23	364.22*
Cutting Speed (*B*)	227.26	1	227.26	74.76*
AB	303.63	1	303.63	99.88*
Blocks	44.36	3	14.79	
Error	27.36	9	3.04	
Total	1709.83	15		

→ These results agree with those from Problem 6.1. Bit size, cutting speed and their interaction are significant at the 1% level.

Design Expert Output

Response: Vibration

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Block	44.36	3	14.79			
Model	1638.11	3	546.04	179.61	< 0.0001	significant
A	*1107.23*	*1*	*1107.23*	*364.21*	*< 0.0001*	
B	*227.26*	*1*	*227.26*	*74.75*	*< 0.0001*	
AB	*303.63*	*1*	*303.63*	*99.88*	*< 0.0001*	
Residual	27.36	9	3.04			
Cor Total	1709.83	15				

The Model F-value of 179.61 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, B, AB are significant model terms.

7.10 Consider the fill height deviation experiment in Problem 6.20. Suppose that each replicate was run on a separate day. Analyze the data assuming that the days are blocks.

Response: Fill Deviation

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Block	1.00	1	1.00			
Model	70.75	4	17.69	28.30	< 0.0001	significant
A	*36.00*	*1*	*36.00*	*57.60*	*< 0.0001*	
B	*20.25*	*1*	*20.25*	*32.40*	*0.0002*	
C	*12.25*	*1*	*12.25*	*19.60*	*0.0013*	
AB	*2.25*	*1*	*2.25*	*3.60*	*0.0870*	
Residual	6.25	10	0.62			
Cor Total	78.00	15				

The Model F-value of 28.30 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case A, B, C are significant model terms.

→ The analysis is very similar to the original analysis in chapter 6. The same effects are significant.