4. ${}_{n}C_{r} = \frac{n}{r} \cdot {}_{n-1}C_{r-1}$

sol) $${}_{n-1}C_{r-1} = \frac{(n-1)!}{(n-r)!(r-1)!}$$

$$= \frac{n! \times r}{(n-r)!\,r! \times n}$$

$$= {}_{n}C_{r} \times \frac{r}{n}$$

$${}_{n}C_{r} = \frac{n}{r} \cdot {}_{n-1}C_{r-1}$$

6. $${}_nC_r = \frac{n-r+1}{r}\,{}_nC_{r-1}$$

sol) $${}_nC_{r-1} = \frac{n!}{(n-r+1)!\,(r-1)!}$$

$$= \frac{n! \times r}{(n-r+1)\times(n-r)!\;r!}$$

$$= {}_nC_r \cdot \frac{r}{n-r+1}$$

$${}_nC_r = \frac{n-r+1}{r}\cdot {}_nC_{r-1}$$

8. $${}_{n}C_{r} = {}_{n-1}C_{r} + {}_{n-1}C_{r-1}$$

sol) $${}_{n-1}C_{r} + {}_{n-1}C_{r-1}$$

$$= \frac{(n-1)!}{(n-1-r)!\,r!} + \frac{(n-1)!}{(n-r)!\,(r-1)!}$$

$$= \frac{n!\,(n-r)}{(n-r)!\,r!\,n} + \frac{n! \times r}{(n-r)!\,r! \times n}$$

$$= \frac{{}_{n}C_{r} \times (n-r)}{n} + {}_{n}C_{r} \times \frac{r}{n}$$

$$= {}_{n}C_{r}\left(\frac{n-r}{n} + \frac{r}{n}\right)$$

$$= {}_{n}C_{r}\,(1) = {}_{n}C_{r}$$

12. ${}_nC_0 + {}_nC_1 + {}_nC_2 + \cdots + {}_nC_n = 2^n$

sol) $(x+1)^n = \sum_{k=0}^{n} {}_nC_k x^k$

$= {}_nC_0 x^0 + {}_nC_1 x^1 + \cdots + {}_nC_n k^n$

if $x=1$.

$\Downarrow$

$2^n = {}_nC_0 + {}_nC_1 + {}_nC_2 + \cdots + {}_nC_n$

포커

모든 : ${}_{52}C_5$

1. straight flush ${}_4C_1 \times {}_{10}C_1 = 40$

2. 4 kind ${}_{13}C_1 \times {}_{48}C_1 = 624$

3. full house

$${}_{13}C_1 \times {}_4C_3 \times {}_{12}C_1 \times {}_4C_2$$

4. flush

$${}_{4}C_{1} \times {}_{13}C_{5}$$

5. straight

$${}_{10}C_{1} \times 4^{5} = 10240$$

6. threekind

$$\frac{{}_{13}C_{1} \times {}_{4}C_{3} \times {}_{12}C_{1} \times {}_{4}C_{1} \times {}_{11}C_{1} \times {}_{4}C_{1}}{3!}$$

7. two pairs.

$$\frac{{}_{13}C_1 \times {}_4C_2 \times {}_{12}C_1 \times {}_4C_2 \times {}_{11}C_1 \times {}_4C_1}{3!}$$

8. one pair

$$\frac{{}_{13}C_1 \times {}_4C_2 \times {}_{12}C_1 \times {}_4C_1 \times {}_{11}C_1 \times {}_4C_1 \times {}_{10}C_1 \times {}_4C_1}{4!}$$

9. nothing

$$5\,2\,(\,5\ \boxed{-(2 \sim 8)+1}$$