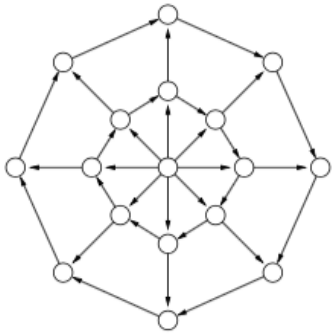
Make-up

You will have **50 minutes to prepare** your answers and **extra 10 minutes to upload** them. You write your answers on paper, scan (or photo capture through a mobile application such as CamScanner) and submit them as a single **.pdf file before 10.30 on February 3(today)**. Your answers have to be handwritten. Also, I accept the solutions of the students having some technical problems sent via email to mosmanoglu@ankara.edu.tr before 10.30. **The answer sheets sent after 10.30 will NOT be graded.**

1.(35p) A wheel graph is a directed graph of the following form, i.e. a wheel graph consists of a center vertex c with k outgoing 'spokes' of s outward oriented edges at each circle; furthermore, all the spokes at each circle are connected to form a directed cycle, and all cycles are oriented the same way (k = 8 and s = 2 for the following figure):



Assume you are given a weighted wheel graph (each edge has a positive integer weight).

a) How long does it take for Dijkstra's algorithm to output the shortest paths from center to all other nodes in a given wheel graph (as a function of n where n is the number of vertices in the graph?

b) Design an efficient algorithm (better than Dijkstra) for finding the shortest paths from center to all other nodes in a given wheel graph. To get full credit, you need to argue the running time.

2.(30p) For the questions, first construct a weighted directed graph using your student id as follows ('14290519' will be used for the following graph):

- take the square of your id

  $14290519^2 = 204218933289361$

- remove all the zeros from the resulting number

  $204218933289361 \longrightarrow 24218933289361$

- cut out the first 9 numbers

  242189332

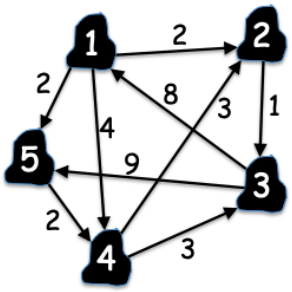
- fill the empty entries in the following adjacency weighted matrix with the obtained ten numbers, with the following order 'from the first row to the last row, and at each row, from left to right.

	1	2	3	4	5
1	0		∞		
2	∞	0		∞	∞
3		∞	0	∞	
4	∞			0	∞
5	∞	∞	∞		0

→

	1	2	3	4	5
1	0	2	∞	4	2
2	∞	0	1	∞	∞
3	8	∞	0	∞	9
4	∞	3	3	0	∞
5	∞	∞	∞	2	0

- construct the corresponding graph



Assume you are using the Floyd-Warshall algorithm to find the shortest paths between every pair of vertices, and $D^{(5)}$ is the final matrix output by the algorithm. Just write the matrices $D^{(3)}$ and $D^{(4)}$.

3. (35p) The airline companies organize their flights from an airport they serve to all possible destinations by considering the fact that the planes may take an emergency landing. So, the flights can only be arranged along predetermined routes between airports. Besides, the airlines companies have to ensure that all the airports in a planned flight will be no more than four links away from the airport that the company serves. Note that a planned flight can consist of multiple stops, and 'link' here refers to the flight from one stop to the next one.

Assume you are given a graph with the set of vertices (denoted by V) representing the airports, the set of edges (denoted by E) representing the flights between the airports, and the set of weights (denoted by W) representing the distance between the pair of adjacent airports. For a specific airline company, let A be the set of airports that the company serves. Your task is to develop an algorithm that calculates the shortest paths from a specific airport s to all other airports with restriction that the airports in a path cannot be more than four edges away from an airport in A.